Whistler wave instabilities of runaway electrons in tokamaks

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Introduction

High energy runaway electron beams can be generated in tokamak experiments in both the flattop and disruption scenarios. Given its potential severe damage in tokamak disruptions, it is importance to have a solid understanding of the physics of runaway electron beams. The kinetic instabilities associated with the runaway electron beam (called fan instability) has been studied previously[1], but the result has not been coupled to a runaway electron model including the avalanche and the radiation. On the other hand, recent experimental observations[2] suggested the existence of fan instability in current tokamak runaway electron experiments. In addition, low frequency whistler waves (100 MHz<ω<150 MHz) have recently been directly observed in tokamak experiments by the first time[3], which is believed to be excited by runaway electrons. This motivate us to develop a self-consistent model to study the interaction of whistler waves and runaway electron beam.

Quasilinear model of wave-particle interactions

The evolution of the electron distribution function f in momentum space is advanced through the kinetic equation. The coordinates for momentum space are (p,ξ), where p is the momentum and $\xi = p_\parallel/p$. The kinetic equation we solve is

$$\frac{\partial f}{\partial t} + \frac{eE_\parallel}{mc}\left(\xi\frac{\partial f}{\partial p} + \frac{1-\xi^2}{p}\frac{\partial f}{\partial \xi}\right) + C[f] + \frac{\partial}{\partial \mathbf{p}}\cdot(\mathbf{F}_{\text{rad}} f) + D[f] = S_A[f], \tag{1}$$

with $E_\parallel$ the parallel electric field, $C[\dots]$ the collision operator. $\mathbf{F}_{\text{rad}}$ is the synchrotron radiation reaction force term. $D[\dots]$ is the diffusion operator from the excited waves. $S_A[\dots]$ is the source term for the avalanche.

Given the distribution function, we can obtain the growth (or damping) rate Γ of every mode, using

$$\Gamma(k,\theta) = \frac{\omega_{pe}^2}{\mathscr{D}}\int d^3p \sum_{n=-\infty}^{n=\infty} Q_n \pi\delta(\omega - k_\parallel v\xi - n\omega_{ce}/\gamma)(p^2/\gamma)\hat{L}f, \tag{2}$$

where

$$Q_n = \left[\frac{n\omega_{ce}}{\gamma k_\perp v}J_n(k_\perp\rho) + E_z\xi J_n(k_\perp\rho) + iE_y\sqrt{1-\xi^2}J_n'(k_\perp\rho)\right]^2, \tag{3}$$

$$\hat{L} = \frac{1}{p}\frac{\partial}{\partial p} - \frac{1}{p^2}\frac{n\omega_{ce}/\gamma - \omega(1-\xi^2)}{\omega\xi}\frac{\partial}{\partial\xi}, \tag{4}$$

Here ω_{pe} and ω_{ce} are the plasma frequency and electron cyclotron frequency, J_n is the nth order Bessel function, v is the particle velocity, γ is the relativistic factor, and $\rho = mp\sqrt{1-\xi^2}/\omega_{ce}$ is the Larmor radius. $\mathscr{D}$ is from Eq. (21) in [4]. E_y and E_z are wave polarization normalized to

E_x. f is normalized so that $\int p^2 dp d\xi f = 1$.

The wave energy $\mathscr{E}(k,\theta)$ then evolves as

$$\frac{d\mathscr{E}(k,\theta)}{dt} = 2\Gamma(k,\theta)\mathscr{E}(k,\theta) + \mathscr{K}(k,\theta), \tag{5}$$

where $\mathscr{K}(k,\theta)$ represents the fluctuation electromagnetic field energy from radiation, which provides the initial amplitudes of the modes. $\mathscr{K}$ can be calculated as

$$\mathscr{K}(k,\theta) = \frac{\omega_{pe}^2}{\mathscr{D}} \int d^3p \sum_{n=-\infty}^{n=\infty} Q_n \pi \delta(\omega - k_\parallel v\xi - n\omega_{ce}/\gamma) m v^2 f. \tag{6}$$

The diffusion of resonant electrons can be calculated using a quasilinear diffusion model,

$$D[f] = \frac{e^2}{2\mathscr{D}} \sum_{n=-\infty}^{\infty} \int d^3\mathbf{k} \hat{L} \left[p_\perp \delta(\omega - k_\parallel v\xi - n\omega_{ce}/\gamma)\mathscr{E}(k,\theta) Q_n p_\perp \hat{L} f \right]. \tag{7}$$

Whistler wave spectrum and runaway electron distribution function

To validate the model and benchmark with experiments, we did a simulation using parameters from a DIII-D QRE experiment. $n_e = 0.6\times10^{19}\mathrm{m}^{-3}$, $T_e = 1.3$keV, and $B = 1.45$T. $E_\parallel = 0.055$V/m, which is about $9E_{\mathrm{CH}}$. The electron distribution is initialized as a Maxwellian distribution, and the runaway electron tail can build up through both Dreicer and avalanche growth. Then after some time as the RE population exceeds certain threshold, we observed that the whistler waves become unstable and the amplitudes grow significantly.

