Filament representation of the plasma in the tokamak disruption studies

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1. Introduction. It is well known that the plasma-produced poloidal field outside the tokamak plasma can be quite accurately approximated by the field of a set of the distributed toroidal currents (filaments or circular loops) [1]. This is justified by referring to the fact that the tokamak plasma looks like a toroidal current and needs a proper electromagnetic treatment for suppressing the outward expansion. An additional argument in favour of such image is that, with a desirable axisymmetry, the magnetic field due to the plasma poloidal current always remains completely 'hidden' inside.

The force balance requires that this current must vary reacting on the plasma changes. It will inevitably generate the *poloidal* electric field outside, which is not accounted for in the models with plasma replaced by current filaments. Such models are often used in calculations of the disruption-induced forces on the tokamak wall [2–9]. Sometimes the plasma current is even modelled by a single filament which position in space is constant [2, 7, 8]. Here we analyze the accuracy of such approaches. The magnetic pressure on the wall during thermal quench (TQ) and current quench (CQ) is analytically calculated by following the approaches described in [10, 11]. Estimates are made for rapid events when the penetration of the plasma-driven perturbation through the vessel outwards is weak due to the skin effect in the wall. Equations are given that include the toroidal effects and wall resistivity.

2. Formulation of the problem. We treat the problem within the standard large-aspect-ratio tokamak model assuming that both the plasma and wall are circular in the perpendicular cross-sections. It follows from $\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t$ that

$$E_\zeta = -\frac{1}{2\pi r}\frac{\partial\psi}{\partial t}, \qquad \oint \mathbf{E}\cdot d\vec{\ell}_p = -\frac{\partial\Phi}{\partial t}, \tag{1}$$

where $\mathbf{B}$ is the magnetic induction, $\mathbf{E}$ is the electric field, r is the radius from the main axis, ζ is the toroidal angle, ψ and Φ are the full poloidal and toroidal magnetic fluxes, ℓ_p is the length of the poloidal contour. Variation of Φ given by the formula derived about 60 years ago

$$2\frac{\Delta\Phi_{pl}}{\Phi_{pl}^0} = \Delta\left(\frac{B_J^2}{B_0^2} - \beta\right) \tag{2}$$

is routinely measured by diamagnetic loops, but is completely ignored in the models with plasma replaced by filaments. Here $\Phi_{pl}^{0} = B_0 S_{pl}$ with $S_{pl} = \pi b^2$ the cross-section of the plasma column, b its minor radius, B_0 the vacuum toroidal field, $B_J = \mu_0 J/(2\pi b)$ the poloidal field at the plasma boundary, J the net toroidal current, β the ratio of the volume-averaged plasma pressure p to the magnetic pressure $B_0^2/(2\mu_0)$. Both TQ and CQ make $\Delta\Phi \neq 0$ and generate poloidal $\mathbf{E}_p$ thereby. Our purpose is to evaluate the contribution from $\mathbf{E}_p$ into the normal force density on the wall with current density $\mathbf{j} = \sigma\mathbf{E} = \nabla\times\mathbf{B}/\mu_0$ (σ is the wall conductivity),

$$f_{wn} \equiv \mathbf{n}_w \cdot (\mathbf{j}\times\mathbf{B}) = j_p B_\zeta - j_\zeta B_\tau = \sigma(E_p B_\zeta - E_\zeta B_\tau) = j_p B_\zeta - rB_\tau \mathrm{div}(\mathbf{B}\times\nabla\zeta)/\mu_0, \tag{3}$$

where $\mathbf{n}_w$ is the outward unit normal to the wall and $\boldsymbol{\tau} = r\nabla\zeta\times\mathbf{n}_w$ so that $B_\tau \equiv \boldsymbol{\tau}\cdot\mathbf{B}$ is the tangential projection. After integration across the wall we obtain

$$p_w \equiv \int f_{wn} d\ell_\perp = \frac{I_w B_\zeta}{2\pi r} - \frac{1}{2\mu_0} B_\tau^2 \Big|_{in}^{out} \tag{4}$$

with I_w the net poloidal current induced in the wall. In the models with either wall or plasma replaced by circular current loops, only the second term is accounted for, while $I_w = 0$. Such wall replacement gives incorrect p_w [11]. Now we prove the same for the filamented plasma.

