The electric field of an electron in a electron-hole plasma with degenerate electrons

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Here we would like to consider the conditions for existence of the electron-hole sound waves in a semiconductor at cryogenic temperatures when electrons (light and of a negative charge) are degenerate and one of the major unsolved problems of the superconductivity theory is determination of the static potential of a point electron. Obviously, the sign of the electron potential in a superconductive medium must be opposite to that in a vacuum. This follows from the Bardeen-Cooper-Schrieffer theory, since the Cooper electron pair can form only when the potential appears to be attractive for both electrons. Below, we will show that in a electron-hole plasma of a semiconductor when the electrons are degenerate and holes are not degenerate, i.e. when

$$\varepsilon_{F-} = \frac{P_{F-}^2}{2m_-} = \frac{3\pi^{2^{2/3}}\hbar^2 n_-{}^{2/3}}{2m_-} >> k_B T_- \geq k_B T_+ >> \frac{3\pi^{2^{2/3}}\hbar^2 n_+{}^{2/3}}{2m_+}, \tag{1}$$

a weakly damped electron-hole sound wave with the speed

$$V_{F-} = \sqrt{\frac{2\varepsilon_{F-}}{m_-}} >> v_s = \sqrt{\frac{\varepsilon_{F-}}{3m_+}} >> V_{T+} = \sqrt{\frac{k_B T}{m_+}} \tag{2}$$

can form in such a plasma . The electron-hole sound wave can be described by the following dispersion relation [1].

$$\varepsilon^l(\omega,k) = 1 + \frac{3\omega_{L-}^2}{k^2 V_{F-}^2}\left(1 + \imath\frac{\pi\omega}{2kV_{F-}}\right) - \frac{\omega_{L+}^2}{\omega^2} = 0. \tag{3}$$

where the following oscillation spectrum comes from:

$$\omega^2 = \frac{\omega_{L+}^2}{1 + 3\omega_{L-}^2/k^2 V_{F-}^2}, \quad \delta = -\frac{3}{4}\frac{\pi m_+}{m_-}\frac{\omega_{L+}^4}{k^3 V_{F-}^3}. \tag{4}$$

In a long range limit at $3\omega_{L-}^2 >> k^2 V_{F-}^2$ or

$$k^2 r_D^2 = k^2 \frac{V_{F-}^2}{3\omega_{L-}^2} << 1. \tag{5}$$

Taking into account that $n_- = n_+$, we can get the linear spectrum from .

$$\omega = k\sqrt{\frac{n_+}{3n_-}\frac{m_-}{m_+}} V_{F-}, \quad \delta = -\frac{\pi}{4}\frac{n_+}{n_-}\sqrt{\frac{n_+}{3n_-}\frac{m_-}{m_+}}\,\omega. \tag{6}$$

In a short range limit, when the inverse to relation is satisfied, we get

$$\omega^2 = \omega_{L+}^2, \quad \delta = -\frac{3}{4}\frac{\pi m_+}{m_-}\frac{\omega_{L+}^2}{k^3 V_{F-}^3}. \tag{7}$$

The interaction potential of two electrons in a electron-hole plasma

The interaction potential of two electrons α and β in a electron-hole plasma can be described by the following equation [2]:

$$U(r) = \int e^{\imath\vec{k}\vec{r}} U(\vec{k}) d\vec{k}, \quad U(k) = \frac{e_\alpha e_\beta}{2\pi^2} \frac{1}{k^2 \varepsilon^l (kV_\alpha, k)}. \tag{8}$$

According to the Eq. (3)

$$k^2 \varepsilon^l (kV_\alpha, k) = k^2 + \frac{3\omega_{L-}^2}{V_{F-}^2} - \frac{\omega_{L+}^2}{V_\alpha^2} + \imath\beta, \ \beta = \frac{3\pi V_\alpha \omega_{L-}^2}{V_{F-}^3}, \tag{9}$$

here V_α is the speed of a test electron with the charge e_α producing the potential ϕ_α at a point $r = 0$ where the charge e_β is located. Taking into account that $d\vec{r}/dt = V_\alpha$, it follows $\vec{r} \uparrow\uparrow V_\alpha$, $V_\alpha > V_{T+}$. As a result, at $e_\alpha = e_\beta = e$ we will get

$$U(k) = \frac{e^2}{2\pi^2} \frac{1}{k^2 + \frac{3\omega_{L-}^2}{V_{F-}^2} - \frac{\omega_{L+}^2}{V_\alpha^2 x^2} + \imath\beta}, \quad U(r) = \frac{e^2}{2\pi^2} \int \frac{e^{\imath krx} d\vec{k}}{k^2 + \frac{3\omega_{L-}^2}{V_{F-}^2} - \frac{\omega_{L+}^2}{V_\alpha^2 x^2} + \imath\beta}, \tag{10}$$

where $x = cos(\theta)$.