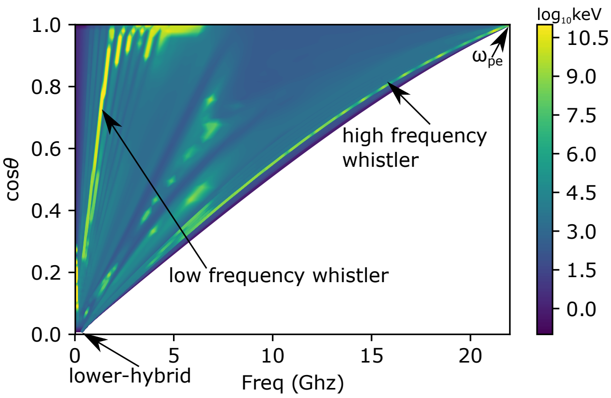

Figure 1: *The spectrum of whistler waves excited by runaway electrons in QRE experiments.*

The spectrum of the whistler waves are shown inf Fig. 1. There are two branches of whistler wave excited in simulation, the low frequency whistler waves (LFWWs, $\omega < 5$ GHz), and high frequency whistler waves (HFWWs). The LFWWs are first excited by the runaway electrons in high energy regime ($p > 10mc$), through the anomalous Doppler resonance. Then in the later time, the HFWWs can also be excited by runaway electrons in the lower energy regime. These waves are close to the whistler wave resonance cone and are mostly electrostatic waves. In addition to these two main branches, we also found that the very low frequency whistler waves ($\omega < 200$ MHz) are also excited in the simulation, which satisfy $k_\perp \gg k_\parallel$.

Due to the quasilinear diffusion from excited whistler waves, the runaway electron distribution in momentum space is strongly altered. As shown in Fig. 2 the runaway electron tail including the quasilinear diffusion effects is very different from the distribution without whistler wave diffusion. The distribution now has a much wider distribution on pitch angle in both high and low energy regime. In addition, the runaway electrons are more cumulated near the low energy regime, because the strong pitch-angle scattering from whistler waves stop the electrons from going to higher energy.

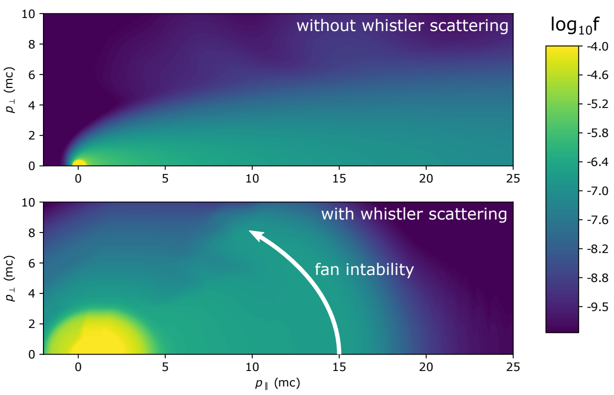

Figure 2: *The RE distribution function affected by fan instability from whistler waves, compared to that without fan instability.*

Increase of critical electric field of RE avalanche

The diffusion from excited whistler waves can affect the growth rate of RE avalanche. Using the simulation and varying the electron density n_e, we study the change of avalanche growth rate for different values of E/E_{CH}[5]. We find that for high values of E/E_{CH}, the whistle waves can enhance the avalanche growth. This is due to the overlapping of Cherenkov resonance region and the runaway-loss separatrix in momentum space. Because the RE distribution satisfies $\partial f/\partial p_{\|} < 0$, the diffusion on $p_{\|}$ can drive more electron to higher energy, crossing the separatrix and become runaway electrons. However, for low values of E/E_{CH}, the whistler waves actually lower the avalanche growth rate or even make the population start to decay. This is because in this parameter regime, the overlapping between the Cherenkov resonance region and the separatrix does not happen. On the other hand, the whistler waves can scatter resonant electron to large pitch angle, which makes them easier to lose energy. This creates another channel for RE to lose energy and become thermal electrons again.

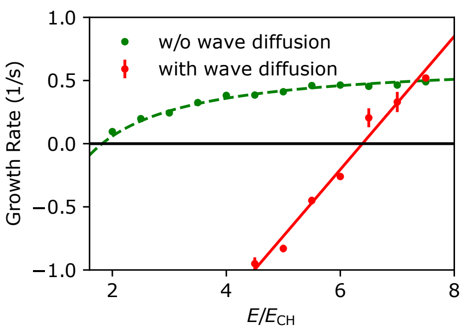

Figure 3: *Growth rate of RE population as a function of E/E_{CH}, including whistler wave diffusion (red), compared to the simulation results without whistler waves (green).*

Combining these two effects, we find that the whistler wave diffusion can raise the threshold value of E/E_{CH} with a positive growth rate to around 6 as shown in Fig. 3. This value is much higher than previously predicted and is close to experimental observations[6]. At this value, the avalanche growth of runaway electrons and the dissipation due to whistler wave scattering reaches a balance. Note that this threshold value depends on the amplitudes of excited whistler modes, which is related to the plasma temperature and magnetic field. Thus determining this value for other tokamaks requires a careful analysis and dedicated simulations.