3. Cylindrical estimates. In this limit, equation (4) gives (at any σ) for a circular plasma

$$2\pi R p_w = I_w B_0 - \pi R\mu_0 \frac{(J+J_w)^2 - J^2}{(2\pi b_w)^2}, \tag{5}$$

where J_w is the toroidal current induced in the wall, b_w and R are its minor and major radii. Only $I_w B_0$ survives here at $J_w = 0$. Consider also rapid events, when the wall resistivity can be ignored, and I_w must provide the flux conservation: $\Phi_w = \Phi_{pl} + \Phi_{gap} = \mathrm{const}$, where Φ_{gap} is the toroidal flux in the plasma-wall gap. With (2) the latter condition yields (see [10] for detail)

$$I_w B_0 = 2\pi R \frac{b^2}{b_w^2} \Delta\left(\overline{p} - \frac{B_J^2}{2\mu_0}\right) = \frac{V_{pl}}{\pi b_w^2}\Delta\overline{p} + \pi R\mu_0 \frac{J_0^2 - J^2}{(2\pi b_w)^2}, \tag{6}$$

where the overhead bar stands for the averaging over the plasma volume $V_{pl} = 2\pi^2 R b^2$. At $J_w = -\Delta J$ the counteracting last terms here and in (5) are equal. Then at $p = 0$ we have

$$j_p B_\zeta = j_\zeta B_p \tag{7}$$

and $p_w = 0$ in (5). This means zero local force during rapid CQ, when the both terms are accounted for in (5), while the disregard of $I_w B_0$ in (5) would result in rather large p_w.

This proves that I_w appearing from the inductive coupling of the wall with the poloidal currents in the plasma (via $\Delta\Phi_{pl}$) must essentially contribute into the wall force even during CQ. Its dominant role in (5) during TQ (when $J_w = 0$ and $j_p B_\zeta = 0$ in the wall) is evident.

4. Toroidal effects. Cancellation of the both terms in (5) in the ideal-wall limit prompts us to look at the first-order toroidal corrections. Since $B_\zeta = B_0 R / r$ in vacuum, and $I_w = 2\pi r_w d_w j_p$, where $r_w = R - b_w \cos u$, u is the poloidal angle and d_w is the wall thickness, we have

$$2\pi R d_w j_p B_\zeta = I_w B_0 (R / r_w)^2 \approx I_w B_0 (1 + 2\varepsilon_w \cos u). \tag{8}$$

Therefore, the force due to I_w (non-accounted for in [2–8]) must be strongly asymmetric. If $\varepsilon_w \equiv b_w / R$ is 1/3, which is a typical value in tokamaks, the ratio in/out will be 4.

In tokamaks, the plasma position relative to the wall is controlled so that ψ_1 in

$$\psi\big|_{wall} = L_w^\psi (J + J_w) + \psi_0^e + \psi_1 \cos u + ... \tag{9}$$

is small in the pre-disruption state. Later it must remain such due to the wall reaction. Here

$$L_w^\psi \equiv \mu_0 R \left(\ln \frac{8R}{b_w} - 2 \right) \tag{10}$$

and ψ_0^e is the slowly-varying flux due to the external sources. With

$$B_\tau = B_J (1 - \varepsilon_w \Lambda_w \cos u) b / b_w \tag{11}$$

at the wall, we obtain

$$2\pi R E_\zeta B_\tau = -\frac{R}{r} \frac{\partial \psi}{\partial t} B_\tau \approx -L_w^\psi B_J [1 - \varepsilon_w (\Lambda_w - 1) \cos u] \frac{b}{b_w} \frac{d}{dt} (J + J_w). \tag{12}$$

The $\Lambda_w \equiv \Lambda + \ln(b_w / b)$ appears from matching the vacuum solution for ψ to the plasma, and

$$\Lambda \equiv \beta_J + \ell_i / 2 - 1 \tag{13}$$

is so-called 'Shafranov's Λ', where $\beta_J \equiv 2\mu_0 \overline{p} / B_J^2$ is the poloidal beta, and $\ell_i \equiv \overline{\mathbf{B}_p^2} / B_J^2$ is the internal inductance per unit length of the plasma column. The expressions above show that $E_\zeta B_\tau$ has smaller asymmetry than $E_p B_\zeta \propto I_w B_0$. Therefore, with perfect cancellation of the CQ-related zero-order amplitudes in (3), the remaining terms will give $p_w \propto \cos u$ on the wall. The results for the plasma and its filamented surrogate will differ as shown in [11].

