# Robust plasma position, current, and shape control system simulated on the plasma evolution code for the spherical tokamak Globus-M

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The majority of the modern tokamaks operate with vertically elongated plasma, which is unstable in the vertical direction. In order to achieve high performance, effective plasma position, current, and shape control systems are mandatory in modern tokamaks. This paper describes design and simulation of these control system for the tokamak Globus-M (Ioffe Inst., S-Petersburg, Russia). Globus-M is a spherical tokamak with major radius $R = 0.36$ m, minor radius $a = 0.24$ m and aspect ratio $A = R/a = 1.5$, and discharge duration $\sim 0.2 \div 0.3$ s. Spherical tokamaks (ST) give a new roadmap to design fusion power plants on the module principle. These power plants based on ST are smaller, faster, and cheaper in comparison with fusion power plants on the base of conventional tokamaks with relatively high aspect ratios [1].

The commonly used tokamak plasma model for the magnetic control problems is a linear model based on linearized Faraday's law equations for coils, vacuum vessel (VV) elements and plasma circuits, and linearized equations of plasma column motion:

$$L_c \delta \dot{i}_c + M_{cv} \delta \dot{i}_v + M_{cp} \delta \dot{i}_p + \frac{\partial \Psi_c}{\partial r} \delta \dot{r} + \frac{\partial \Psi_c}{\partial z} \delta \dot{z} + R_c \delta I_c = \delta U, \quad \frac{\partial F_r}{\partial r} \delta r + \frac{\partial F_r}{\partial z} \delta z + \frac{\partial F_r}{\partial I_c} \delta I_c + \frac{\partial F_r}{\partial I_v} \delta I_v + \frac{\partial F_r}{\partial I_p} \delta I_p = 0,$$

$$M_{vc} \delta \dot{i}_c + L_v \delta \dot{i}_v + M_{vp} \delta \dot{i}_p + \frac{\partial \Psi_v}{\partial r} \delta \dot{r} + \frac{\partial \Psi_v}{\partial z} \delta \dot{z} + R_v \delta I_v = 0, \quad \frac{\partial F_z}{\partial z} \delta z + \frac{\partial F_z}{\partial r} \delta r + \frac{\partial F_z}{\partial I_c} \delta I_c + \frac{\partial F_z}{\partial I_v} \delta I_v + \frac{\partial F_z}{\partial I_p} \delta I_p = m \delta \ddot{z},$$

$$M_{pc} \delta \dot{i}_c + M_{pv} \delta \dot{i}_v + L_p \delta \dot{i}_p + \frac{\partial \Psi_p}{\partial r} \delta \dot{r} + \frac{\partial \Psi_p}{\partial z} \delta \dot{z} + R_p \delta I_p = 0.$$

(1)

Here $L$ and $M$ are self and mutual inductance matrices, $R$ is the diagonal resistance matrix, $I$ is the vector of circuit currents and $\Psi$ is the vector of magnetic fluxes through the circuits due to the plasma. Indexes $c$, $v$ and $p$ denote coil, VV and plasma, respectively. $F_r$ and $F_z$ are components of the force acting on the plasma column; $r$ and $z$ are coordinates of magnetic axis of the plasma. A small plasma mass $m$ is neglected in the equation of horizontal motion, but is retained in the vertical motion equation to avoid erroneous stability results [2]. The control signal is the voltage vector $U$ applied to coils.

The described model is based on linearized equations, and thus the model is valid only for small deviations from the initial plasma state. However, plasma characteristics in the Globus-M tokamak change significantly during a discharge. The changes in plasma current density

distribution lead to changes in the inductances, forces and fluxes in the system. That causes the unstable pole of the plasma linear model to increase by order of magnitude during the pulse.

The plasma current density distribution $J$ satisfies the Grad-Shafranov equation:

$$r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}\Psi + \frac{\partial^2}{\partial z^2}\Psi = -2\pi\mu_0 rJ, \quad J = \frac{r}{2\pi}\frac{d}{d\Psi}p(\Psi) + \frac{1}{4\pi\mu_0 r}\frac{d}{d\Psi}F^2(\Psi). \tag{2}$$

