The helicity and the generation of large scale flows in confined plasma

F. Spineanu, M. Vlad

National Institute of Laser, Plasma and Radiation Physics, Bucharest, Romania

The helicity plays an essential role in mediating the spectral transfer of energy leading to generation of cuasi-coherent structures from turbulence in 3*D* flows. In two dimensions the inverse cascade naturally obtains at asymptotic relaxation a large scale ordered structure of the flow but in 3*D* it seems that only the breaking of the parity symmetry can induce inverse spectral transfer. For a turbulent fluid/plasma the parity non-invariance means that small scale filaments with helical geometry results from nonlinear mode coupling. It has been shown that the helical filaments evolve through coalescence to even larger helical structures. We suggest that the formation of large scale quasi-coherent pattern of flow, including the zonal flows and the H-mode rotation layer, out from turbulence is only possible if the turbulence is parity-non-invariant. The presence of helicity extends the self-organization process, which is typical only for 2*D*, somehow further in 3*D* where the natural expectation would be direct cascade to dissipative scales.

In the following we discuss briefly the fundament of the helicity-mediated large scale organization, the statistical correlation when helicity is present and we formulate a proposal for the study of the "cancellation exponent" of helicity fluctuations in numerical simulations.

Nature of structure formation in a parity-breaking turbulence

For plasma there are three measures of helicity: kinetic $\mathbf{v}\cdot\omega$, magnetic $\mathbf{A}\cdot\mathbf{B}$ and mixed $\mathbf{v}\cdot\mathbf{B}$. We confine to the first type. The importance of the vorticity in plasma flows at all scales is known. The most familiar quasi-three-dimensional model of plasma is Hasegawa-Mima, for which the Boltzmann distribution of electrons along the field line suppresses the convective nonlinearity $(-\nabla\phi\times\widehat{\mathbf{n}}\cdot\nabla\phi=0)$ and puts emphasis on the vectorial nonlinearity. However in many cases the two-dimensional Euler equation is a good description $d\omega/dt=0$. For the ideal 2*D* Euler fluid the dynamics of the vorticity is reducible to the motion of a discrete set of point-like vortices interacting as: $\dot{x}_{s,i}=\varepsilon_{sj}\partial_j\sum_{j=1,j\neq i}^{N}\omega_j\ln\left(\left|\mathbf{x}_i-\mathbf{x}_j\right|\right)$ with $x_{s=1,2}\equiv(x,y)$, $i=1,N$, and where $\omega_j=\pm\omega_0$. Each point-like vortex can be represented as a spinor and the continuum field as a mixed spinor $x^{\alpha\dot{\beta}}$, a 2×2 complex matrix which incorporates the time-forward propagating spinors (positive elementary vortex) and time-backward propagating spinors (negative elementary vortex). $x^{\alpha\dot{\beta}}\equiv\phi$ is a representation of the algebra $sl(2,\mathbf{C})$. Physical vorticity should not exist in 2*D* since $\mathbf{v}$ is in plane and $\omega=\nabla\times\mathbf{v}$ is perpendicular on the plane, resulting $\mathbf{v}\cdot\omega=0$.

However reduction to 2D means that any translation along the direction perpendicular leave invariant the physics and then we can define, as for the *massless fermions*, $\frac{\sigma\cdot\mathbf{p}}{|\mathbf{p}|}$ =helicity which identifies the helicity with the chirality. We have formulated a field theoretical model with a Lagrangian where the matter field ϕ (mixed spinor) has non-relativistic dynamics and self-interaction of the type $\mathrm{tr}\left(\left[\phi^{\dagger},\phi\right]^{2}\right)$ and the gauge field is governed by the Chern-Simons term, a generalization of the *helicity*, $\varepsilon^{ijk}A_i\partial_j A_k \sim \mathbf{A}\cdot\mathbf{B}$. We have demonstrated the following property of this system. It can be reduced to a local problem of interaction between an elementary vortex and a fixed, large scale vortex, a scattering problem that reduces the equations of motion to usual Dirac equations for a scattering problem. Then the difference between the positive vortices and the negative vortices becomes clear: the like sign vortices are attracting and the opposite-sign vortices a repelled. This is very important. We then understand that the dynamics of the 2D ideal Euler fluid consists of separation and clusterization of vortices of like signs.

We naturally extend such conclusion to the elementary helical filaments and assert that the like-sign helical filaments will attract mutually and will coalesce, increasing the separation of the two kinds of helicity and their clusterization into large scale patterns of flow.

We consider that this is the fundament for the property of parity-noninvariant turbulent fields to evolve to large scale structures, via separation and clusterization of helicity.

The statistical transfer of energy from turbulence to coherent structure in a parity-breaking turbulence.

