# Global modes of gradient drift instability in Hall plasma thruster

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Plasma in coaxial Hall thrusters is subject to a lot of instabilities driven by azimuthal $\mathbf{E}\times\mathbf{B}$ electron flow [1, 2]. Such instabilities directly affect the operational capabilities of plasma thrusters due to their impact on anomalous electron mobility across the external magnetic field [1, 3, 4]. In this paper the instability analysis of global electrostatic modes in inhomogeneous partially magnetized plasmas (unmagnetized ions and magnetized electrons) is performed in the framework of two fluid model. Assuming that plasma is inhomogeneous along the axial direction of ions acceleration, $x$, such modes can be described by the following eigenvalue equation

$$\frac{d^2\Psi}{dx^2}+\kappa_n\frac{d\Psi}{dx}-k_y^2\Psi-\frac{\omega^2}{\omega^2-\omega_{lh}^2}\left[2\kappa_B\frac{d\Psi}{dx}-\frac{k_y\Lambda(x)}{\omega-\omega_E}\Psi\right]=0. \tag{1}$$

Here $\Psi$ is the eigenfunction of the perturbed electrostatic potential, $\phi'$ ($\mathbf{E}'=-\nabla\phi'$): $\phi'=\sum\Psi(x)\exp(-i\omega t+ik_y y)$; $\omega$ and $k_y$ are the frequency and azimuthal wave-number of oscillations, respectively; $\kappa_{(n,B)}=d\ln(n(x),B(x))/dx$ are the parameters characterizing axial gradients of plasma density and magnetic field; $\omega_{lh}=\sqrt{\omega_{Be}\omega_{Bi}}$ is the lower-hybrid frequency, $\omega_{B(e,i)}$ are the electron and ion cyclotron frequencies; $\omega_E=k_yV_E$, $\mathbf{V}_E=-(cE/B)\,\mathbf{e}_y$ is the velocity of the azimuthal $\mathbf{E}\times\mathbf{B}$-electron flow and

$$\Lambda(x)=\omega_{Be}(\kappa_n-2\kappa_B)+(\kappa_n-2\kappa_B)\left(\frac{dV_E}{dx}\right)+\frac{d^2V_E}{dx^2}.$$

Equation (1) describes electrostatic oscillations in the frequency range $\omega_{Bi}\ll\omega\ll\omega_{Be}$ propagating strictly across the external radial magnetic field $\mathbf{B}=B(x)\mathbf{e}_z$ and includes the effects of electron inertia and shear of equilibrium electron flow. For the derivation of the Eq. (1) see, e. g., Ref. [5].

To formulate the eigenvalue problem we use the zero boundary conditions for the perturbed plasma potential on electrodes, $\Psi(0)=\Psi(d)=0$ ($x=0$ corresponds to the location of anode and $x=d$ – to cathode), and consider, as an example, the profiles of plasma parameters for SPT-100 plasma thruster obtained in simulations [6], approximating them by smoothed analytical curves. The resulting normalized profiles of the magnetic field, $B(x)$, plasma density, $n(x)$, and plasma potential, $\Phi(x)$, are shown in Fig. 1. Below the following absolute values of these parameters are used: $B_m = 200$ G, $n_m = 0.5 \cdot 10^{12}$ cm$^{-3}$ and $\Phi_m = 300$ V; and we consider xenon atoms in the acceleration channel with the half-radius $R = 4$ cm and the length $d = 2.5$ cm.

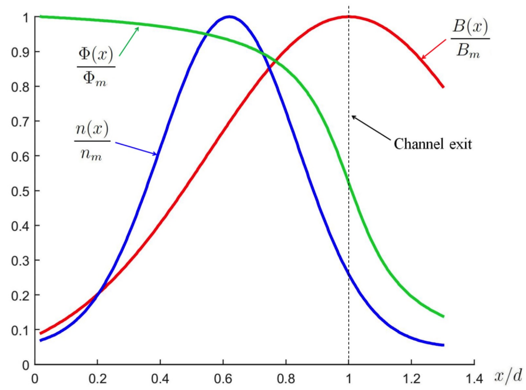


Figure 1: Dependencies of $B(x)/B_m$, $n(x)/n_m$ and $\Phi(x)/\Phi_m$ on normalized coordinate along the thruster channel, $x/d$

Results of the numerical solution of the corresponding eigenvalue problem are shown in Fig. 2. The spectrum of unstable eigenmodes, $\omega = 2\pi f + i\gamma$, consists of five modes with frequencies $f \simeq 0.23 \div 0.67$ MHz and azimuthal wavenumbers $m = k_y R = 1 \div 5$ – see Fig. 2 (a). All modes have the same axial structure of eigenfunctions – shown in Fig. 2 (b) – with the predominant localization in the near anode region. This structure of eigenfunctions arises from the local criterion of instability, which fulfills only near the anode for the considered plasma profiles – see Ref. [7]. The stabilization of the shorter wave-lengths is provided by the inertia of electrons [8].