It follows from (2) that plasma current density may be expressed by plasma pressure function $p$ and poloidal current function $F$ that depends only on the magnetic flux $\Psi$. To model evolution of a plasma current density profile one needs to solve plasma transport equations. In order to do that various plasma evolution codes (in particular DINA [3]) have been developed.
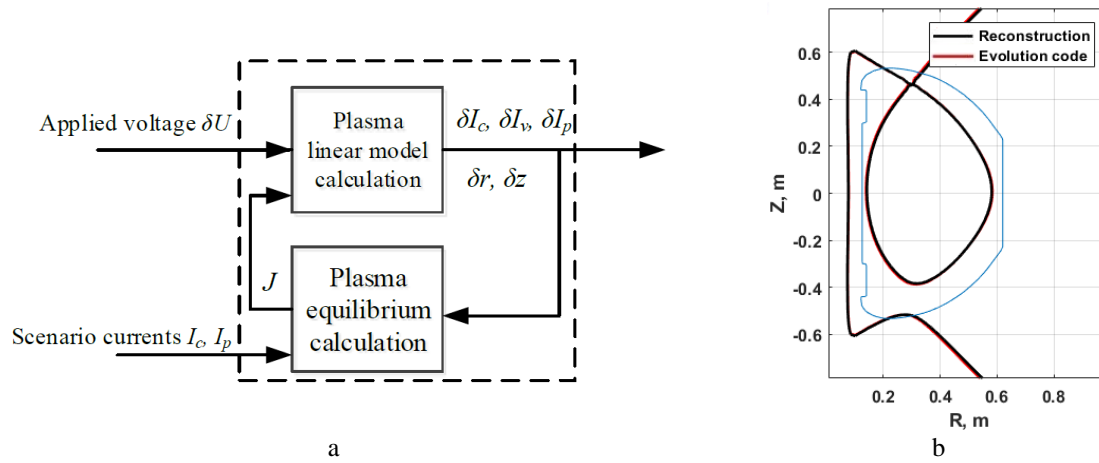


Fig. 1. (a) Plasma model as a feedback system. (b) Comparison between plasma equilibrium obtained by reconstruction from experimental data (shot №31648, $t$=180 ms) and modeled by magnetic evolution code.

To simulate plasma dynamics during the entire discharge without solving complex transport equations, a new non-stationary numerical Plasma Magnetic Evolution Code (PMEC) has been developed in MATLAB environment (Fig. 1a). On each time step of the numerical code the plasma linear model is calculated and then used to advance tokamak currents and plasma position. Then new currents and plasma position are used to calculate new plasma equilibrium. The plasma current density is modeled by approximating functions $p$ and $F$ from Grad-Shafranov equation (2) by polynomial functions of magnetic flux $\Psi$. The coefficients of polynomials are calculated from boundary conditions on plasma current density, total plasma current $I_p$, and plasma parameters $\beta_p$ and $l_i$ specified by the user. The updated plasma equilibrium is then used to create the plasma linear model on the next time step, accounting for the changed plasma parameters in (1) and closing the feedback inside the model (Fig. 1a). To verify the code, the equilibria obtained by the PMEC and the equilibria reconstructed from the experimental data have been compared. As shown in Fig. 1b the resulting plasma shapes are practically identical. The error estimate is less than 1%.

The hierarchical multi-loop control system includes loops and cascades for controlling position and current of the plasma and currents in the Poloidal Field (PF) coils with robust SISO PID controllers (green blocks in Fig. 2) tuned by Quantitative Feedback Theory (QFT) [4]. In these cascades the full thyristor current inverter models as fast actuators for plasma position control in self-oscillation mode with frequency of about 3 kHz [5] and the multiphase thyristor rectifier models as actuators for PF coils [6] are used (blue blocks in Fig. 2) in order to approximate the experimental conditions.
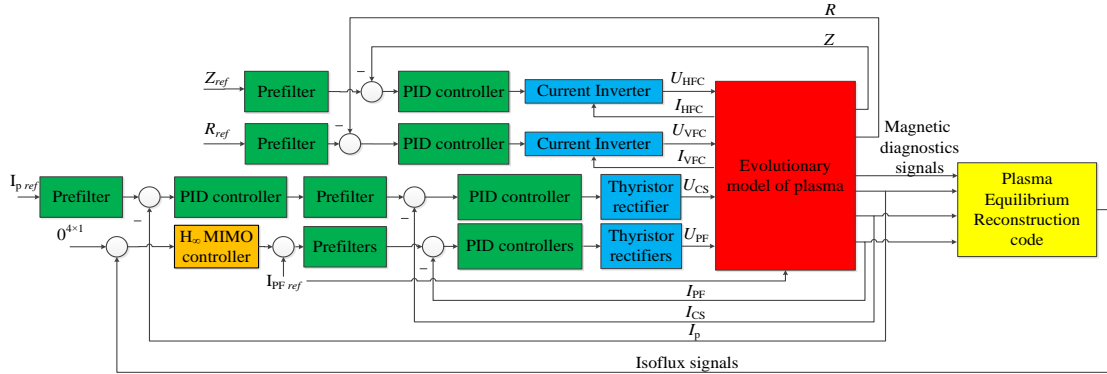


Fig. 2. System diagram for controlling position, current, and shape of the plasma in the Globus-M tokamak.

