Threshold Effect In Tearing Mode Stabilization

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Since the suggestion that magnetic islands produced by tokamak tearing modes might be stabilized by non-inductive currents [1], a great number of experimental, theoretical, and computational efforts have been exerted. The most studied non-ohmically produced currents for stabilizing the tearing mode, particularly the neoclassical tearing mode, are the lower hybrid current drive (LHCD) [2] or the electron cyclotron current drive (ECCD) [3]. Both exploit the fact that a high current drive efficiency is obtained when the rf waves are damped in plasma by superthermal electrons. The stabilization effect relies upon rf waves driving current preferentially at the island center (O-line) as opposed to the island periphery (X-line) (see Fig. 1.).

The usual technique to concentrate the current near the O-line exploits the geometrical advantage of absorbing power at the minor radius that includes the O-line. If the power absorption is narrow compared to the island width, then there is more overlap of the deposition on surfaces near the O-line than the X-point. To improve the differential absorption might require precise steering and modulation of the rf waves to coincide with a rotating island.

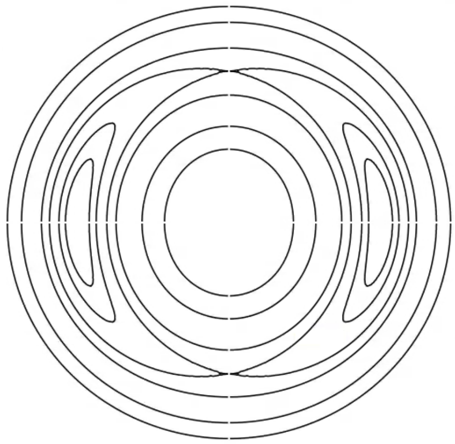

Figure 1: Minor cross section view of (2,1) large magnetic island. The usual technique is to arrange for wave damping in a narrow region around the minor radius that includes the O-line, thereby overlapping most completely with magnetic surfaces near the O-line.

The theoretical efforts that describe rf wave propagation and deposition in a magnetic geometry that includes the islands generally approach the deposition of the power absent the islands. The driven current then is proportional to the power absorbed, and it follows the power deposition profile. However, the islands do affect the wave propagation and damping. Most importantly, the islands thermally insulate the plasma contained within them. Absent radiation, the island is always hottest in the center, since it can only lose heat through its boundary, while the heating occurs internal to the boundary. The magnetic surface that forms the island boundary is thus necessarily at one temperature, namely the lowest temperature. For lower hybrid waves, this produces increased damping at the island center, leading to preferential driving of current in the island center [1]. This effect occurs also for electron cyclotron waves, since both electron cyclotron damping and lower hybrid wave damping involve superthermal electrons.

This insulation effect also provides an important positive feedback mechanism for the dissipation, with stark differentiation between the island interior and the island periphery. Because of the very sensitive dependence of the rf wave power deposition on the electron temperature, either in the case of LHCD or ECCD, the hot center attracts more power deposition which in turn makes it hotter yet. This positive feedback then causes the power deposition profile to narrow. The current profile, following the power deposition profile, similarly narrows with a maximum on the O-line, with the feedback giving what we call the *rf current condensation* effect [4]. The current condensation is illustrated in Fig. 2. In fact, the strength of the stabilizing effect of rf-driven currents displays a sharp threshold in the rf power density, with the stabilizing effect dramatically enhanced when the threshold power density is exceeded. When this threshold is exceeded, the current profile within the island also becomes even more peaked on the island O-line [4].

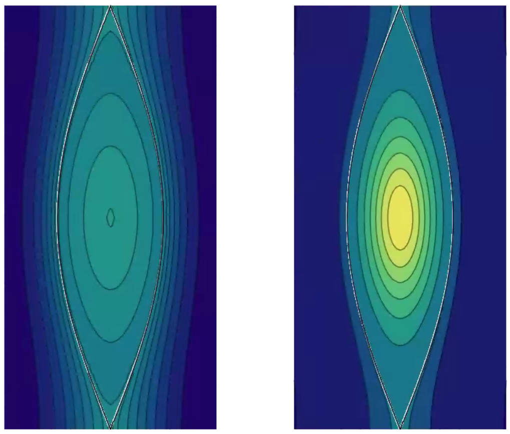

Figure 2: Left: island with uniform current density. Right: current condensation effect, with current peaked on O-line.

An important aspect of the current condensation is that the rf power drives current mainly near the rational surface, where its utility is greater. This is particularly significant for large islands for two reasons. First, the insulation effect is larger for large islands, so the current becomes even more peaked as the island size grows, thereby being even more efficient in stabilizing the island. In other words the current condensation utility is larger for larger islands. Second, the bootstrap current lost is proportional to the size of the island, so the rf current needed to make up for the bootstrap current also grows with island size. In other words, absent the current condensation effect, the power required for stabilization grows with the island size. For large islands, the required power could be impractical. However, because the current condensation utility increases with island size, the more efficient use of the current greatly reduces the power requirement. Note that because of the large power thought to be required for stabilizing large islands, greater effort has been placed on stabilizing islands while they are small, which requires more accurate steering of the RF wave deposition layer. Clearly this effort can be relaxed if there is an efficient way of stabilizing an island that is not caught when it is small.

To see how the condensation effect arises, consider either lower hybrid waves or electron cyclotron waves, with resonant phase velocities parallel to the magnetic field between v_1 and

$v_2 > v_1$. Both for LHCD [5] and for ECCD [6], the power dissipated by an intense wave spectrum is exponentially sensitive to the lower resonant velocity.

