Manifold Tracing for Symplectic Maps of Magnetic Field Lines

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Invariant manifolds of unstable closed magnetic field lines (periodic saddles), organize the dynamics of chaotic field lines in magnetically confined plasmas. In this work we illustrate their fundamental role to understand the structure of the chaotic field lines and provide insight into the mechanisms of transport at the plasma edge.

In some situations, the geometry of the manifolds can be estimated through the mapping of a large collection of orbits close to the periodic saddle. However, without an ordering scheme and refinement this method is computationally expensive and limited in resolution. To determine the desired manifolds, we use an interpolation approach based in the curve decomposition in bare and fine details [1]. The bare details of the curve are determined by a discretization procedure that determines an appropriate set of nodes containing most of the curve information, while the fine details are contained in a normal displacement function or shape function which ensures the smoothness of the manifold. The resulting approximation method gives the invariant manifold as a continuous parametric curve.

Here, we apply the mentioned method introduced in [1] to obtain the invariant manifolds of the unstable periodic orbits of the Ullmann Map, a symplectic map describing large aspect-ratio tokamaks perturbed by a magnetic limiter. We show examples of the manifold tangles obtained for magnetic islands at the plama edge and how they change with the perturbation amplitude.

Bidimensional symplectic maps have been introduced to describe Poincaré maps of magnetic field lines in plasmas confined in tokamaks [2, 3]. One of these maps, the Ullmann map [4, 5] has been applied to analyze field lines transport and nonlinear dynamics. In this map, the equilibrium magnetic field lines are determined by the toroidal current density profile inside the plasma and the poloidal current in the external toroidal field coils. In the limit of small inverse aspect ratio, the magnetic axis is surrounded by KAM invariant curves in the Poincaré maps, without chaos. However, additional external resonant perturbations create magnetic islands and chaos at the plasma edge.

In this work, a chaotic magnetic field is generated by the magnetic limiter, which is an external current configuration whose magnetic field resonates with the equilibrium field yielding a magnetic island structure and invariant flux surfaces [4, 5]. Chaotic field lines result from interactions among magnetic islands, with the progressive destruction of magnetic flux surfaces as the perturbation strength increases, yielding a magnetic island structure and invariant flux surfaces. The non-uniformity of the outer chaotic region due to a magnetic limiter determines the complex distribution of field lines in the plasma edge.

The Ullmann map is written in terms of the cylindrical coordinates r and θ, respectively, the radius from the geometric axis and the poloidal angle, and ϕ is the toroidal angle. Accordingly, the large aspect ratio tokamak has a cylindrical geometry, with period $2\pi R_0$. We set the magnetic limiter rings at the same radius of the tokamak wall.

The analytical expressions for the map are obtained as a composition of two maps. One map for the equilibrium with toroidal correction, and the other for the external perturbation [4]. Since the map is derived from a generating function, interpreted as a canonical transformation between the previous and the next coordinates, the Jacobian for this map is unitary and, consequently, the map is symplectic.

Here, we use rectangular coordinates $x = b\theta/2\pi,\ y = 1 - r/b$, and $z = R_0\phi$. Thus, $y = 0$ corresponds to the plasma edge and $y = 1$ corresponds to the plasma center. For the numerical applications, we use the main TCABR tokamak parameters, as $a = 0.18m, b = 0.21m$ and $R_0 = 0.61m$ [6]. We consider a magnetic limiter with 4 rings equally spaced in the toroidal angle, perturbation strengths, indicated in the Fig. 1 and Fig. 2, are typical of magnetic limiters, EML helical excitation modes $(M,N) = (7,1)$. The safety factor q_0 represents the unperturbed orbit helicity, independent of the coordinate θ. However, the toroidal correction introduces a dependence on θ and a ripple in the modified invariant curves.

The topology of the magnetic field lines are mapped by their intersections in the section $z = constant$, with variables $\{x_n, y_n\}$ denoting the coordinates of the nth intersection of the field line on the considered section. Figures are obtained from the Ullmann map, simulating tokamak Poincaré sections with many field line intersections. The figures show a divided phase space with resonant periodic islands, which appear due to the breaking of equilibrium invariants with rational safety factors, KAM tori, corresponding to deformed flux surfaces for which the safety factor is irrational, and a chaotic area-filling lines, which appear due to the homoclinic and/or heteroclinic crossings of the invariant manifolds of unstable periodic orbits. Locally chaotic orbits at the resonance borders may fuse, as the perturbation strength builds up, and a globally chaotic region merges as a consequence (Fig. 2).

The magnetic limiter perturbation creates a chaotic region extending from the plasma edge to the tokamak inner wall. This chaotic region is not uniform and presents densely intertwined regions: one from where the field lines hit the tokamak wall with relatively short connection and another one where the chaotic field lines take a large number of turns before eventually hitting the tokamak wall.

For a small perturbation strength, the monotonic Ullmann map presents a chaotic field line layer at the tokamak edge, and is mostly regular in the remaining of the phase space, as shown in Figure 1. The $q = 5$ invariant surface is destroyed and the ressonant island chain is due to the main mode $(m,n) = (5,1)$.

