Anomalous absorption in O1 ECRH experiments due to parametric excitation of localized UH waves

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Electron cyclotron resonance heating (ECRH) is widely used in toroidal plasmas and is considered for application in ITER for heating and neoclassical tearing mode control. In contradiction to the theory predictions [1] during the last decade many experiments utilizing both X2 and O1 heating scheme have demonstrated excitation of the anomalous phenomena at the ECRH in tokamaks TEXTOR, FTU, TCV, ASDEX-U and stellarators L-2, TJ-II, LHD. The clearest evidence of the nonlinear effects was obtained at TEXTOR [2] where the strong backscattering signals down–shifted in frequency and amplitude modulated by the magnetic island were observed X2 ECRH experiment. To explain excitation of nonlinear wave phenomena at a low pump power a theoretical model was developed [3] taking into account, as distinct from the standard theory [1], the presence of a non-monotonous density profile, which always exists on the discharge axis or is often present due to the magnetic island or the density pump-out effect. The model interprets the generation of backscattering signal, as a result of secondary nonlinear processes that accompany a primary low – threshold two–upper-hybrid (UH) – plasmon PDI of the pump X mode. The threshold of the primary PDI is shown to be smaller than the one predicted in [1] due to the trapping of at least one UH wave in the presence of the non-monotonous density profile. The primary PDI is absolute due to the finite-size of the pump beam. Its growth enhancing the UH wave fluctuations from the thermal noise level is saturated in the model due to both the pump wave depletion and the decay of the trapped daughter UH wave that leads to excitation of the secondary UH wave, which is also localized and ion Bernstein (IB) wave. The coupling of different daughter UH waves is responsible in the theory for generation of the backscattering signal. This mechanism appears capable of reproducing the fine details of the frequency spectrum of the anomalously reflected X wave and the absolute value of the observed backscattering signal in TEXTOR experiment. It also predicts substantial (up to 20%) anomalous absorption in the electron channel. This makes important the detailed analysis of also the parametric decay instabilities accompanying the O1-mode ECRH scenario planned for utilization in ITER.

In the present paper the anomalous absorption of the O-mode pump in the O1 ECRH experiment due to parametric excitation of the trapped UH wave and IB wave is discussed. We consider the parametric decay of the ordinary wave pump in the presence of a hollow density profile. In the narrow decay layer we introduce the local Cartesian coordinate system

(x, y, z) with the $x-$ axis being along the flux coordinate, y, z standing for the coordinates perpendicular to and aligned with the magnetic field line on the magnetic surface. In this layer we represent the ordinary wave beam propagating obliquely at a small angle $\theta << \pi / 2$ by means of the WKB approximation $\mathbf{E} = \mathbf{e}(x)\frac{a_0}{2}\exp\left(i\int^x k_x(x', k_z)dx' + ik_z z - i\omega_0 t\right) + c.c.$, where $a_0 = \sqrt{8P_0 / \left[w^2 \upsilon_{gx}(x)\right]}\exp\left(-\left(y^2+z^2\right)/\left(2w^2\right)\right)$; υ_{gx} is the x component of the wave group velocity; P_0 stands for the pump wave power and w is the beam waist; $k_x \simeq \omega_0\sqrt{\eta}\left(1 - k_z^2 c^2 / \left(2\omega_0^2\right)\right)/c >> k_z = \omega_0 \sin\theta / c$ and $\mathbf{e}$ is a polarization vector possessing components $e_x / e_y = -i\omega_{\text{ce}} / \omega_0$, $e_x / e_z \approx \sqrt{\eta}\, n_z$, $\eta = 1 - \omega_{\text{pe}}^2 / \omega_0^2$. In figure 1 we demonstrate the UH frequency profile (thick solid curve) for the hollow density profile corresponding to the typical value of the line average density in the O1-mode ECRH experiments in the deuterium discharge at FTU ($T_{\text{i}} = 0.25$ keV, $T_{\text{e}} = 0.35$ keV in the decay layer, the toroidal magnetic field and density at the magnetic axis are $B_0 = 4.6$ T, $n_0 = 0.8 \times 10^{14}\text{cm}^{-3}$) and the dispersion curves of the waves involved into the primary decay. The thin solid curve corresponds to the dispersion curve of the UH wave (q_{Ex}) upshifted by the wavenumber k_x of the pump wave ($f_0 = 140$ GHz, $k_z = 2.55$ cm^{-1}). The wavenumber q_{Ex} here is a localized solution of the dispersion relation $D(\omega_{\text{E}}) = q_{\text{Ex}}^2 + q_{\text{Ez}}^2 + \chi_{\text{e}}'(\omega_{\text{E}}) + \chi_{\text{i}}'(\omega_{\text{E}}) = 0$ corresponding to the eighth mode $n = 8$, $q_{\text{Ey,z}} = 0$ and $f_{\text{E}} = 138.5$ GHz. Functions χ_{e}', χ_{i}' are the real parts of expressions for the electron and ion susceptibilities. The dash-dotted line is the dispersion

