Application of the free-boundary SIESTA MHD equilibrium code to bootstrap control scenarios in the W7-X stellarator

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SIESTA is a recently developed [1] MHD equilibrium code that has been designed to perform fast and accurate calculations of ideal MHD equilibria for three-dimensional magnetic configurations. It is an iterative code that uses the solution previously obtained by the well-known VMEC code [2] (for the same problem) to provide an Eulerian background coordinate system and an initial guess of the equilibrium solution, from which the iteration starts. Since VMEC assumes well-defined closed magnetic surfaces, the solution that VMEC provides provides a good, non-singular, polar-like generalized coordinate system. But in contrast to VMEC, SIESTA does not assume the existence of closed magnetic surfaces. Thus, the final equilibrium solution that SIESTA converges too may include other magnetic topologies such as magnetic islands and stochastic regions. Magnetic flux and mass conservation are the only constraints imposed on the solution. Numerically, SIESTA iterates through a series of plasma displacements ξ while it looks for a minimum of the total MHD energy, given by:

$$W = \iiint \left(\frac{B^2}{2\mu_0} + \frac{p}{\gamma - 1} \right) d^3\mathbf{r}'. \tag{1}$$

The minimum energy state (i.e., equilibrium) is obtained when the displacement satisfies $\mathbf{F}(\xi) = \partial W / \partial \xi = 0$, where the ideal MHD force is $\mathbf{F} = \mathbf{J} \times \mathbf{B} - \nabla p$. The displacement required is obtained iteratively, by applying a Newton method and solving the associated linear problem by a combination of preconditioning and iterative algorithms such as GMRES, among others.

In its original implementation back in 2010, SIESTA addressed only fixed-boundary problems. That is, the shape of the plasma edge was fixed and equal to the one obtained by the VMEC code (and therefore, a magnetic surface), remaining unchanged as the solution iteratively converges to equilibrium. This fixed boundary condition has somewhat restricted the possible applications of SIESTA in the past, limiting it to problems in which a possible variation of the plasma boundary was not of interest. In order to circumvent these limitations, SIESTA has been recently extended to deal with free-plasma-boundary problems [3], opening up the possibility of addressing situations in which the plasma boundary is perturbed.

The way in which the extension has been done combines two elements. First, the computational domain has been extended outwards of the plasma, with the possibility of including (if desired) all the volume available up to the vacuum vessel. The only requirement that must be satisfied is that no coil is included. The extension of the domain has also required the extension of the background coordinate system all the way to the boundary, since it is no longer provided by the VMEC solution. Several techniques have been applied to build such coordinate system, including 1) the extrapolation of a selected set of poloidal rays of the VMEC solution until they intersect with the boundary; 2) the rearrangement of these intersections to avoid crossings that might lead to singularities/non-differentiability of the coordinate system; and 3) the proper partitioning of the poloidal rays to define new radial-like surfaces in the intermediate space between the plasma and the boundary. Exact details about how all these procedures have been implemented can be found in Ref. [3].

Secondly, an initial guess for the magnetic field and the plasma pressure must also be provided over the extended region, so that SIESTA can iterate from it. In regards to the magnetic field, it is obtained by solving:

$$\nabla \times \nabla \times \mathbf{A} = \mu_0 \mathbf{J}_{\mathrm{VMEC}}, \tag{2}$$

over the extended volume. Here, $\mathbf{J}_{\mathrm{VMEC}}$ is the current density provided by VMEC and $\mathbf{A}$ is the vector magnetic potential, from which $\mathbf{B}$ is obtained by using $\mathrm{B} = \nabla \times \mathbf{A}$. The boundary conditions are given by prescribing the values of the vector potential $\mathbf{A}$ at the extended boundary and very close to the origin. These are estimated by directly integrating Biot-Savart's law,

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \iiint_{\mathrm{plasma}} \frac{J_{\mathrm{VMEC}}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}', \tag{3}$$

and adding to it the contribution to the vector potential originating from the currents flowing in the external coils. Regarding the pressure, the VMEC pressure solution is considered in the plasma region while a finite, fast-decaying pressure profile is used in the extended volume between the plasma and the boundary. This non-zero pressure is required in the extended volume to avoid the possibility of having non-zero plasma displacements that leave the total system energy unchanged, which would lead to a singular Hessian for the numerical problem that complicates the convergence of the iterative solvers in SIESTA.

Application of SIESTA to W7-X bootstrap current scenarios

The free-boundary capabilities of SIESTA have been applied to several scenarios for control of the bootstrap current in the W7-X stellarator. W7-X relies on a vacuum magnetic island chain (the 5/5 island) that sits just outside of the plasma edge to control the outflow of particles and energy toward its divertor. Self-generated bootstrap currents may, particularly at low to mid densities, lead to a deterioration of this topology [5]. Although some studies using the

VMEC+EXTENDER tool [6] have considered ECCD to compensate these currents and maintain the topology and location of the 5/5 island, it is difficult to predict the consequences of these schemes inside of the plasma. The reason is that EXTENDER uses the so-called casing principle to estimate the total magnetic field in the region between the plasma edge and the vacuum vessel, but the result is not a self-consistent ideal MHD equilibrium. Here, we show that SIESTA can be useful to determine in which cases this approach may not be sufficiently good.