Let us analyse this expression in two limiting cases:

1) In a **short range** limit at $r \sim \frac{1}{k} \leq \frac{V_{F-}}{\omega_{L-}} << \frac{V_\alpha}{\omega_{L+}} \sim \frac{V_{F-}}{\omega_{L+}}$

2) In a **long range** limit at $r \sim \frac{1}{k} >> \frac{V_{F-}}{\omega_{L-}}$.

The short range limit interaction

$$U(r) = \frac{e^2}{2\pi^2} \int \frac{e^{\imath\vec{k}\vec{r}} d\vec{k}}{k^2 + \frac{1}{r_{D-}^2}} = \frac{e^2}{r} e^{-r/r_{D-}}, \tag{11}$$

where $r_{D-} = \sqrt{\frac{V_{F-}^2}{3\omega_{L-}^2}}$

The long range limit interaction

$$U(r) = \frac{e^2 r_{D-}^2}{2\pi^2} \int \frac{e^{\imath krx} d\vec{k}}{1 - \frac{A}{x^2} + \imath\beta_1}, \tag{12}$$

where $\beta_1 = \beta r_{D-}^2 > 0$, $A = \frac{\omega_{L+}^2}{3\omega_{L-}^2} \frac{V_{F-}^2}{V_\alpha^2} \sim \frac{m_+}{m_-} << 1$.

Taking into account that

$$\lim_{\beta_1 \to 0} \frac{1}{1 - \frac{A}{x^2} + \imath\beta_1} = -\imath\pi\delta(1 - \frac{A}{x^2})$$

the potential (12) will get the following view

$$\begin{aligned} U_D(r) = -\imath\pi \frac{e^2 r_{D-}^2}{\pi} \int_0^\infty k^2 dk \int_{-1}^{+1} dx e^{\imath krx} \delta(1 - \frac{A}{x^2}) = \\ -\imath e^2 r_{D-}^2 \int_0^\infty k^2 dk [\frac{e^{\imath krx_0}}{2/x_0} - \frac{e^{-\imath krx_0}}{2/x_0}] = \\ -\imath \frac{e^2 r_{D-}^2 x_0}{2} 2\imath \int_0^\infty sin(krx_0) k^2 dk = \\ \frac{e^2 r_{D-}^2}{r^3 A} \int_0^\infty sin(x) x^2 dx = -2 \frac{e^2 r_{D-}^2}{r^3 A} \end{aligned} \tag{13}$$

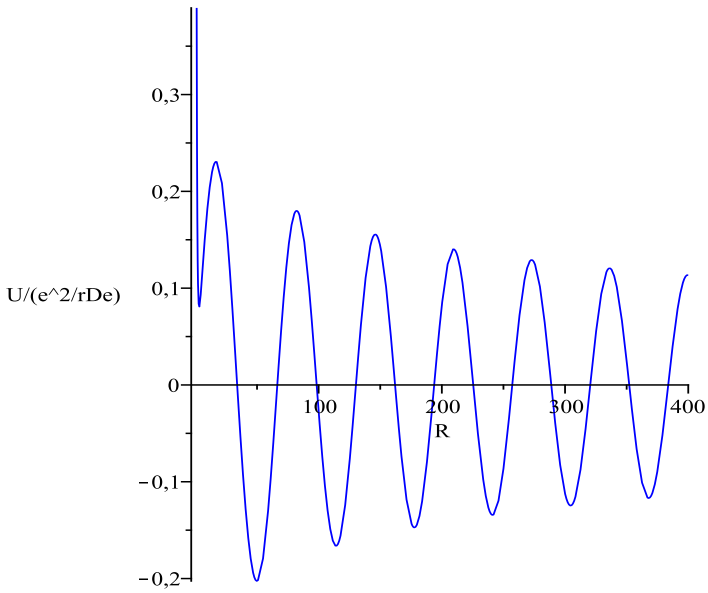

Figure 1: The sum of two potentials (13) and the potential (14) of two electrons in a degenerate electron-hole plasma, here $R = r/r_{De}$

MIN	R_1	R_2	R_{min}	$U(R_{min})/U_D$	$U(R_{min})$, eV	U_{minL}, eV	U_{minL}, K	$U(R_{min})$, K
1	33.85	66.36	50.46	-0.202	-0.053	$6*10^{-6}$	0.07	622
2	98.41	130.06	114.17	-0.166	-0.044	$6.47*10^{-6}$	0.0751	511
3	161.95	193.66	177.65	-0.147	-0.039	$6.45*10^{-6}$	0.0749	453
4	225.35	257.12	240.88	-0.134	-0.035	$6.4*10^{-6}$	0.0746	413
5	288.68	320.3	304.71	-0.125	-0.033	$6.4*10^{-6}$	0.075	385

Table 1: The potential wells

Analysis

It can be clearly seen that at a long range distances the potential has an opposite sign and decays slowly as $-1/r^3$ compared to that.

