Modification of the Alfvén spectrum by 3D density inhomogeneities

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1. Introduction

Alfvén eigenmodes (AEs) driven unstable by energetic particles are routinely observed in axisymmetric tokamak and 3D stellerator plasmas. The most frequently observed Alfvén eigenmodes are weakly damped (and easily excited) gap modes, where poloidal harmonics are coupled by inhomogeneity in the magnetic field. For example, the toroidal Alfvén eigenmode (TAE) is caused by toroidicity-induced coupling of poloidal harmonics in tokamaks. In 3D magnetic configurations, toroidal harmonics are also coupled, producing helicity-induced gaps and helical Alfvén eigenmodes.

In this paper, we study the possible coupling of harmonics introduced by 3D inhomogeneities in the plasma density. Such coupling is transiently caused by the injection of frozen deuterium pellets, which are used to refuel the core of the plasma, control edge localised modes (ELMs), and mitigate disruptions [1]. The material introduced by the pellet breaks the toroidal and poloidal symmetry of the density profile, coupling poloidal and toroidal eigenmode harmonics, and significantly modifying the Alfvén continuum and discrete eigenmode spectrum.

We modified the 3D MHD codes Stellgap [2] and AE3D [3], which were developed to study Alfvén waves in stellarators, to incorporate 3D density profiles. Hence, we obtain the Alfvén continuum and eigenmodes for tokamak plasmas with pellet injection. We compare this work to analytical results of mode coupling due to density inhomogeneities.

These results complement efforts to use Alfvén eigenmodes observed during pellet injection for MHD spectroscopy [4]. From changes in the frequency and amplitude of Alfvén eigenmodes – both of which are modified by the coupling of harmonics – information about the changes in the plasma density and fast particle distribution function due to pellets can be inferred.

2. Experiment

Once injected, pellets break down on timescales of several milliseconds. We observe a significant change in the Alfvén eigenmode spectrum during this short period, as shown in Figure 1. New toroidal mode numbers appear after pellet injection.

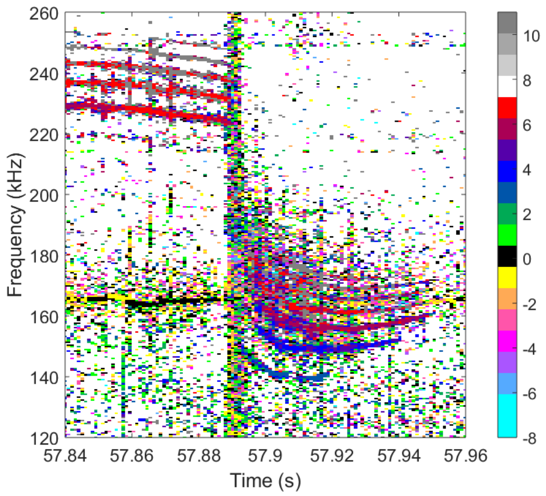

Figure 1: Phase magnetic spectrograph showing the toroidal mode numbers of a toroidicity-induced Alfvén eigenmode (TAE) before and after pellet injection. New toroidal harmonics appear when the pellet is injected at $t = 57.89$ s.

Additionally, in some JET discharges, TAEs are observed after pellet injection where none were excited prior. In some of these discharges, negative toroidal mode numbers appeared after the pellet was injected, suggesting the pellet produced positive velocity gradients in the fast ion distribution function [4].

3. Alfvén continuum

We introduced 3D density profiles to the MHD code Stellgap [2], which is usually used to study the Alfvén continuum in stellarators. Introducing poloidal and toroidal modulation couples poloidal and toroidal harmonics, modifying the continuum, as demonstrated in Figure 2. The poloidal inhomogeneity widens existing continuum gaps, while the toroidal inhomogeneity produces new gaps where the continua of the two toroidal harmonics were degenerate in the homogeneous case.

The size of the TAE continuum gap increases with the size of poloidal modulation of the density. As Figure 3 shows, the numerical results agree well with our analytical estimate, which was obtained by introducing poloidal modulation of the equilibrium magnetic field (i.e. toroidicity ε) and ion density n_i: $B_0 \propto 1 + \varepsilon \cos\theta$, and $n_i \propto 1 + \delta \cos\theta$ into the Alfvén velocity, $v_A = B_0/\sqrt{\mu_0 m_i n_i}$, in the MHD wave equation.

When the pellet is located on the low field side (LFS) of the tokamak, the TAE gap size increases with size of poloidal inhomogeneity. Conversely, the TAE gap size reduces with the size of poloidal inhomogeneity for a pellet on the high field side (HFS). The poloidal inhomogeneity must extend to the radial position of the TAE gap to affect it. We find the effect of the poloidal inhomogeneity on the gap size is more dramatic near the plasma edge, where the equilibrium density is lowest, effectively amplifying the size of poloidal modulation.