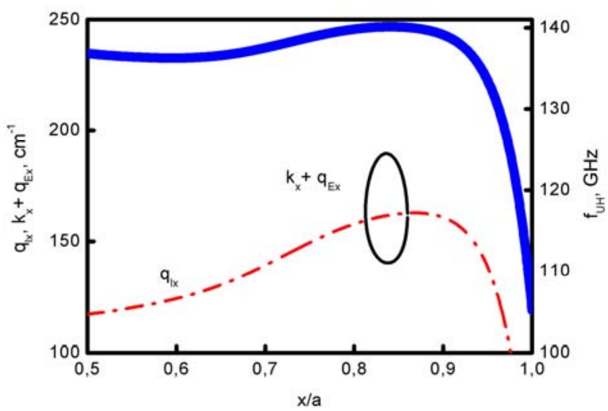

Figure 1. UH (black) and LH (red) wave dispersion curves and the UH frequency distribution (blue).

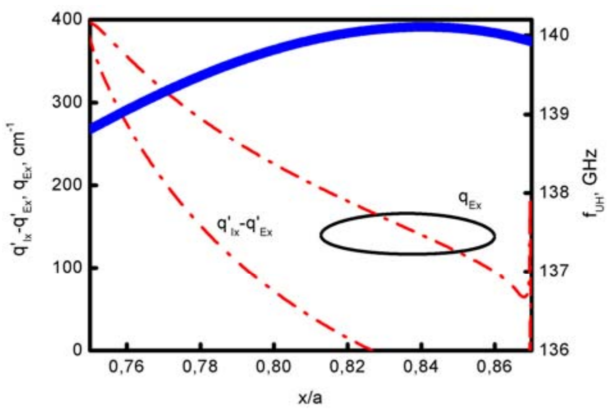

Figure 2. UH (black) and IB (red) wave dispersion curves and the UH frequency distribution (blue).

