Spectrum broadening and degradation of the O-X mode coupling efficiency due scattering of a microwave beam on plasma density fluctuations

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Linear transformation of the ordinary (O) and extraordinary (X) electromagnetic modes in the vicinity of the plasma cutoffs in magnetized plasma is a basis for advanced schemes of microwave heating and diagnostic of overdense plasmas (where the central density exceeds the cutoff) in magnetic fusion research. A significant influence of density fluctuations, inherent of hot magnetized plasma in toroidal magnetic traps, on the process of linear transformation was first noted in [1], where a simple phenomenological model of this phenomenon was considered. Recently, by a more rigorous analysis, this picture has been substantially revised [2]. As a result of solving the truncated Maxwell equations in the linear interaction region in the presence of density fluctuations, it was established that the dominant effect in the O-X coupling efficiency is due to the small-angle scattering of the wave beam on the fluctuations along the propagation path towards the cutoff, at least fluctuations for the parameter range corresponding to modern experiments. Outside the coupling region, the scattering modifies the incident O-wave beam, which affects its subsequent transformation and reflection. At this, the so-called enhanced scattering, characteristic of the plasma cut-off, does not appear due to peculiar features of wave propagation near the point of polarization degeneracy (where the X and O modes can not be treated as independent) [3]. In this case, perturbations of the averaged coupling efficiency due to the fluctuations inside the localized coupling region are usually negligible.

The results of Refs. [2,3] were obtained basing on the assumption that regular (without fluctuations) gradients of the plasma density and magnetic field modulus are parallel. Analysis of influence of density fluctuations on the process of linear O-X mode coupling in magnetized plasma inhomogeneous in two dimensions for simplest 2D model of the coupling region was done in [4]. The effect of a random phase modulation induced by density fluctuations along the beam propagation path on the O-X mode coupling efficiency in 2D geometry was considered. To verify the analytical theory of Ref. [4], in the present paper we do a straightforward numerical simulation of the tunnelling of the incident O wave through the 2D-inhomogeneous layer of a fluctuating overdense plasma.

According to basic 2D model, the magnetic field (directed along the z-axis) and plasma density is assumed to vary in (x, y) plane. Their regular profiles inside the simulation domain $[-x_0, x_0]\times[-y_0, y_0]$, are chosen as

$$n_e./n_{\mathbf{cr}} = (1+\tanh\frac{x\cdot\cos\alpha - y\cdot\sin\alpha}{L_n})\tanh\frac{x+x_0}{3L_n},$$

$$B/B_{\text{res}} = Y - 2(Y+1)y\sin\alpha/L_n,$$

where n_{cr} and B_{res} are critical plasma density and magnetic field corresponding to the electron cyclotron resonance for given wave frequency. Locally, near the coordinate origin, these profiles form the evanescent region with two nearly plane cutoff surfaces intersecting by the angle 2α. The distribution of density fluctuations is defined as a sum over a discrete set of harmonics,

$$\delta n_e/n_{\text{cr}} = A\cdot w(x)\times\sum_{\kappa_x,\kappa_y}\exp(-\frac{\kappa_x^2\lambda_x^2}{8}-\frac{\kappa_y^2\lambda_y^2}{8})\cos(\kappa_x x+\kappa_y y+\xi_{\kappa_x,\kappa_y}).$$

Here A is the normalization factor, and $w(x) = \exp(-(x-x_{\delta n})^2/(2l_{\delta n}^2))$ is the spatial distribution of fluctuation amplitude. For each realization, a sum over 200x100 harmonics with random phases ξ is generated, and then results of electromagnetic simulations are averaged over 100 realizations.

An incident electromagnetic wave, $\mathbf{E},\mathbf{H}\propto\exp(-i\omega t + ik_{\|}z)$, is defined as monochromatic quasioptical Gaussian beam with O-mode polarization. The longitudinal wave vector is chosen close to the optimal one $k_{\|} = \sqrt{Y/1+Y}\cdot\omega/c$ [1]. The distribution of the complex amplitude over y at the plasma boundary is given by

$$\tilde{E}(y) = E_0\exp(-y^2/2a_y^2).$$

For this beam, the analytical theory [4] predicts the averaged O-X mode coupling efficiency $<T>$ described by the following expressions:

$$<T> = (1-\eta)\tilde{T} + \eta T,\ \eta = \exp(-<\phi^2>),$$

$$T = (1+\tanh\tilde{\alpha})/(1+\varepsilon\tanh|\tilde{\alpha}|+\tanh\tilde{\alpha})^{1/2},\quad \varepsilon = a_y^2/a_0^2 + a_0^2/a_y^2,$$

$$\tilde{T} = (1+\tanh\tilde{\alpha})/(1+\tilde{\varepsilon}\tanh|\tilde{\alpha}|+\tanh\tilde{\alpha})^{1/2},\quad \tilde{\varepsilon} = \varepsilon + K_y^2 a_0^2,$$

$$K_y^2 = \frac{4}{\lambda_y^2}(1+<\phi^2>)\left(1+\tanh\left|\tilde{\alpha}\right|\cdot a_y^2 / a_0^2\right), \quad a_0^2 = \sqrt{\frac{Y}{2}}\frac{L_n}{k_0 \sin\left|\alpha\right|}, , \ \tilde{\alpha} = \pi \tan\alpha .$$