In the case of LHCD, for intense rf waves, a plateau is formed in parallel velocity space, with the number of electrons resonant with the wave far exceeding the initial number of resonant electrons [5]. Figure 3b displays contours of the electron density in normalized parallel w and perpendicular x velocity space. The power absorbed is then determined essentially by the lowest parallel phase velocity which "grabs" electrons from the bulk Maxwellian distribution. The case of ECCD is shown in Fig. 3a. Here, it is not exactly a plateau that is formed in velocity space; rather the intense rf accelerates electrons largely in the perpendicular velocity direction. However, similar to case of LHCD, the number of resonant electrons is significantly increased due to the rf waves. Also, similar to the case of LHCD, over a wide range of parameters describing the wave parameters, the damping decrement for the waves is determined essentially by the lowest parallel phase velocity which also "grabs" electrons from the bulk Maxwellian distribution [6]. Thus, for both LHCD and ECCD, the damping decrement to the wave then goes as $\gamma \propto \exp[-w_1^2]$, where the normalized parallel velocity $w_1 \equiv v_1/v_T$; where the electron thermal velocity is defined by $mv_T^2/2 = T$, and where T is the electron temperature. This is important for the feedback mechanism; other types of current drive, such as neutral beam current drive, minority species current drive, or Alfven wave current drive, do not display this sensitivity to electron temperature [7].

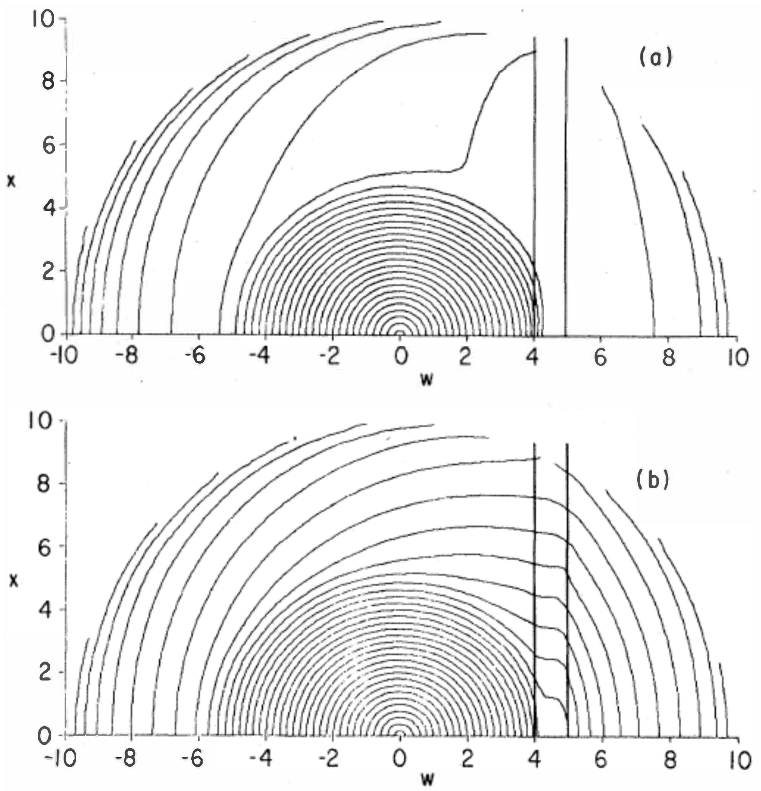

Figure 3: (a) Contour plot of electron density for electron cyclotron current drive with normalized parallel resonant velocity between 4 and 5. (b) Contour plot for lower hybrid current drive with normalized parallel resonant velocity between 4 and 5. In both cases, the intense rf limit is taken. Figure taken from [6].

For simplicity, to model this sensitivity, we ignore both the depletion of the power and changes in the resonant parallel phase velocity along the ray trajectory. Thus, the sensitivity of the power dissipated on the temperature is captured by modeling the available power density P as constant in space, with the absorbed power density P_{rf} proportional to the local wave damping, i.e., $P_{rf} \propto P\exp[-w_1^2 T_x/T_I]$, where we normalized the parallel phase velocity to the temperature at the X-line, T_x; and T_I is the local electron temperature in the island. While this crude model can be improved by ray tracing both for ECCD or LHCD, it captures the critical temperature dependency. Suppose the island is sufficiently large that the temperature can be taken to be constant on the flux surfaces in the island interior. Then, if we also model the diffusion of heat in 1D, the temperature in the island T_I obeys a diffusion equation of the form:

$$\frac{\partial T_I(x,t)}{\partial t} = D\frac{\partial^2 T_I}{\partial x^2} + cPe^{-w_1^2 T_x/T_i}, \qquad (1)$$

where D is a diffusion coefficient; x is the length across the island; c is a proportionality constant; and $w_1 \gg 1$ can be treated as a constant. Eq. (1) is to be solved with boundary conditions $T_I = T_x$ at say $x = +d$ and $x = -d$, where d measures the island width.

The temperature in the island typically equilibrates on a time scale short compared to the island growth time, so we can solve for the steady-state heat diffusion by taking the LHS of Eq. (1) to vanish. The steady state solution, however, displays bifurcation [4]. For low power, the temperature is largest at the island center ($x = 0$). With increasing power, the temperature rises and the current condenses sharply around the temperature maximum [4]. At some point, for increasing power density P, or increasing island width d, the temperature rises so that no steady solution exists. This occurs even if maximum temperature rise in the island is small, so long as $w_1 \gg 1$. At that point, additional physics, can be included in the nonlinear diffusion equation, leading to saturation of the temperature in the island.

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