curve of LH wave q_{Ix}. The wavenumber q_{Ix} appears to be a solution to the local dispersion relation $D(\omega_{\mathrm{I}}) = q_{\mathrm{Ix}}^2 + q_{\mathrm{Iz}}^2 + \chi'_{\mathrm{e}}(\omega_{\mathrm{I}}, q_{\mathrm{Iz}}, q_{I\perp}) + \chi'_{\mathrm{i}}(\omega_{\mathrm{I}}, q_{\mathrm{Iz}}, q_{I\perp}) = 0$ at $f_{\mathrm{I}} = 1.5$ GHz , $q_{\mathrm{Iz}} = 2.55$ cm^{-1}, $q_{I\perp} = \sqrt{q_{\mathrm{Ix}}^2 + q_{\mathrm{Iy}}^2}$, $q_{\mathrm{Iy}} = 0$. In the vicinity of points $x_{d1,2}$, where solid and dash-dotted curves intersect, the decay conditions $\Delta K(x_{d1,2}) = q_{\mathrm{Ex}} + k_{\mathrm{x}} - q_{\mathrm{I}} = 0$ and $\omega_0 = \omega_{\mathrm{E}} + \omega_{\mathrm{I}}$ are fulfilled and the parametric decay is possible. Highly likely the secondary low-threshold decay leading to the excitation of a localized UH wave could provide the saturation mechanism to the instability [3]. In figure 2 we illustrate the possibility of secondary decay. The dispersion curve of primary UH wave is shown by the solid line. The dispersion curve of secondary IB wave q'_{Ix} down-shifted by the wavenumber of secondary UH wave q'_{Ex} is depicted by the dash-dotted line. The wavenumber q'_{Ix} is a solution to the equation $D(\omega'_{\mathrm{I}}) = q'^2_{\mathrm{Ix}} + q'^2_{\mathrm{Iz}} + \chi'_{\mathrm{e}}(\omega'_{\mathrm{I}}, q'_{\mathrm{Iz}}, q'_{I\perp}) + \chi'_{\mathrm{i}}(\omega'_{\mathrm{I}}, q'_{\mathrm{Iz}}, q'_{I\perp}) = 0$,where $f'_{\mathrm{I}} = 0.95$ GHz , $q'_{\mathrm{Iy}} = 0$ and $q'_{\mathrm{Iz}} = 0.15$ cm^{-1}. In its turn the wavenumber q'_{Ex} can be found out solving the dispersion relation of the UH wave $D(\omega'_E) = q'^2_{\mathrm{Ex}} + q'^2_{\mathrm{Ez}} + \chi'_{\mathrm{e}}(\omega'_E, q'_{\mathrm{Ez}}, q'_{E\perp}) + \chi'_{\mathrm{i}}(\omega'_E, q'_{\mathrm{Ez}}, q'_{E\perp}) = 0$. At $f'_{\mathrm{E}} = 137.55$ GHz and $q'_{\mathrm{Ey,z}} = 0$ it corresponds to the fourteenth mode ($n = 14$). In the points $(x'_{d1,2})$ where the solid and dashed curves intersect the secondary decay resonance conditions $\Delta K'(x'_{d1,2}) = q'_{\mathrm{I}} - q'_{\mathrm{Ex}} - q_{\mathrm{Ex}} = 0$ and $\omega_{\mathrm{E}} = \omega'_{\mathrm{E}} + \omega'_{\mathrm{I}}$ are fulfilled.

A system of equations describing spatial and temporal behaviour of primary (b_n) and secondary (b_m) UH waves derived using the approach of [3] takes a form

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + i\Lambda_{ny}\dfrac{\partial^2}{\partial y^2} + i\Lambda_{nz}\dfrac{\partial^2}{\partial z^2}\right) b_n = \left(\gamma_0 \exp\left(-\dfrac{y^2}{w^2} - \dfrac{z^2}{w^2}\right) - \gamma'_0(b_m)\right) b_n \\ \left(\dfrac{\partial}{\partial t} + i\Lambda_{my}\dfrac{\partial^2}{\partial y^2} - \upsilon_{mz}\dfrac{\partial}{\partial z}\right) b_m = \gamma'_0(b_n) b_m \end{cases} \qquad (1)$$

where $\gamma_0 = \dfrac{|a_0(0)|^2}{4H^2 D_{\mathrm{E}\omega}} \int\limits_{-\infty}^{\infty} dx \dfrac{\kappa^{2*}(x)\varphi_n(x)^*}{|D_{\mathrm{Iq}}(\omega_{\mathrm{I}}, x)|^{1/2}} \int\limits_{-\infty}^{x} d\xi \dfrac{\kappa^2(\xi)\varphi_n(\xi)}{|D_{\mathrm{Iq}}(\omega_{\mathrm{I}}, \xi)|^{1/2}} \exp\left(i\int_\xi^x (q_{\mathrm{Ix}}(\omega_{\mathrm{I}}, x') - k_x(x'))dx'\right)$,