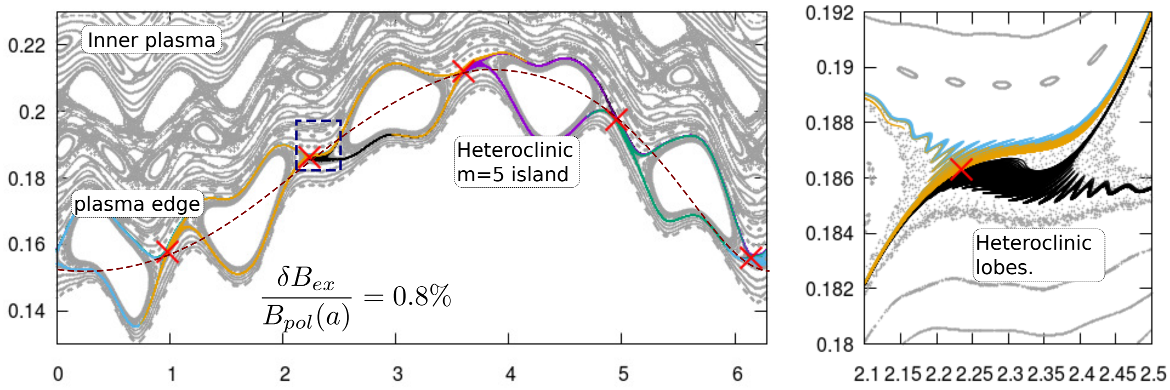

Figure 1: Ressonant islands near the plasma edge for $\delta B_{ex} = 0.008 B_{pol}(a)$. Manifolds of the period-5 unstable orbit present a characteristic heteroclinic structure.

The chaotic region increases with the perturbation amplitude and becomes more homogeneous as can be seen in the Figure 2. Moreover, the perturbation enhancement gives rise to a peculiar bifurcation where a secondary island emerges in the chaotic region between the dominant island chain.

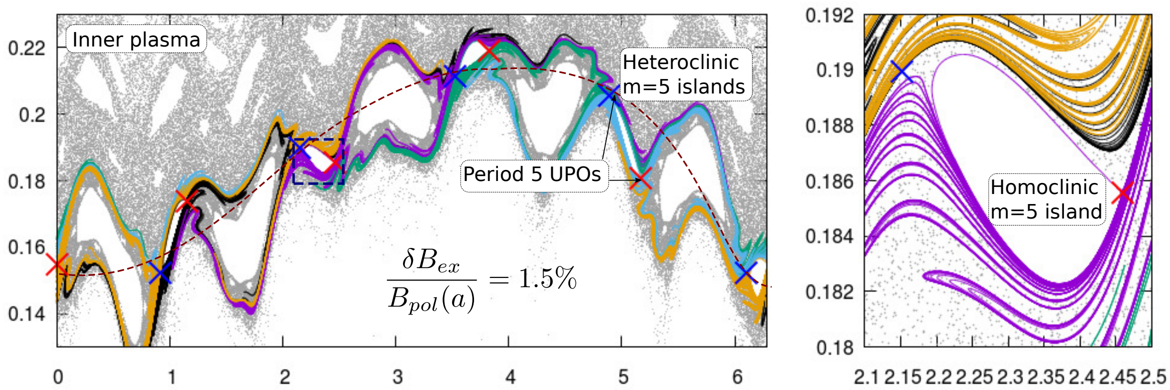

Figure 2: Ressonant islands near the plasma edge for . For this perturbation intensity a secondary set of islands with homoclinic structure emerges.

The new secondary chain has the same mode numbers $(5,1)$ of the original chain. Thus, in the Figure 2, the resonance $(5,1)$ presents two sets of period-5 unstable orbits known as isochronous [4, 7], which are expected according to the Poincaré-Birkhoff Fixed Point Theorem [7]. The first set belongs to the primary chain of islands and the second to the emerging secondary chain. Lines transport alteration due to this observed onset of the second isochronous island chain are presently under investigation. Meanwhile, in Fig. 2 it is observed that the manifold tangle around the new isochronous island presents a homoclinic structure, while continues to be embeded in the heteroclinic structure of the dominant mangetic island. This type of behavior suggests local magnetic reversing respect to the irregular helical pattern of lower perturbations. Manifold tracing for the presented mappings allows a detailed bifurcation analysis. While the manifold intersections in the original island are heteroclinic, the manifold intersections in the smaller second island are homoclinic.

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References

[1] D. Ciro, T. Evans, I. L. Caldas, arXiv:1710.10140 (2018).

[2] P. J. Morrison, Physics of Plasmas **7**, 2279 (2000).

[3] S. S. Abdullaev, Lecture Notes in Phyics **691**, 219 (2006).

[4] K. Ulmann, I.L. Caldas, Chaos, Solitons, and Fractals **11**, 2129 (2000).

[5] J. S. E. Portela, I. L. Caldas, R. L. Viana, Eur. Phys. J. Spec. Top. **165**, 195 (2008).

[6] J. H. F. Severo, I. C. Nascimento, Yu. K. Kuznetsov, R. M. O. Galvão, Z. O. Guimarães-Filho, F. O. Borges, O. C. Usuriaga, J. I. Elizondo, W. P. de Sá, E. K. Sanada, M. Tendler, Nuclear Fusion **49**, 115026 (2009).

[7] M. C. de Souza, I. L. Caldas, A. M. Ozorio de Almeida, F B Rizzato and R Pakter, Journal of Physics: Conference Series **641**, 012003 (2015).