In QFT frequency analysis, a set of curves (QFT bounds) are considered on the Nichols diagram (Fig. 3) for a finite number of representative frequencies. These curves describe the frequency constraints for an open-loop system at each characteristic frequency taking into account various specifications (stability specification, reference tracking specification, etc.) for the whole set of linear plasma models at once. This is the main advantage of the QFT methodology. It is only necessary to determine nominal plant $P_0(s)$ and perform tuning of the controller $G(s)$ for the nominal open-loop transfer function $L_0(s) = P_0(s)G(s)$ without violation of the frequency constraints (as shown in Fig. 3).
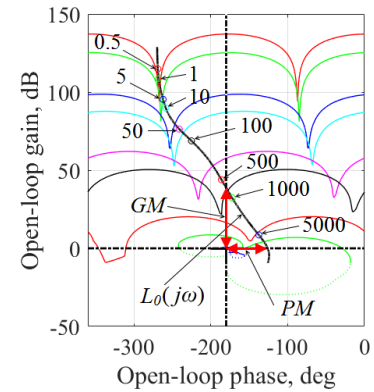


Fig. 3. Loop shaping procedure for the controller of the plasma vertical position $Z$ with QFT bounds for characteristic frequencies (rad/s), which are marked with numbers.



a                                       b                                       c
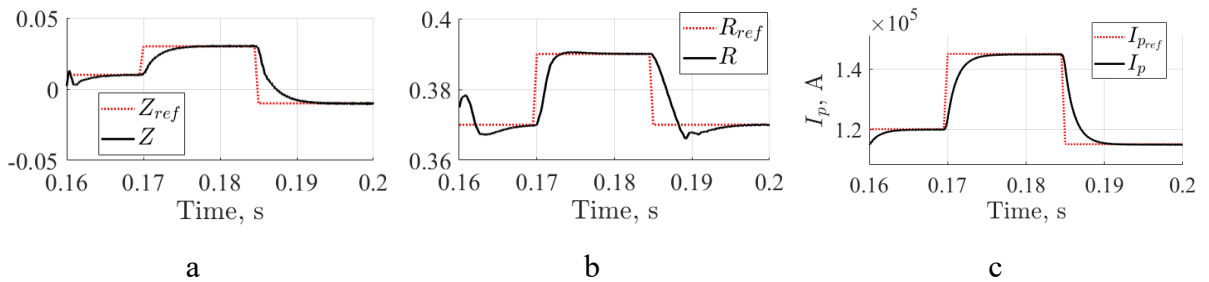
Fig. 4. Control of plasma position and current with a stepped reference value of (a) the vertical position $Z$ of the plasma, (b) the horizontal position $R$ of the plasma, (c) the plasma current $I_p$.

The closed control loop of the vertical position Z of the plasma has the gain margin GM = 37.8 dB at 690.1rad/s and the phase margin PM = 53.6 deg at 10304.5 rad/s (Fig 3).

The test results are shown in Fig. 4, where different stepped input signals were used in each individual model scenario. The voltages on the actuators did not exceed their maximum values during the simulation.

The outer cascade of plasma shape control (yellow and orange blocks in Fig. 2) is based on the isoflux control and incorporates the Improved Moving Filaments method for plasma equilibrium recon-



Fig. 5. Plasma shape control: $B_r$, $B_z$ at $X$-point, $\delta\Psi_1 = \Psi_X - \Psi_1$, $\delta\Psi_2 = \Psi_X - \Psi_2$, $\Psi_X$ is the flux at $X$-point, $\Psi_1$, $\Psi_2$ are fluxes at the separatrix.

struction, which has sufficient accuracy and speed of response [7]. A plasma shape MIMO controller was designed by the robust loop-shaping approach [8] for the magnetic field $B_r$, $B_z$ at $X$-point and poloidal fluxes on the plasma separatrix $\delta\Psi_1$, $\delta\Psi_2$. The plasma shape control loop was turned on at 0.18 s (Fig. 5) of the discharge and transferred the plasma boundary to the desired position in less than 5 ms during the divertor phase.
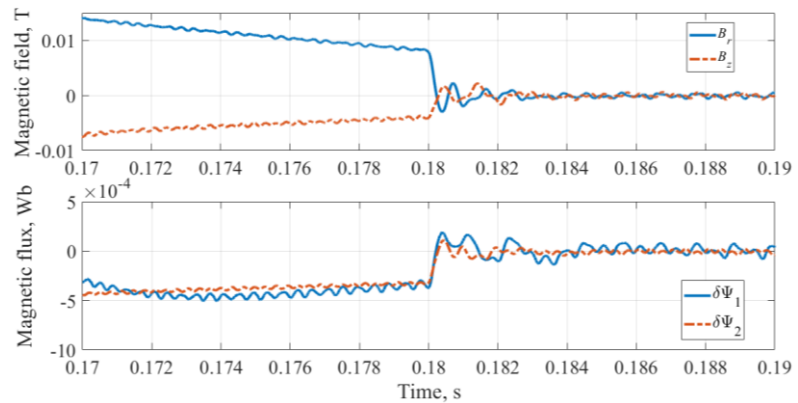
In this paper, the plasma evolution code was developed and the robust plasma magnetic control system was applied to it with plasma reconstruction code in the feedback in simulations to achieve maximum speed of response. This system can be used to control ST plasma position, current, and shape in real time using appropriate equipment. Our next step will be the installation of the developed models and reconstruction codes [7] on the Globus-M2 ST [9].

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