Anisotropic heat diffusion on stochastic magnetic fields

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Introduction

The heat transport on the magnetic island and stochastic field is an important topic in the magnetic confinement fusion plasma research, because the magnetic island and stochastic field simultaneously appear in the fusion plasma. In tokamaks, the magnetic island is generated by the MHD instability like the tearing mode. Sometimes, in the reversed shear, two magnetic island chains are overlapped each other and then the magnetic field becomes stochastic. In stellarators, because of no symmetry along the toroidal direction, the magnetic field can become stochastic easily.

According to the classical theory by Rechester and Rosenbluth [1], the electron temperature is flattening, in other words, no gradient, on the magnetic islands and stochastic field. However, in the experiments, although the stochastic magnetic field is expected, the finite temperature is observed from the measurement. Thus, someones believe existing the finite temperature gradient is an evidence where the magnetic field does '*not*' become stochastic. However, this is not true. Hudson *et al*. studies the anisotropic heat transport on strongly stochastic field [2]. If some KAM surfaces exist on the stochastic field, those KAM surfaces can be worked as transport barriers. In addition, depending on the ration of the parallel and perpendicular heat conductivity, $\chi_\parallel/\chi_\perp$, the finite temperature gradient can be sustained on the stochastic field.

In this study, we study numerically the anisotropic heat diffusion on the stochastic magnetic field of the perturbed tokamak and stellarator. The anisotropic heat diffusion is given by a following equation,

$$\frac{3}{2}n\frac{\partial}{\partial t}T = -\nabla\cdot q + Q, \tag{1}$$

where n and T are electron temperature and density, q is the electron heat flux, and Q is the heat source. Here, the convection term is neglecting. We discuss initial results from a new code to solve the anisotropic heat diffusion based on the explicit and implicit schema.

Numerical scheme to solve anisotropic heat diffusion equation

In this study, the equation 1 is numerically solved. Here, the strong anisotropy is giving by the heat flux,

$$q = -n\left(\chi_{\parallel}\nabla_{\parallel}T + \chi_{\perp}\nabla_{\perp}T\right). \tag{2}$$

Gradients of the T along the parallel and perpendicular directions can be defined by,

$$\nabla_{\parallel}T = \mathbf{bb}\nabla T, \tag{3}$$

and

$$\nabla_{\perp}T = \left(\mathbf{I}-\mathbf{bb}\right)\nabla T, \tag{4}$$

where, $\mathbf{b}$ is the unit vector of the magnetic field and I is the unit tensor. In this study, these gradients are approximated by the finite difference scheme on the cylindrical coordinate (R,ϕ,Z). In the previous studies, the parallel and perpendicular gradients are approximated by metrics of the magnetic field. However, to keep high numerical accuracy, in particular, along the parallel direction, it is very difficult. Therefore, in this study, to improve the numerical accuracy to calculate the parallel gradient, the field line tracing method is used in. At first, the anisotropic heat diffusion equation is simplified in the given magnetic field as

$$\frac{3}{2}n\frac{\partial T}{\partial t} = n\left(\chi_{\parallel}\nabla_{\parallel}^{2}T + \chi_{\perp}\nabla_{\perp}^{2}T\right). \tag{5}$$

In the next, two magnetic field lines are traced from a computational grid to b and $-$b directions, respectively. In figure 1, the schematic view of the field line trace method is shown. Two magnetic field lines colored with the green along b and $-$b directions are traced from the computational grid. If the length L of the field line tracing from the computational grid can be defined, the parallel gradient at $L/2$ can be defined as

$$\frac{\partial T}{\partial l}\Big|_{l=+\frac{L}{2}} = \frac{T|_{l=+L} - T|_{l=0}}{\Delta l}, \tag{6}$$

where l is the arch length of the field line from the grid and $T|_{l=+L}$ is the temperature at $l=+L$. Here, the $l=0$ is the grid. Thus, the parallel diffusion on the grid can be defined as

$$\frac{\partial^2 T}{\partial l^2}\Big|_{l=0} = \frac{\frac{\partial T}{\partial l}\Big|_{l=+\frac{L}{2}} - \frac{\partial T}{\partial l}\Big|_{l=-\frac{L}{2}}}{\Delta l}. \tag{7}$$

The magnetic field line is traced by the 4th order Runge-Kutta method and the temperature is interpolated by the 4th order scheme. At the beginning of the simulation, the field line is traced and two end points of the field line at $l=+L$ and $-L$ starting from $l=0$ are stored on the

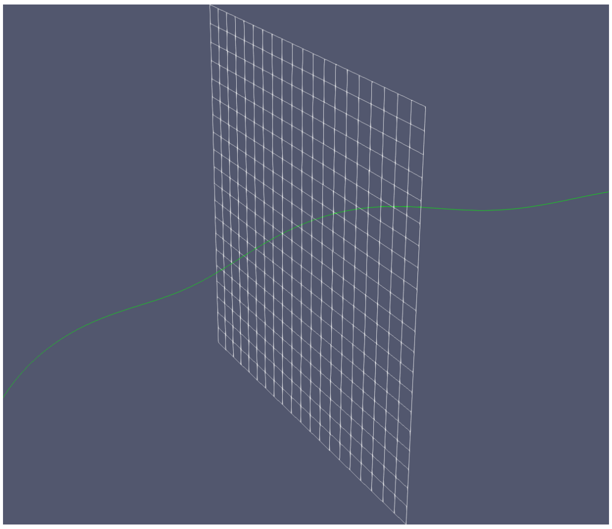

Figure 1: A schematic view of the field line tracing method. Green lines indicate two magnetic field lines from a grid. White lines indicate meshes of a fixed *phi* plane.