The W7-X scenario examined corresponds to a medium-density W7-X plasma in the standard configuration. The evolution of the profiles and the related bootstrap current is estimated with the NTSS code [8] after considering appropriate sources (see Fig. 1, frames a) and b)). Plasma beta is increased to about 2% and a significant variation of the rotational transform is observed. Poincaré plots of the magnetic field obtained by VMEC+EXTENDER show that, in this freely-evolving case, the 5/5 rational surface that was initially at the plasma edge enters the plasma and relocates to about $s = 0.6$ ($s = 1$ signals the plasma edge, $s = 0$, the magnetic axis). In principle, the VMEC+EXTENDER field should not show any islands inside of the plasma, due to the details of its approach to the problem. The fact that it does is in itself rather puzzling [7]. When SIESTA is applied to the same problem, the resulting field is very similar to the VMEC+EXTENDER solution, although additional island chains appear inside the plasmas. They are small, though, so a relatively low impact on confinement should be expected.

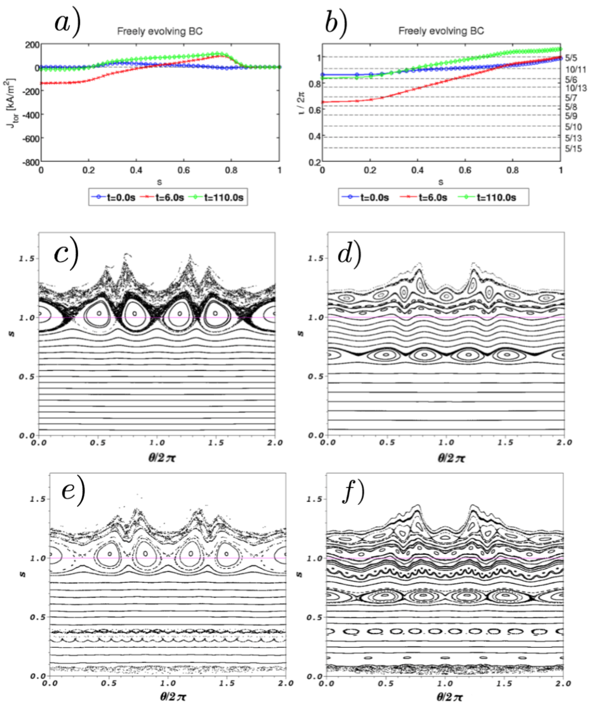

Figure 1: *a) estimated bootstrap current density and b) rotational transform for the free-evolving case at various times; Poincare plots (at toroidal plane* $\phi = 0$*) computed for the VMEC+EXTENDER magnetic field at times c)* $t = 6s$ *and d)* $t = 110s$*, and for the SIESTA field at times e)* $t = 6s$ *and f)* $t = 110s$*.*

The same scenario has also been analyzed after including off-axis ECCD to bring the net toroidal current to zero and compensate the bootstrap current effect outside of the plasma. Both EC power deposition and current drive were calculated with the TRAVIS code [9]. Since ECCD provides a very localized drive, a very large negative current spike appears close to the center that leads to a significant modification of the rotational transform. A large number of low-order rationals do enter the plasma, some even resonating at more than one location. The Poincare plots computed with the magnetic field obtained by VMEC+EXTENDER show that the main goal of the ECCD-compensation scheme is achieved. Indeed, the $5/5$ island remains clamped at its original location by the plasma edge. In this case, no significant island chain is seen within the plasma, particularly at the later times. Some minor islands are appreciated at $t = 6$ seconds, though. The magnetic field obtained by SIESTA in this case is rather different inside the plasma, though, since a large stochastic region is found at the center. This region corresponds with the location where lower-order rationals accumulated according to the rotational transform profile, suggesting that Chirikov's overlapping criterion is probably violated there. One should thus expect that plasma profiles would flatten across that region, probably impacting core confinement.

These results suggest that broader deposition profiles might be required to drive a less peaked toroidal current density near the axis in the scenarios examined, whilst still keeping the net toroidal current close to zero. The optimization of the deposition profiles required might be possible with ECRH, but is outside of the scope of this work.

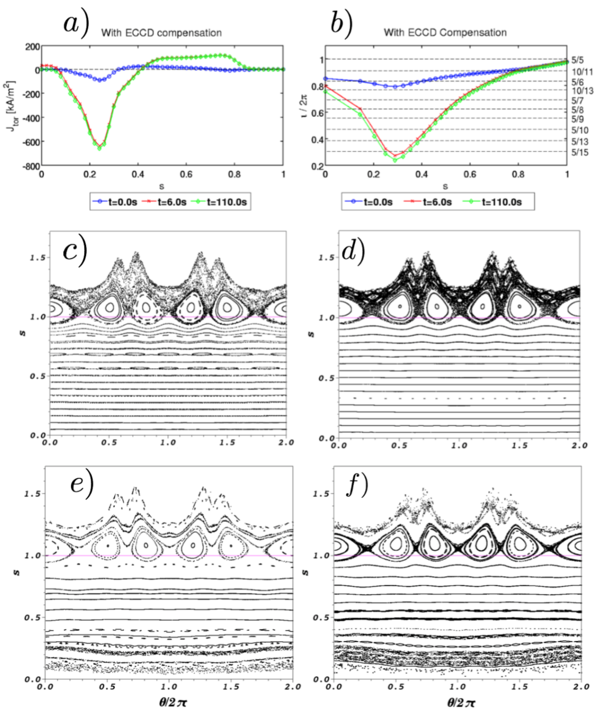

Figure 2: *a) estimated bootstrap current density and b) rotational transform for the ECCD case at various times; Poincare plots (at $\phi = 0$) computed for the VMEC+EXTENDER magnetic field at times c) $t = 6s$ and d) $t = 110s$, and for the SIESTA magnetic field at times e) $t = 6s$ and f) $t = 110s$.*

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