Promptly growing ECE from runaway electrons with whistler waves

The pitch-angle scattering from the exited whistler waves can transfer electrons' energy from parallel to perpendicular direction. Given that the ECE comes from the gyro-motion of electrons, the radiation power depends highly on the electrons' pitch angle. Thus it is expected that the whistler wave can enhance the ECE from runaway electrons. To validate that, we develop a synthetic diagnostic tool to calculate the emission and absorption of electron cyclotron waves from runaway electrons given a distribution function[7]. The ECE power obtained at the antenna is calculated using the reciprocity theorem, which simplifies the calculation by solving the backwards propagation equation.

Using this new tool, we find that for RE distribution formed by avalanche but without whistler wave diffusion, the ECE from them is very weak and the total ECE power still mainly comes from thermal electrons. However, with whistler waves included in the model, the ECE signals can have a prompt growth after the HFWWs are excited, as shown in Fig. 3. In this case, the ECE from runaway electrons actually dominate the radiation from thermal electrons, and the radiation power is much higher than that from a black-body radiation. The higher frequency electron cyclotron waves are more enhanced by the runaway electrons than the lower frequency ones, resulting a flattening of ECE spectrum, which has been observed in experiments.

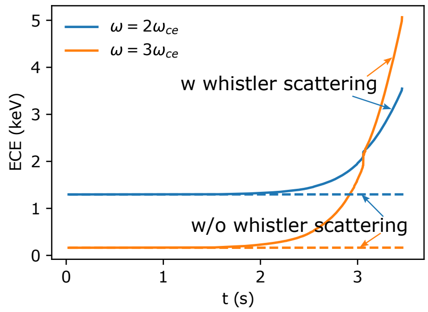

Figure 4: *The prompt growth of ECE signals affected by the whistle waves, compared to the simulation result without whistler waves (dashed line).*

Conclusion

After taking into account the whistler wave excitation and quasilinear diffusion effects, we find several open questions regarding the experimental observations about runaway electrons can be partly explained, including the increase of critical electric field for avalanche, and the prompt growth of ECE from runaway electrons. In addition to QRE regimes, the whistler waves may also be important for runaway electrons generated in disruptions, which may be helpful to find new paths for runaway electron mitigation in ITER.

References

[1] V.V. Parail and O.P. Pogutse, "The kinetic theory of runaway electron beam instability in a tokamak", Nucl. Fusion **18**, 303 (1978).

[2] C. Paz-Soldan, C.M. Cooper, P. Aleynikov, D.C. Pace, N.W. Eidietis, D.P. Brennan, R.S. Granetz, E.M. Hollmann, C. Liu, A. Lvovskiy, R.A. Moyer, and D. Shiraki, "Spatiotemporal Evolution of Runaway Electron Momentum Distributions in Tokamaks", Phys. Rev. Lett. **118**, 255002 (2017).

[3] D.A. Spong, W.W. Heidbrink, C. Paz-Soldan, X.D. Du, K.E. Thome, M.A. Van Zeeland, C. Collins, A. Lvovskiy, R.A. Moyer, M.E. Austin, D.P. Brennan, C. Liu, E.F. Jaeger, and C. Lau, "First Direct Observation of Runaway-Electron-Driven Whistler Waves in Tokamaks", Phys. Rev. Lett. **120**, 155002 (2018).

[4] P. Aleynikov and B. Breizman, "Stability analysis of runaway-driven waves in a tokamak", Nucl. Fusion **55**, 043014 (2015).

[5] C. Liu, E. Hirvijoki, G. Fu, D.P. Brennan, A. Bhattacharjee, and C. Paz-Soldan, "Role of kinetic instability in runaway electron avalanche and elevated critical electric fields", ArXiv:1801.01827 (2018).

[6] C. Paz-Soldan, C.M. Cooper, P. Aleynikov, N.W. Eidietis, A. Lvovskiy, D.C. Pace, D.P. Brennan, E.M. Hollmann, C. Liu, R.A. Moyer, and D. Shiraki, "Resolving runaway electron distributions in space, time, and energy", Phys. Plasmas **25**, 056105 (2018).

[7] C. Liu, L. Shi, E. Hirvijoki, D.P. Brennan, A. Bhattacharjee, C. Paz-Soldan, and M.E. Austin, "The effects of kinetic instabilities on the electron cyclotron emission from runaway electrons", ArXiv:1803.09897 (2018).