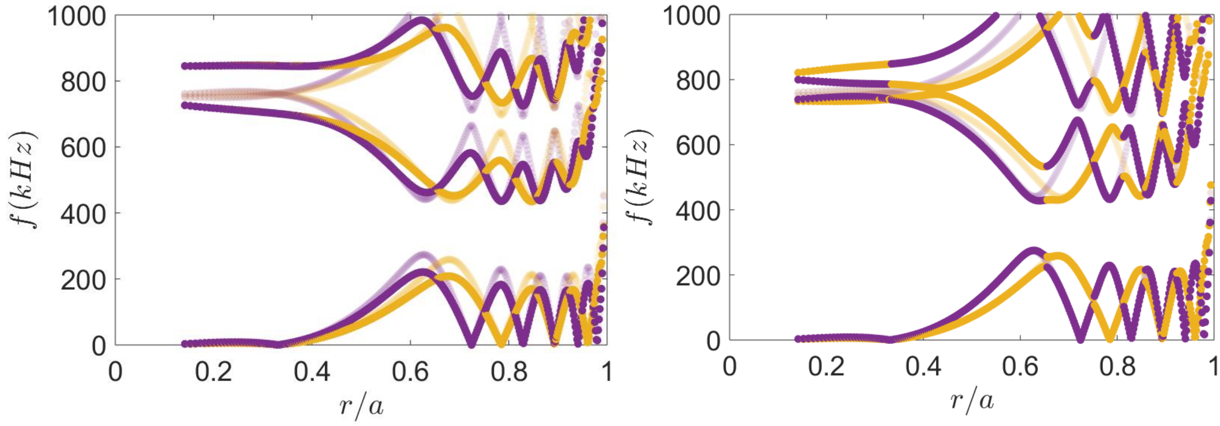

Figure 2: The Alfvén continua for the $n = 2$ (yellow) and $n = 3$ (purple) toroidal harmonics for a tokamak with circular poloidal cross-section and (a) poloidal or (b) toroidal modulation of density. The continuum for a completely homogeneous plasma is shown in the background.

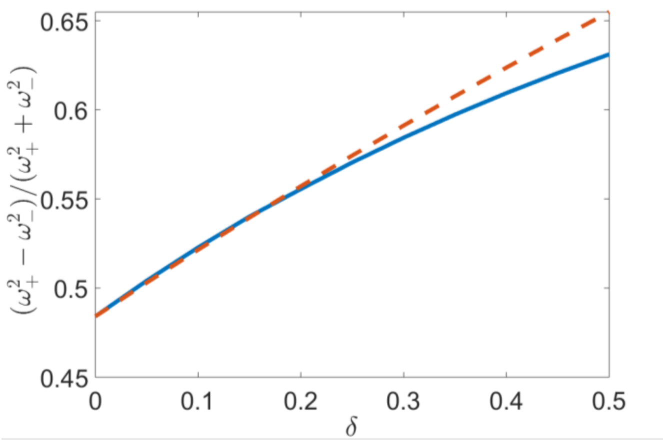

Figure 3: The size of the $n = 2$ TAE continuum gap increases with the size of poloidal modulation, given by δ. The frequency at the top (bottom) of the TAE gap is denoted by $\omega_+(\omega_-)$. The numerical estimate (solid blue) agrees well with our analytical estimate (dashed orange).

4. Alfvén eigenmode

We solve the Alfvén wave equation for a toroidal plasma with poloidal modulation of the mass density. The inner layer of the mode – where coupling terms dominate – is most strongly affected by the density inhomogeneity. Increasing the size of the poloidal inhomogeneity located on the LFS of the plasma reduces the mode eigenvalue and increases the width of the inner region of the mode.

We introduced 3D density profiles to the MHD code AE3D [3], which calculates the Alfvén eigenmodes present in 3D equilibria. The decrease in eigenvalue with size of poloidal modulation found by AE3D closely matches that of the analytical estimate, as shown in Figure 5. Additionally, we find the width of the mode inner layer increases with the size of poloidal modulation, as predicted analytically.

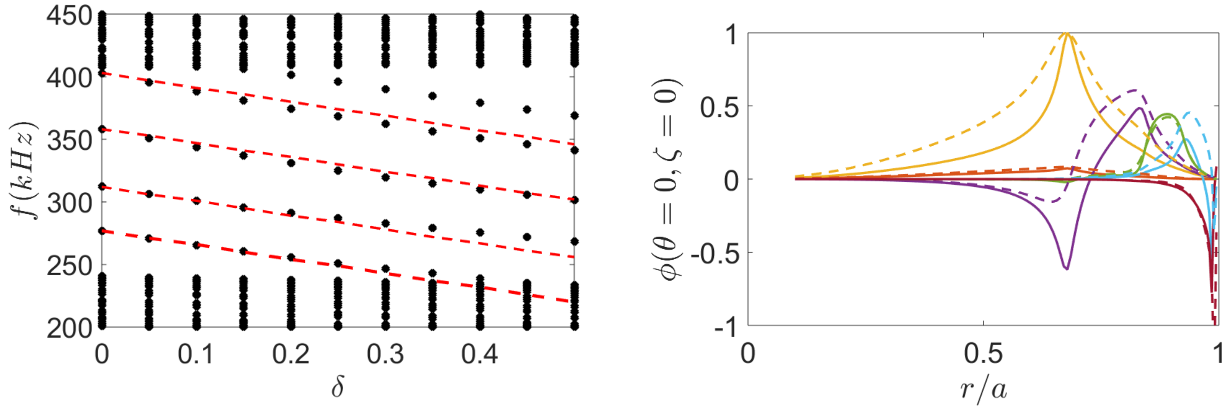

Figure 5: With increasing poloidal modulation on the low field side, given by δ: (a) the eigenvalue decreases, and (b) the mode widens. The analytical estimate is shown in red in (a). In (b) poloidal harmonics $m = 2 - 6$ of the wave potential ϕ are shown for $\delta = 0.0$ (solid) and $\delta = 0.5$ (dashed).

5. Conclusions

3D density inhomogeneities couple eigenmode harmonics, modifying the Alfvén spectrum. For a poloidal inhomogeneity on the LFS, increasing the degree of inhomogeneity: (i) widens the TAE gap, (ii) reduces the TAE frequency, and (iii) increases the TAE inner mode width. The reverse is true for an inhomogeneity located on the HFS. The decrease in TAE frequency due to increased coupling of harmonics is comparable to the decrease in frequency due to increased density. We find these results analytically, as well as numerically. Next, we will attempt MHD spectroscopy to obtain information about the changes in the plasma density due to pellets.

Acknowledgements

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