memory. On the other hand, the perpendicular diffusion on the cylindrical coordinate may be simplified as

$$\nabla_\perp^2 T \sim \frac{\partial^2 T}{\partial R^2} + \frac{\partial^2 T}{\partial Z^2}, \tag{8}$$

where the metric along R direction is ignored. The second derivate is approximated by the second order finite difference scheme and the derivative along R direction defines as

$$\frac{\partial^2 T}{\partial R^2} = \frac{T_{i+1,j,k} - 2T_{i,j,k} + T_{i-1,j,k}}{\Delta R^2}. \tag{9}$$

The time evolution of the anisotropic heat diffusion is integrated by an explicit and implicit schema. The explicit scheme is a usual FTCS and the implicit scheme is used to ADI (Alternating Direction Implicit) scheme.

Results

We applied a new code implemented above ideas to a perturbed tokamak and stellarator. Here, we mainly discussed about an application to the Large Helical Device, which is an L/M = 2/10 Heliotron configuration. Figure 2 shows a result of the anisotropic heat diffusion of $\chi_\parallel/\chi_\perp = 10^8$. A top of figure 2 shows a Poincaré plot of the horizontally elongated cross section for the vacuum standard configuration. In this study, the plasma response effect does not include and the magnetic field is fixed in the heat diffusion. The color of the Poincaré plot means the logarithm of the connection length between 1m to 1km. In the outside of the last closed flux surfaces, the stochastic magnetic field lines appear and opening field lines appear. A bottom figure shows a color map of contours of the electron temperature corresponding to the top figure.

Since the parallel electron transport is extremely strong in this case, the electron heat propagates along divertor legs. Thus, in the inward of the torus, many lobes from the separatrix appear and the finite temperature gradient can be kept on those lobes. In addition, on the stochastic region, many KAM surfaces exist and those surfaces can be worked to the transport barrier [**?**]. The finite temperature gradient cal also appear on the stochastic region.

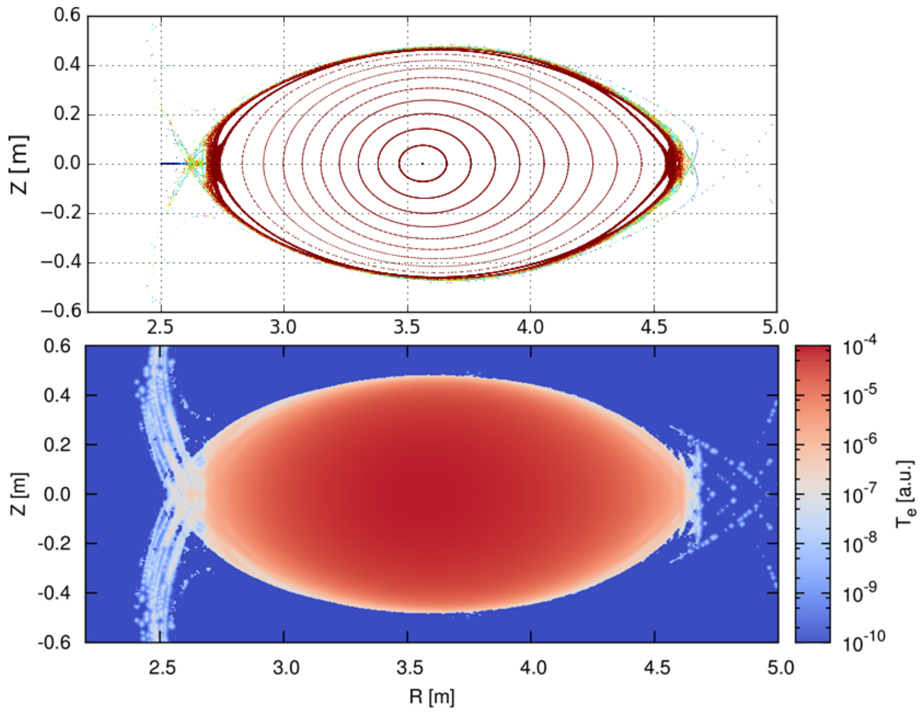

Figure 2: A result of the anisotropic heat diffusion of the LHD. A top figure shows a Poincaré plot of the horizontally elongated cross section. The color of the Poincaré plot means the logarithm of the connection length between 1m to 1km. A bottom figure shows a color map of contours of the electron temperature corresponding to the top figure. The finite temperature can be kept on the stochastic field.

Summary

We numerically study the anisotropic heat diffusion with the stochastic field. We study tow cases, one is the perturbed tokamak and another is the stellarator. In the realistic ratio of $\chi_{\parallel}/\chi_{\perp} = 10^8$, the parallel electron heat transport propagates along the stochastic field but the finite temperature gradient can be kept.

References

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