Heat flux in fusion plasmas, and variations in velocity space

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Understanding and being able to predict core turbulent (or anomalous) transport, in order to mitigate it, is crucial to achieve controlled fusion energy[1]. We investigate the energy dependance of the radial heat flux with the gyrokinetic code TERESA (Trapped Element REduction in Semi lagrangian Approach)[2, 3, 4, 5, 6, 7, 8]. The model enables the processing of the full f problem for trapped ions and electrons at very low numerical cost, although the study here focuses on trapped ions. We then compare the results to the predictions from the quasi-linear theory (QLT). Both heat fluxes from TERESA and the QLT are in qualitative agreement but shows a quantitative error. The source of discrepancy is that the QLT neglect some nonlinear coupling that are in fact of the same order of magnitude of some non-neglected terms. Both heat fluxes present a peak at a resonance energy and resonant particles account for the majority of the heat flux.

Trapped particles

The motion of a single trapped particle in a tokamak can be divided into three parts: The fast cyclotron motion (ω_c, ρ_c), the bounce (or "banana") motion (ω_b, δ_b), and the precession drift along the toroidal direction (ω_d, R), with $\omega_d \ll \omega_b \ll \omega_c$ and $\rho_c \ll \delta_b \ll R$ (See Fig.1).

The turbulence driven by trapped particles is characterized by frequencies of the order of the precession frequency ω_d. Averaging over both cyclotron and bounce motions filters the fast frequencies ω_c and ω_b and the small space scales ρ_c and δ_b. It reduces the dimensionality of the kinetic model from 6D to 4D:

$$\bar{f}_s = \bar{f}_{\kappa,E}(\psi, \alpha)$$

with $\bar{f}_s$ the "banana center" distribution function, $\alpha = \varphi - q\theta$ and ψ the poloidal flux ($d\psi \sim -rdr$). φ and θ are the toroidal and poloidal coordinates, and q is the safety factor. Only two kinetic variables appear in the differential operators. The two other variables appear as parameters - two exact invariants, namely particle kinetic energy E and κ the pitch angle.

Model

Although TERESA allows kinetic trapped ions and electrons, here we solely focus on ions. The Vlasov equation for the ion banana center distribution function f (we omit the $\bar{f}$ notation

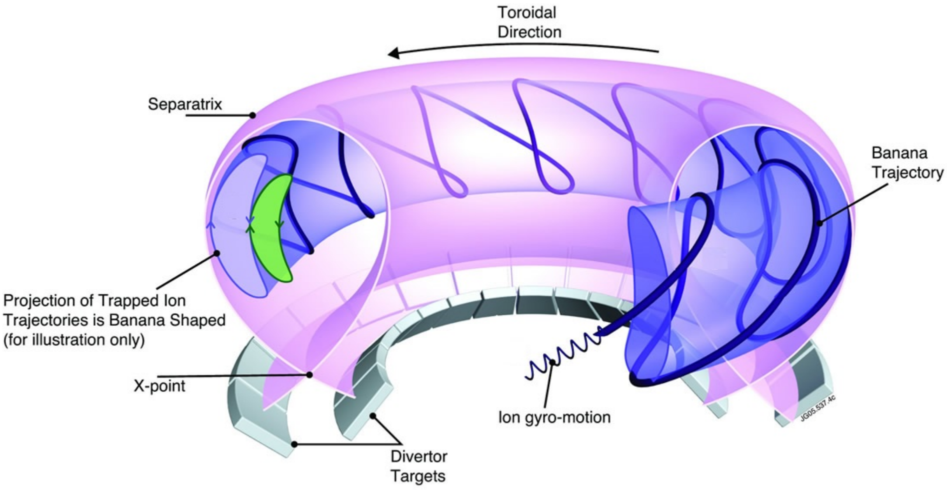

Figure 1: *Motion of trapped particles in a tokamak (source : EUROfusion).*

for clarity) writes:

$$\frac{\partial f}{\partial t} - [\mathscr{J}_0\phi, f]_{\alpha,\psi} + E\Omega_d \frac{\partial f}{\partial \alpha} = 0 \quad (1)$$

where ϕ is the plasma electrostatic potential, $E\Omega_d$ is the energy-dependent precession frequency, $[g,h]_{\alpha,\psi} = \partial_\alpha g \partial_\psi h - \partial_\alpha h \partial_\psi g$ are the Poisson bracket, and $\mathscr{J}_0$ is the gyro-bounce-averaging operator.

The normalized quasi-neutrality constraint writes:

$$C_1 \left[\phi - \langle\phi\rangle_\alpha + \mathscr{F}^{-1}\left(i\delta_m\hat{\phi}_m\right)\right] - C_2\bar{\Delta}\phi = \frac{2}{\sqrt{\pi}} \int_0^\infty \mathscr{J}_0(E) f \sqrt{E} \mathrm{d}E - 1 \quad (2)$$

where $\mathscr{F}^{-1}$ is the inverse Fourier transform, δ_m is the electron dissipation which takes into account the effects of electrons-ions collisions, expressed as a phase-shift between electron density and perturbed electric potential[8], $\hat{\phi}_m$ is the m-th component of the Fourier decomposition in α of ϕ. $C_1 = \tau C_2/f_p$ and $C_2 = e\omega_{d,0}L_\psi/T_0$ are dimensionless, constant input parameters, which account for the fraction of trapped particles f_p and ion/electron temperature ratio τ, $\bar{\Delta}\phi$ is the polarization term of the quasi-neutrality equation, with $\bar{\Delta} = \left(\frac{q_0\rho_0}{a}\right)^2 \frac{\partial^2}{\partial\alpha^2} + \delta_b^2 \frac{\partial^2}{\partial\psi^2}$.