Here $<\phi^2>$ is the variance of the random phase obtained by the beam while passing through plasma density fluctuations. It is may be determined as

$$<\phi^2> = n_{cr}^2 \int_{x_0}^{x_m}\int_{x_0}^{x_m} \beta(x')\beta(x'')w(x')w(x'')\exp\left(-(x'-x'')^2/\lambda_x^2\right)dx'dx'' , \quad \beta = k_0 \frac{\partial N_x}{\partial x} ,$$

where β determines the dependence of wave refractive index on the plasma density. Upper limits of integrations are defined as a root of the following equation

$$<\delta k^2>(x_m) = k_\perp^2(x_m) \ .$$

The difference between x_m and zero reflects the saturation of small-angle scattering near the transformation region, when the characteristic width of the spatial spectrum amplified by fluctuations becomes comparable with the transverse wave vector [2,4].

Maxwell equations were solved with the full-wave finite-difference time-domain code IPF-FDMC [5], which already is proven to be efficient in modeling the scattering of radiation by density fluctuations in magnetized plasma [6].

To demonstrate the effect of fluctuations, we study the dependence of the averaged O-X coupling efficiency on the width $l_{\delta n}$ of the region with density perturbations, see figure 1. The figure compares the results of numerical simulations of $<T>$ with analytical results. Solid lines correspond to the theory with saturation of the small-angle scattering being taken into account and dotted lines correspond to the simplest theory without saturation. we find a relatively good agreement of the analytical results and the results obtained with full wave modeling, and demonstrate the role of two physical effects related to

(1) non-one-dimensionality of the non-perturbed plasma

(2) stopping of the stochastic spectral broadening of the wave beam near the O-X mode coupling region.

Thus, the analytical theory proposed in [4] and now verified with a full-wave code for realistic beam distributions, may be suggested as a universal tool for modeling the O-X mode coupling in fluctuating plasmas.

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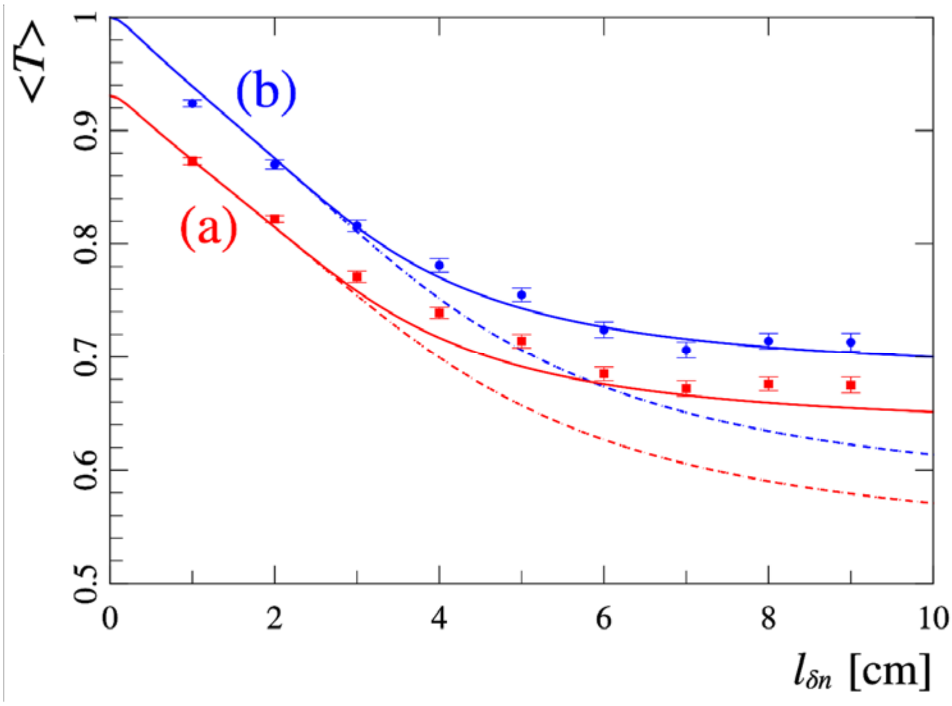

Figure 1. The averaged O-X mode coupling efficiency <*T*> of the Gaussian beam with the width $a_y = 3\ cm$ depending on the size $l_{\delta n}$ of the region occupied by fluctuations for (a) $\alpha = 0$ (b) $\alpha = 0.05$ Lines show predictions of the analytical theory with (solid lines) and without (dotted lines) the saturation of small-angle scattering being taken into account. Points correspond to full-wave numerical simulations. Parameters of the model are $k_0 = 12cm^{-1}$ $\sqrt{<(\delta n_e / n_{cr})^2>} = 3\%$, $\lambda_x = \lambda_y = 0.5cm$, $Y = 0.8$, $L_n = 10cm$, $x_{\delta n} = -6cm$, $x_0 = 10\,cm$, $y_0 = 16cm$.

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