5. Effects of the wall resistivity. Equation (6) was derived for the ideal wall reaction. With

$$\Phi_w = \Phi_w^0 + \Delta\Phi_{pl} = L_w^\Phi (I_{tc} + I_w) + \Delta\Phi_{pl} \tag{14}$$

and allowing for the wall resistivity, we obtain from (1) and Ohm's law for the wall [10]:

$$I_w = -\frac{\tau_w R}{\mu_0 b_w^2}\frac{\partial \Phi_w}{\partial t} = -\frac{\tau_w}{2}\frac{d}{dt}\left[I_w + \frac{\pi R B_0}{\mu_0}\frac{b^2}{b_w^2}\left(\frac{B_J^2}{B_0^2} - \beta\right) + I_{tc}\right], \quad (15)$$

where $L_w^\Phi \equiv 0.5\mu_0 b_w^2 / R$, I_{tc} is the full poloidal current in the toroidal coils, and

$$\tau_w \equiv \mu_0 \sigma b_w d_w \quad (16)$$

is the standard resistive wall time used in the Resistive Wall Mode (RWM) theory.

The toroidal current induced in the resistive wall is described by

$$J_w = -\tau_w\left(\ln\frac{8R}{b_w} - 2\right)\frac{d}{dt}(J_w + J + \psi_0^e / L_w^\psi). \quad (17)$$

These equations show that the currents I_w and J_w in the resistive wall react differently on the change in the plasma current J. Then the ideal-wall CQ balance (7) will be broken, but the I_w term will always be comparable to that with B_τ or J_w in (4) or (5).

6. Conclusion. The derived formulas allow comparison of the disruption-induced forces calculated differently: with plasma described by the MHD equilibrium equations as opposed to the plasma modelled by a set of filaments. It is explicitly demonstrated that the filamentary model of the plasma (or disregard of the poloidal current in the plasma) gives unacceptably large errors in the simulated forces for both TQs and CQs. The earlier results obtained with the use of such models for EAST, J-TEXT, JET, CFETR and ITER [2–8] should be revised. It is also proved that incorporation of the toroidal effects in the wall force calculation is essential. Equations (15) and (17) for I_w and J_w in the resistive wall allow analysis of the events with arbitrary characteristic times τ, covering in particular the typical slow CQs with $\tau = O(\tau_w)$.

[1] B. J. Braams, Plasma Phys. Control. Fusion **33**, 715 (1991).

[2] Xu W., Liu X., Song Y., Li J., and Lu M., Fusion Eng. Des. **88**, 1848 (2013).

[3] R. Albanese, et al., Fusion Eng. Des. **94**, 7 (2015).

[4] V. Rozov and A. Alekseev, Nucl. Fusion **55**, 083022 (2015).

[5] S. N. Gerasimov, et al., Nucl. Fusion **55**, 113006 (2015).

[6] R. Roccella, et al., Nucl. Fusion **56**, 106010 (2016).

[7] Weiwei Xu, Xufeng Liu, Shuangsong Du, Yuntao Song, Fusion Eng. Des. **114**, 131 (2017).

[8] Xuebin Ma, Min Li, Songlin Liu, Fusion Eng. Des. **131**, 21 (2018).

[9] S. L. Chen, F. Villone, Y. W. Sun and B. J. Xiao, Nucl. Fusion **58**, 076009 (2018).

[10] V. D. Pustovitov, Fusion Eng. Des. **117**, 1 (2017).

[11] V. D. Pustovitov and D. I. Kiramov, Plasma Phys. Control. Fusion **60**, 045011 (2018).