A semi-Lagrangian scheme is used in order to solve the Vlasov equations. To solve the quasi-neutrality, the fields are first projected in the Fourier space along the periodic direction α and then the electric potential ϕ is a solution of a second order differential equation in ψ.

Quasi-linear theory (QLT)

The QLT main hypothesis are : a weak turbulence, particles are not trapped inside "potential wells", and a small correlation time of the electric field compared to the evolution time of the profiles.

Once we inject the linear response of f and ϕ in the Vlasov equation we obtain :

$$\frac{\partial \langle f \rangle}{\partial t} = \frac{\partial}{\partial \psi}\left[D_{\mathrm{QL}} \frac{\partial \langle f \rangle}{\partial \psi} \right], \tag{3}$$

with

$$D_{\mathrm{QL}}(\psi, E, t) \; = \; \sum_l l^2 \left| \hat{\phi}_l(\psi, t) \right|^2 \frac{1 - e^{-i(\omega_{R,l}-\omega_l)t - \gamma_l t}}{i(\omega_{R,l} - \omega_l) + \gamma_l}, \tag{4}$$

where the sum $\sum_l$ is over the l components of the Fourier decomposition in α of ϕ, noted $\hat{\phi}_l(\psi, t)$. γ_l and ω_l are respectively the growth rate and frequency of each Trapped Ion Mode (TIM) l obtained from the linear dispersion relation[5, 6], and $\omega_{R,l}(\psi, E, t) = l\left(\frac{\Omega_d E}{Z} + \frac{\partial \hat{\phi}_0}{\partial \psi}\right)$ is homogeneous to a pulsation and takes into account the Doppler effect from the zonal flow.

The QL heat flux is then given by

$$q^{QL}(\psi, t) = \int_E D_{QL} \frac{\partial \langle f \rangle}{\partial \psi} E' \sqrt{E'} \mathrm{d}E' \tag{5}$$

The velocity space is linked to the energy space as $d^3 v = C\sqrt{E}\mathrm{d}E$, therefore we include a $\sqrt{E}$ factor inside the integral over energy space to physically integrate over the velocities.

QL and NL heat fluxes in real and in energy space

We compare the heat flux obtained from a nonlinear simulation with TERESA, to the QL prediction in the real space (ψ), Fig. 2a, and in the velocity/energy space, Fig. 2b.

The QL and NL heat fluxes are in qualitative agreement but the QL calculation shows a significant quantitative error. Both are negative for $E < 1.5$ because of the Maxwellian nature of the equilibrium function f_{eq}. The resonant particles contribute the most to the heat fluxes. The position of the resonant peak in energy E is determined by the resonance condition of the most intense toroidal modes at $t = 5$ and $\psi = 0.5$.

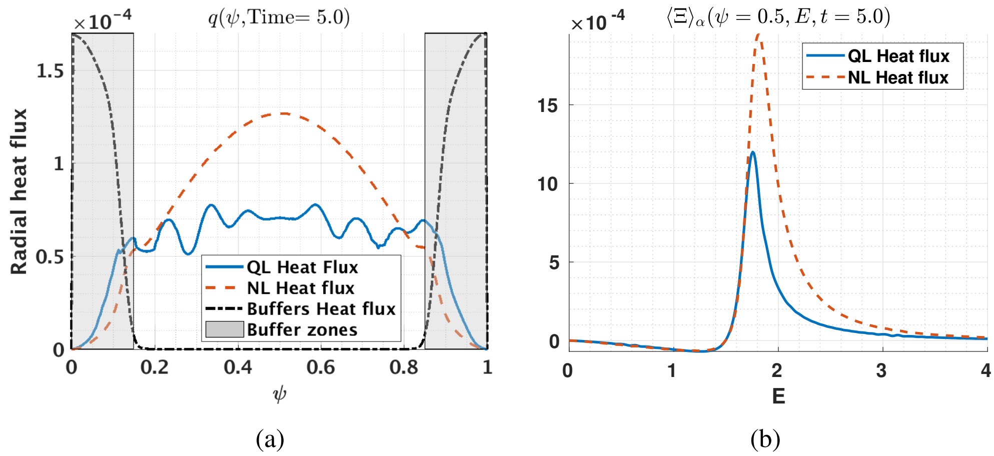

Figure 2: *Fig. 2a QL and NL heat fluxes at $t = 5$ as a function of ψ. The buffer zone consist in an artificial diffusion in the greyed out area. Fig. 2b QL and NL heat fluxes at $t = 5$ and $\psi = 0.5$ in the energy space.*

Conclusion

We took advantage of the TERESA code to run a simulation with great accuracy in E space without sacrificing the precision in real space. We thus obtained from the nonlinear (NL) simulation the heat flux in real space (ψ), and its details in energy space. We compared these NL results to the predictions from the QLT which take into account the effect of the zonal radial electric field. The QL and NL heat fluxes are in qualitative agreement although the QL calculation shows a significant quantitative error. The source of discrepancy is that the QLT neglect some nonlinear coupling that are in fact of the same order of magnitude of some non-neglected terms (as checked directly in the simulation). In the energy space, both present a peak at a resonant energy and the resonant particles are the main contribution to the heat flux. The position of the resonance in E space is determined by the most intense toroidal modes and by the intensity of the zonal radial electric field.

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