Effect of externally applied resonant magnetic perturbations on the stability of magnetic island

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1. Introduction

It is well known that externally applied resonant magnetic perturbations (RMPs) of a sufficiently large amplitude can generate magnetic islands at the resonant surfaces in tokamak experiments even if the plasma is originally stable to tearing modes. Static RMPs of a moderate amplitude, however, are found to stabilize rotating islands [1,2]. Theoretical results based on single-fluid model have shown that moderate RMPs cause a non-uniform island rotation and a corresponding net stabilizing effect [1-3].

As the two-fluid physics is important for the stability of a small magnetic island [4,5], based on Ref. [5] the effect of RMPs on the growth of an m/n=2/1 island is investigated numerically in this paper using the four-field equations (m and n the poloidal and toroidal mode numbers), including the mass conservation equation, the generalized Ohm's law, the plasma vorticity equation, and the equation of motion in the parallel (to magnetic field) direction [6]. Using the large aspect-ratio tokamak approximation, the magnetic field is defined as $\boldsymbol{B}=B_{0t}\boldsymbol{e}_t-(k_t/k_\theta)B_{0t}\boldsymbol{e}_\theta+\nabla\psi\times\boldsymbol{e}_t$, where ψ is the helical flux function, $k_\theta=m/r$ and $k_t=n/R$ are the wave vector in $\boldsymbol{e}_\theta$ (poloidal) and $\boldsymbol{e}_t$ (toroidal) direction, R is the major radius, and the subscript 0 denotes an equilibrium quantity. The ion velocity $v=\boldsymbol{v}_{||}+\boldsymbol{v}_\perp$, including both the parallel and perpendicular components. A constant electron temperature T_e and cold ion are assumed. Normalizing the length to the minor radius a, the time t to the resistive time $\tau_R=a^2\mu_0/\eta$, ψ to aB_{0t}, $\boldsymbol{v}$ to a/τ_R, and the electron density n_e to its value at the magnetic axis, the four-field equations become [5]

$$\frac{dn_e}{dt}=d_1\nabla_{||}j-\nabla_{||}(n_e v_{||})+\nabla_\perp(D_\perp\nabla_\perp n_e)+S_n, \tag{1}$$

$$\frac{d\psi}{dt}=E_0-\eta_N(j-j_b)-\frac{\eta_N m_e}{n_e e^2}\frac{dj}{dt}+\Omega\nabla_{||}n_e, \tag{2}$$

$$\frac{dU}{dt}=S^2\nabla_{||}j+\mu\nabla_\perp^2 U+S_m, \tag{3}$$

$$\frac{dv_{||}}{dt}=-C_s^2\nabla_{||}P/n_e+\mu\nabla_\perp^2 v_{||}, \tag{4}$$

where $d/dt=\partial/\partial t+\mathbf{v}_\perp\cdot\nabla_\perp$, j is the parallel plasma current density, η_N the normalized resistivity, $j_b=-c_b\sqrt{\varepsilon}(\partial p_e/\partial r)/B_p$ the bootstrap current density, c_b a constant of order of unity, $\varepsilon=r/R$ the inverse aspect ratio, $P_e= n_eT_e$, B_p the poloidal magnetic field, and m_e the electron mass. E_0 is the equilibrium electric field, $U=-\nabla_\perp^2\phi$ the plasma vorticity, ϕ the stream function, S_n the particle source, and S_m the poloidal momentum source leading to an equilibrium poloidal plasma rotation frequency ω_{E0}. $\Omega=\beta d_1$, $d_1=\omega_{ce}/\nu_e$, $\beta=4\pi P_e/B_{0t}^2$, ω_{ce} and ν_e are the electron cyclotron and the collisional frequency, $S=\tau_R/\tau_A$, where $\tau_A=a/V_A$, and V_A is the Alfven velocity defined using B_{0t}. C_s, μ and $D_\perp$ are the normalized ion sound velocity, plasma viscosity and perpendicular particle diffusivity.

Equations (1) - (4) are solved simultaneously using the initial value code TM1 [4]. The effect of RMP with a m/n component is taken into account by the boundary condition

$$\psi_{m/n}|_{r=a} = \psi_{a,m/n}\, aB_{0t}\cos(m\theta+n\phi+\omega_{RMP}\, t), \qquad (5)$$

where $\psi_{a,m/n}$ is the normalized amplitude of the m/n component helical flux at $r=a$, and ω_{RMP} is the angular frequency of the applied RMP.

2. Numerical results

The radial profile of the equilibrium plasma current density is of the form $j \sim [1-(r/a)^2]^2$, leading to a monotonic profile of the safety factor q with the $q=2$ surface located at $r_{2/1}=0.628a$. The $m/n=2/1$ magnetic island is unstable with this q-profile for $\psi_a=0$ and $j_b=0$.