In a general case the Eq.(8) can be deduced to the following integral

$$U(r) = \frac{e^2 r_{D-}^2 \sqrt{A}}{r^3} \int_0^{\infty} \frac{y^2 dy}{(y^2 \frac{r_{D-}^2}{r^2} + 1)^{3/2}} sin(\frac{y\sqrt{A}}{\sqrt{y^2 \frac{r_{D-}^2}{r^2} + 1}}) = \frac{e^2 r_{D-}^2 \sqrt{A}}{r^3} \int_0^{R \cdot \frac{r_{D-}}{r_{D+}}} \frac{y^2 dy}{(\frac{y^2}{R^2} + 1)^{3/2}} sin(\frac{y\sqrt{A}}{\sqrt{\frac{y^2}{R^2} + 1}})$$
$$= \frac{e^2 r_{D-}^2}{r^3} \int_0^{\sqrt{A} r/r_D} \frac{z^2 sin(z) dz}{(A - z^2 \frac{r_{D-}^2}{r^2})^{5/2}} \quad (14)$$

Having solved the integral (14), we get the following form of the potential shown in Fig.1, where the integration till the $k \leq 1/r_{Di}$ was performed. Here, we see the multiple minima revealing the quantum nature of the phenomenon: many energy levels, see the Table 1. Here, for the existence of the energy level the following condition must be satisfied :

MIN	R_1	R_2	U_{max}, eV	$n=1$	E_{min1}, eV	$n=N$	E_{minN}, eV
1	33.85	66.36	-0.03235	1	$-1.228\cdot 10^{-5}$	51	$-51^2\cdot 1.228\cdot 10^{-5}$
2	98.41	130.06	- 0.0267	1	$-1.2956\cdot 10^{-5}$	45	$-45^2\cdot 1.2956\cdot 10^{-5}$
3	161.95	193.66	-0.0236	1	$-1.29\cdot 10^{-5}$	42	$-42^2\cdot 1.29\cdot 10^{-5}$
4	225.35	257.12	-0.0215	1	$-1.2859\cdot 10^{-5}$	40	$-40^2\cdot 1.2859\cdot 10^{-5}$
5	288.68	320.3	- 0.01997	1	$1.298\cdot 10^{-5}$	39	$-39^2\cdot 1.298\cdot 10^{-5}$
.	.	.	.	.	.	.	.
45	2812.6	2844.12	0.0072	1	$1.306\cdot 10^{-5}$	23	$-23^2\cdot 1.306\cdot 10^{-5}$
.	.	.	.	.	.	.	.
120	7538.75	7570.26	0.00362	1	$1.307\cdot 10^{-5}$	16	$-16^2\cdot 1.307\cdot 10^{-5}$

Table 2: Estimated energy levels of particles moving in a series of potential wells

$$U(R_{min}) > U_{minL} = \frac{\pi^2\hbar^2}{4m_e a^2}, \tag{15}$$

here a - potential well width, U_{minL} - its height [3].

Let us estimate the energy levels of particles moving in a series of potential wells shown in Fig. 1. The energy levels in a rectangular potential wells:

$$E_{min_n} = \frac{\pi^2\hbar^2}{2m_e a^2}n^2, n = 1,2\cdots,N \tag{16}$$

where the following condition must be satisfied $U_{max} > E_{min_n}$, here $S = (R_2 - R_1)\cdot U_{max}$, S - area of the rectangular well with the width $R_2 - R_1$ and height U_{max}.

In the following Table 2 the estimated energy levels are given for a series of potential wells.

Discussion of the results and quantitative estimations

We have observed a **superconductivity state** in a semiconductor or in a **electron-hole plasma** with the **degenerate electrons** due to the attractive forces between the electrons as a result of the exchange effects through the **electron-hole sound wave** by analogy to the phonon waves in a solid state.

Interaction between two electrons at the distances much larger than the Debye electron radius changes its sign, i.e. becomes **attractive**, and decreases as $1/r^3$. This potential amplitude is much higher than that, repulsive one, at the short distances and can lead to the **bound state of two electrons** (Cooper pair).

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