Now we examine the behavior of the wave packet formed by the revealed unstable modes. Its spatio-temporal dependence can be presented by means of function

$$\Phi_b(y,t) = \sum_{m=1}^{5} \exp\left[-i(2\pi f_m + i(\gamma_m - \gamma_3))t + im\frac{y}{R}\right], \qquad (2)$$

entering into the perturbation of plasma potential in the form

$$\phi'(x,y,t) = \Psi(x)\Phi_b(y,t)\exp(\gamma_3 t).$$

Considering the dynamics of $\Phi_b$ instead of $\phi'$ we exclude the exponential growth of plasma perturbations from the consideration focusing on the “internal structure” of the wave packet. The time-evolution of function $\Phi_b$ in $y = \text{const}$ cross-section is shown in Fig. 3. It is clearly

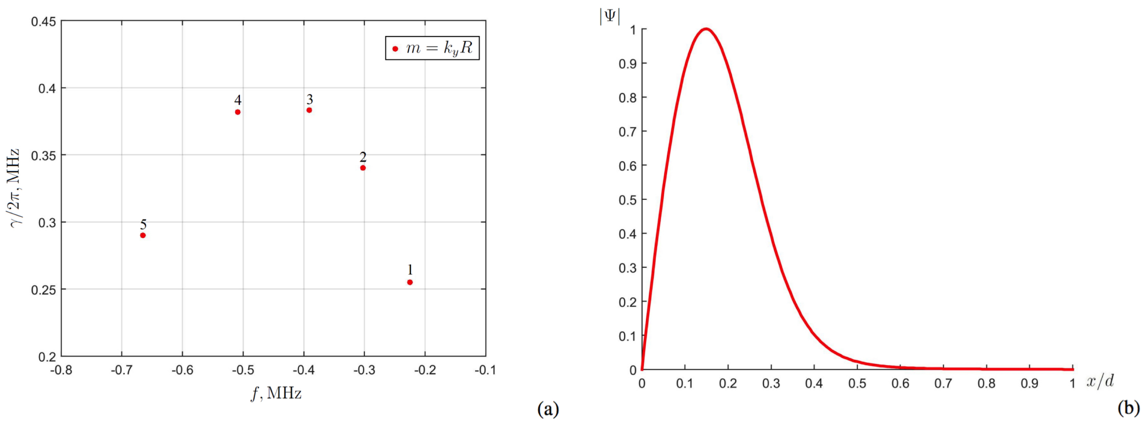


Figure 2: Eigenspectrum of unstable modes with different azimuthal wave-numbers, $m = k_y R$, (a); axial structure of eigenfunctions (b)

seen that the wave packet (2) relaxes to the beats formed by the most unstable modes $m = 3$ and $m = 4$ with almost identical growth rates $\gamma_3 \simeq \gamma_4$. The frequency of the envelope of wave packet equals 50 kHz, which is much less than the frequencies of single modes.

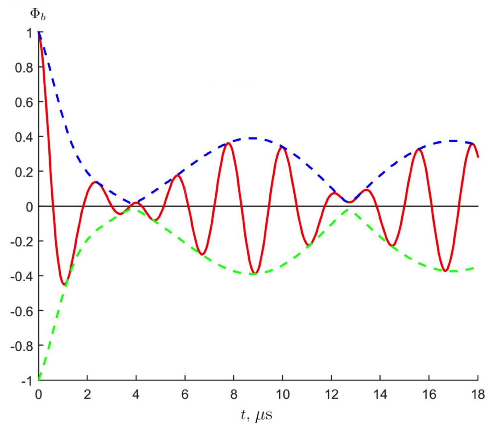


Figure 3: Time-dependence of function $\Phi_b$ in $y = \mathrm{const}$ cross-section. The amplitude is normalized on unity. Dashed curve shows the envelope of the wave packet, $\Phi_b$

To demonstrate the dynamics of the wave packet (2) in $x = \mathrm{const}$ cross-section we add formal radial parabolic dependence on $z$: $\Phi_b^*(y,z,t) = [z_0^2 - (z-R)^2]\Phi_b(y,t)$, providing zeroth values of perturbations on the outer and inner walls of the acceleration channel. The contour lines of the envelope of wave packet $\Phi_b^*$ for different moments of time are shown in Fig. 4. One can see, that at the linear stage of instability the wave packet forms a large-scale $m \sim 1$ structure rotating in the direction of stationary electron drift with group velocity $V_g \simeq 0.02\,V_E$. The characteristics of the obtained beats, i.e.: the near anode localization, $\sim 10$ kHz frequency, low $m$, slow azimuthal

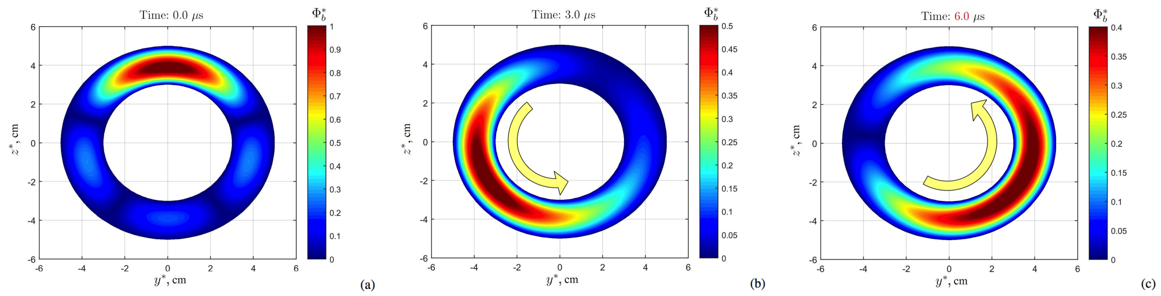


Figure 4: Contour lines of the envelope of wave packet, $\Phi_b^*$, in $x = \text{const}$ cross-section for different moments of time, $t$: (a) 0.0, (b) 3.0 $\mu$s, (c) 6.0 $\mu$s. On figure the local Cartesian system $\{y^*, z^*\}$ is introduced in $x = \text{const}$ plane

rotation in $[\mathbf{E}\times\mathbf{B}]$-direction – are the same as for the quasicoherent macroscopic structures observed in a variety of experiments with Hall thrusters, known as spokes – see, e.g., Ref. [9]. Thus, the gradient-drift instability can be regarded as the possible mechanism for the formation of such structures in Hall plasmas.

The publication was partly supported by the grant of Russian Science Foundation (RSF), project # 17-12-01470.

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