The input parameters are as the following: the toroidal magnetic field is $B_{0t}=2T$, the plasma minor and major radius are $a=0.5m$ and $R=1.7m$, $T_e=2$ keV, and $n_e=3\times10^{19}m^{-3}$. These parameters lead to $S=2.6\times10^8$, $C_s=2\times10^7(a/\tau_R)$, $\eta_N=1$, and $d_1=3.1\times10^7$. A parabolic profile for the electron density is taken, leading to an electron diamagnetic drift frequency $\omega_{*e0}=1.5\times10^5/\tau_R$ ($f_{*e0}=1$ kHz) at $r_{2/1}$ for $\Omega=2\times10^4$. The local bootstrap current density fraction $f_b=0.35$. The perpendicular plasma momentum and particle transport is assumed to be at an anomalous transport level of $\mu=0.2m^2/s=18.8$ (a^2/τ_R) and $D_\perp=\mu/5$. In tokamak experiments the plasma rotation is essentially toroidal [1], while in Eqs. (1)-(4) only the poloidal rotation is included, so that a larger plasma viscosity for the $m/n=0/0$ component, $\mu_{0/0}=18.8\times10^2\,(a^2/\tau_R)$, is used to guarantee a reasonable balance between the electromagnetic and viscous force, based on the following considerations [1,5]: (a) The electromagnetic force in the toroidal direction is smaller than that in the poloidal direction. (b) To have the same mode frequency due to the plasma rotation, the toroidal rotation velocity should be larger than the poloidal one. These two effects lead to a larger viscous force compared to the electromagnetic force for the toroidal rotation case by a factor $[(m/n)(R/r_s)]^2$, which is of the order 10^2.

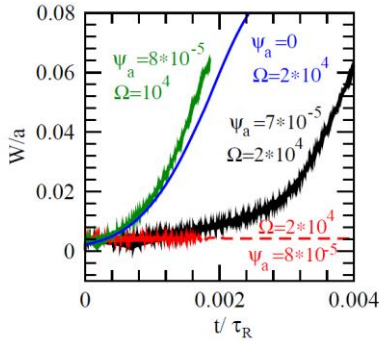

Figure 1 Time evolution of W/a ($\omega_{RMP}=0$) for $\psi_{a,2/1}=$ 0, 7×10^{-5}, and 8×10^{-5} with $\Omega=2\times10^{-4}$ ($\omega_0= 4.5$), The green curve is for $\psi_{a,2/1}=8\times10^{-5}$ and $\Omega=10^{-4}$.

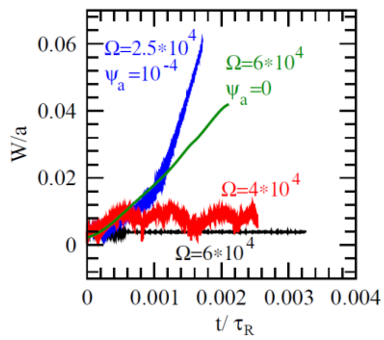

Figure 2 Time evolution of W/a ($\omega_{RMP}=0$ and ω_{E0}=-$2.6\times10^5/\tau_R$) for $\psi_{a,2/1}=10^{-4}$, Ω=2.5×, 4× and 6×10^{-4}. The green curve is for $\psi_{a,2/1}=0$ and $\Omega= 6\times10^{-4}$.

The equilibrium electron rotation frequency at $r=r_{2/1}$ can be defined as $\omega_{e\perp}\equiv(1-\omega_0)\,\omega_{*e0}$, where $\omega_0\equiv-\omega_{E0}/\omega_{*e0}$, and ω_{E0} is the frequency due to the plasma rotation (electric drift). For $\omega_{RMP}=0$, $\Omega=2\times10^4$ and $\omega_0=4.5$ ($\omega_{e\perp}/\omega_{*e0}$=-3.5), corresponding to a plasma rotation in the ion drift direction with a frequency being 4.5 times larger than $|\omega_{*e0}|$, the time evolution of the 2/1 island width, calculated from $W=4[\psi_{2/1}/(B_p\, q/q')]^{1/2}$ at $r_{2/1}$, is shown in Fig. 1. The island grows for $\psi_{a,2/1}=0$ or 7×10^{-5} but saturates at a width being smaller than $0.01a$ for $\psi_{a,2/1}=8\times10^{-5}$, indicating a threshold in the 2/1 RMP amplitude for suppressing the island growth. The island growth is suppressed by a static 2/1 RMP for $\psi_{a,2/1}=8\times10^{-5}$ - 5×10^{-4}. A too large RMP amplitude, e.g. $\psi_{a,2/1}=6\times10^{-4}$, however, is found to be strongly destabilizing. For a smaller electron diamagnetic drift frequency, $\Omega=10^{-4}$, the island still grows for $\psi_{a,2/1}=8\times10^{-5}$.