$\kappa^2 = \dfrac{\chi_i(\omega_{\mathrm{I}})\chi_e(\omega_{\mathrm{E}})}{q_{\mathrm{I}} q_{\mathrm{E}}} \dfrac{ck_z\omega_{\mathrm{ce}}}{\omega_0^2}\left(\left(1 + \dfrac{ck_x\sqrt{\omega_0^2 - \omega_{\mathrm{pe}}^2}}{\omega_0^2 - \omega_{\mathrm{ce}}^2}\right) + i\left(\dfrac{ck_x\sqrt{\omega_0^2 - \omega_{\mathrm{pe}}^2}}{\omega_0^2 - \omega_{\mathrm{ce}}^2}\right)\right)$, $D_q = \dfrac{\partial D}{\partial q_x}$, $D_\omega = \dfrac{\partial D}{\partial \omega}$,

$$\gamma_0'\left(b_{n,m}\right)=\frac{\left|b_{n,m}\right|^2\left(n_eT_e\right)^{-1}}{\omega_{\mathrm{E}}^{n,m}D_{\mathrm{E}\omega}D'_{\mathrm{E}\omega}r_{\mathrm{de}}^2}\int_{-\infty}^{\infty}dx\frac{\chi_{\mathrm{e}}^{\mathrm{nl}*}(x)\varphi_n^*(x)\varphi_m^*(x)}{\left|D_{\mathrm{Iq}}\left(\omega_{\mathrm{I}}',x\right)\right|^{1/2}}\int_x^{\infty}d\xi\frac{\chi_{\mathrm{e}}^{\mathrm{nl}}(\xi)\varphi_n(\xi)\varphi_m(\xi)}{\left|D_{\mathrm{Iq}}\left(\omega_{\mathrm{I}}',\xi\right)\right|^{1/2}}\mathrm{e}^{i\int_{\xi}^{x}q_{\mathrm{I}x}(\omega_{\mathrm{I}}',x')dx'}$$

$\varphi_n(\xi)$ -eigenfunction of the localized UH wave; $\chi_{\mathrm{e}}^{\mathrm{nl}}(\xi)$ is the coupling coefficient describing the secondary decay, which is determined in [4].

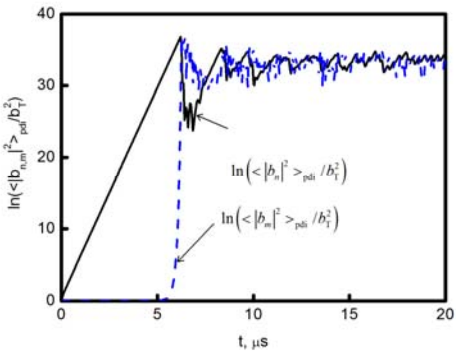

Figure 3. The temporal evolution of the averaged density of the primary (solid curve) and secondary (dashed curve) UH plasmon energies.

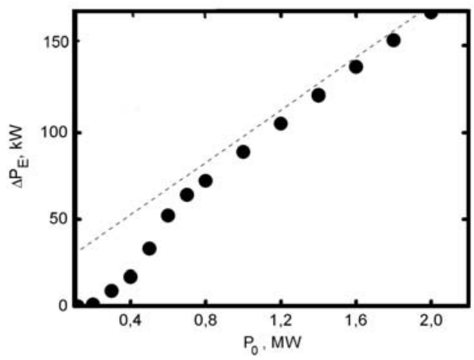

Figure 4. The dependence of the power fraction absorbed anomalously on the pump power (scattered symbols).

Solving (1) for the incident pump power $P_0 = 600$ kW and the beam width $w = 1$ cm we get the dependence the primary (solid curve) and secondary (dashed curve) UH plasmon energy shown in figure 3. In the early stage of primary decay the growth of the primary UH wave energy is perfectly described by the exponential dependence typical for the absolute instability. For the parameters under consideration the instability growth-rate is $2\nu_{0,0} = 5.9\ 10^6\mathrm{s}^{-1}$ and the power threshold of primary decay is $P_0^{\mathrm{th}} = 125$ kW . When the amplitude of the primary UH wave exceeds the threshold for the secondary instability onset the quick growth of the secondary UH wave starts leading to saturation of the primary wave amplitude growth and its decay to the values close to the secondary instability threshold. That results in the saturation of the secondary instability growth. The dependence of the power fraction gained by the daughter UH waves is shown in Fig.4. This power is deposited into the electron component far from the pump wave absorption region predicted by the linear theory. The asymptotic value of the anomalous absorption rate at the high pump power is about 8.2 %. This value is lower than in the case of the X-mode decay [3] due to the specific direction of the pump electric field (almost parallel to the magnetic field).

The financial support of the RSF grant 16-12-10043 and of Ioffe Institute is acknowledged.

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