For the plasma rotation in the electron drift direction with $\omega_{E0}=-2.6\times10^5/\tau_R$, $\omega_{RMP}=0$ and $\psi_{a,2/1}=10^{-4}$, the time evolution of the 2/1 island width is shown in Fig. 2. The island grows for $\Omega=2.5\times10^{-4}$ but is suppressed for $\Omega=4\times10^{-4}$ or larger. The island suppression by the static 2/1 RMP is asymmetric on the two sides of $\omega_{e\perp}=0$, being more effective for $\omega_{e\perp}<0$, i.e. the plasma rotation in the ion diamagnetic drift direction with a frequency being lager than $|\omega_{*e0}|$. In this case a lower diamagnetic drift frequency is required for stabilizing the island growth by the 2/1 RMP. Such an asymmetry was also found in field penetration experiments [7].

Static 2/1 RMP is found to suppress the 2/1 island growth only for a sufficiently large $|\omega_{e\perp}|$ [5]. Rotating 2/1 RMPs can however stabilize the island even for $\omega_{e\perp}=0$. For $\Omega=2\times10^{-4}$ and

$\omega_0=1$ ($\omega_{e\perp}=0$), the time evolution of the 2/1 island width is shown in Fig. 3. The island grows for $\psi_{a,2/1}=0$ or for a too low RMP angular frequency ω_{RMP}, but is suppressed for $\omega_{RMP}=5\times10^4$ and $10^5/\tau_R$ with $\psi_{a,2/1}=6\times10^{-5}$. The RMP rotation is in the electron diamagnetic drift direction, being more effective for the island suppression than the rotation in the ion drift direction.

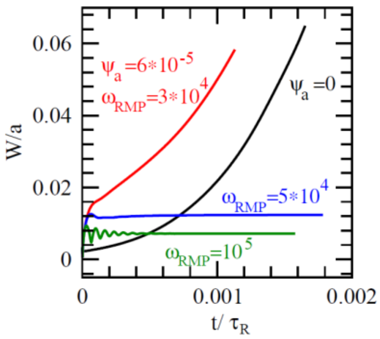

Figure 3 Time evolution of W/a ($\Omega=2\times10^{-4}$ and $\omega_{e\perp}=0$) for $\psi_{a,2/1}=6\times10^{-5}$ with $\omega_{RMP}=3\times10^4$, 5×10^4 and $10^5/\tau_R$. The black curve is for $\psi_{a,2/1}=0$.

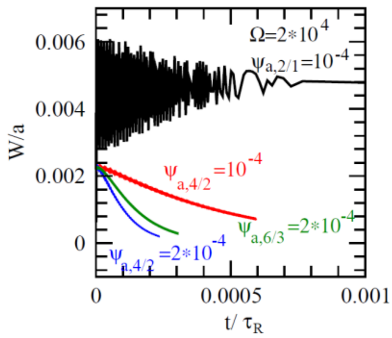

Figure 4 Time evolution of W/a ($\Omega=2\times10^{-4}$, $\omega_0=4.5$, $\omega_{RMP}=0$) by applying the RMP with /m/n=2/1, 4/2 or 6/3 components.

In addition to the 2/1 RMP, a static RMP with a m/n=4/2 or 6/3 component is also found to be able to stabilize the 2/1 island growth. The time evolution of the 2/1 island width is shown in Fig. 4 for $\omega_{RMP}=0$, $\Omega=2\times10^{-4}$ and $\omega_0=4.5$ with different mode numbers of the applied RMP, m/n=2/1, 4/2 or 6/3. The 4/2 or 6/3 RMP also stabilizes the 2/1 island in this case due to the change of the equilibrium plasma current density gradient by RMPs [5].

3. Summary

The effect of an externally applied RMP of the same helicity on the *m/n=2/1* island growth is studied based on two fluid equations. If the local electron diamagneitic drift frequency and bi-normal electron fluid velocity at $q=2$ surface is sufficiently large, the island growth is found to be suppressed by a static 2/1 RMP of moderate amplitude even with a significant fraction of local bootstrap current density. Rotating 2/1 RMPs of a sufficiently high frequency are found to stabilize the island growth for zero electron fluid velocity. A static RMP of m/n=4/2 or 6/3 component is also found to stabilize the 2/1 island growth for $\omega_{e\perp}<0$.

[1] Hender T.C. *et al* 1992 *Nucl. Fusion* **32**, 2091

[2] Hu Q., Yu Q., Rao B., Ding Y. H. *et al* 2012 *Nucl. Fusion* **52**, 083011

[3] Fitzpatrick R. 1993 *Nucl. Fusion* **33**, 1049

[4] Yu Q. 2010 *Nucl. Fusion* **50** 025014

[5] Yu Q., Günter S. and Lackner K. 2018 *Nucl. Fusion* **58** 054003

[6] Hazeltine R. D. *et al* 1985 *Phys Fluids* **28** 2466.

[7] Koslowski H.R., Liang Y., Krämer-Flecken A. *et al* 2006 *Nucl. Fusion* **46**, L1.