This is illustrated by a classical Rayleigh Benard setting, according to the treatment of **Moiseev** *et al* [1]. The gradient of temperature $\rho=\rho_0\left(1-\beta T\right)$ is along the z direction, perpendicular on the plane (x,y). The equilibrium state consists of the profiles $T_0\left(z\right)$ and $p_0\left(z\right)$, with $T_0\left(z\right)=-A\widehat{\mathbf{e}}_z$. The equation for velocity

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x^k}=-\frac{1}{\rho}\frac{\partial p}{\partial x^i}+\beta T g e_i+\nu\Delta v_i$$

together with the equation for temperature perturbation and the incompressibility. A random external force to create tubulence $\langle F_i\rangle=0$. The correlation function

$$Q_{ij}^{T}\left(t_1-t_2,\mathbf{k}\right)=B\left(t_1-t_2,k\right)\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)+iG\left(t_1-t_2,k\right)\varepsilon_{ijl}k_l$$

where $\langle\mathbf{v}\cdot\left(\nabla\times\mathbf{v}\right)\rangle\sim\int k^2dk\;G\left(\tau_1-t_2,k\right)$. Note the presence of the pseudotensor ε_{ijl} and the pseudoscalar $G\left(t,k\right)$. In a similar context, Chechkin [2] shows explicitely the content of the correlation of the fluctuating field, with the occurence of the imaginary term in k space, associated to the parity-non-invariance. Assume the electric field is fluctuating, the correlations are, in real space

$$\left\langle E_i E_j \right\rangle = A(R,\tau)\,\delta_{ij} + B(R,\tau)\,R_i R_j + G(R,\tau)\,\varepsilon_{ijm} R_m$$

or in Fourier space

$$\left\langle E_i E_j \right\rangle_{\mathbf{k},\omega} = A(k,\omega)\,\delta_{ij} + B(k,\omega)\,\frac{k_i k_j}{k^2} + iG(k,\omega)\,\varepsilon_{ijm}\frac{k_m}{k^2}$$

The set of equations is solved for the temperature which is then replaced in the velocity equation

$$L_{ij} v_j = -D_\chi P_{im} \frac{\partial}{\partial x^k}(v_k v_m) - \beta A g P_{im} e_m e_j \frac{\partial}{\partial x^k}\left(v_k \frac{1}{D_\chi} v_j \right) + F_i$$

with the operators $L_{ij} = D_\nu D_\chi \delta_{ij} - \beta A g P_{im} e_m e_j$, $D_\nu = \frac{\partial}{\partial t} - \nu\Delta$, $D_\chi = \frac{\partial}{\partial t} - \chi\Delta$, and $P_{im} = \delta_{im} - \frac{1}{\Delta}\frac{\partial}{\partial x^i}\frac{\partial}{\partial x^m}$. The fluctuating field of velocity must be decomposed $v_i = \langle v_i \rangle + v_i^T + \widetilde{v}_i$.

For the average $L_{ij}\left\langle v_j \right\rangle$ and the fluctuating part $L_{ij}\widetilde{v}_j$ we calculate the correlations, using the functional dependence of $\widetilde{v}_k$ on $\left\langle v_j \right\rangle$ and on v_j^T, using the Furutsu-Novikov formula

$$\begin{aligned} & \left\langle v_k^T(t,\mathbf{x})\,\widetilde{v}_m(t,\mathbf{x}) \right\rangle \\ = & \lim_{t_1 \to t, \mathbf{x}_1 \to \mathbf{x}} \int ds \int d\mathbf{y} \left\langle v_k^T(t,\mathbf{x})\, v_r^T(s,\mathbf{y}) \right\rangle \left\langle \frac{\delta \widetilde{v}_m(t_1,\mathbf{x}_1)}{\delta v_r^T(s,\mathbf{y})} \right\rangle \end{aligned}$$

The second average, of the functional derivative, can be calculated formally from the solution of the equation of $\widetilde{v}_m$.

$$\begin{aligned} \left\langle \frac{\delta \widetilde{v}(t,\mathbf{x})}{\delta v_s^T(s,\mathbf{y})} \right\rangle = & -L_{ji}^{-1}\left\{ D_\chi P_{im} \langle v_k \rangle \frac{\partial}{\partial x^k} \left\langle \frac{\delta v_m^T(t,\mathbf{x})}{\delta v_r^T(s,\mathbf{y})} \right\rangle \right. \\ & \left. + \beta A g P_{im} \langle v_k \rangle \frac{\partial}{\partial x^k} D_\chi^{-1} \left\langle \frac{\delta v_j^T(t,\mathbf{x})}{\delta v_r^T(s,\mathbf{y})} \right\rangle \right\} \end{aligned}$$

Now we will assume an explicit form $G(t-s,k) = G_0 \frac{u^2\lambda^4}{\left(1+\lambda^2 k^2\right)^2} \exp\left[-\frac{|t-s|}{\tau} \right]$ and it is possible to calculate

$$M_{kj}(t,\mathbf{x}) = \left\langle \widetilde{v}_k(t,\mathbf{x}) D_\chi^{-1}\, v_j^T(t,\mathbf{x}) \right\rangle + \left\langle v_k^T(t,\mathbf{x})\; D_\chi^{-1}\widetilde{v}_j(t,\mathbf{x}) \right\rangle$$

with the result

$$M_{kj}(t,\mathbf{x}) = \frac{4\pi}{3}\varepsilon_{kja}\langle v_a \rangle \frac{G_0}{(2\pi)^3}\frac{\pi}{2}\frac{u^2\tau^2}{\lambda} \times W(\nu,\lambda,\chi,\tau)$$

where W is an expression of the viscosities. This correlation shows the coupling and energy transfer between the turbulent fluctuations v^T and the correction $\widetilde{v}$ to the main flow.

The sign-singularity of the helicity fluctuations

We argue that there may be a systematic statistics of the helicity fluctuations and that it can be "measured", most easily, from numerical simulations of plasma turbulence.

The local values of the velocity and vorticity produce a local fluctuating kinetic helicity h, which can be seen as helical lines of flow with positive or negative linking relative to a fixed reference line. This *fluctuating chirality* can be reduced to a change of sign of this linking. The strong oscillations of the signs (*i.e.* positive and negative helicity) are an indication of the "sign singularity" [3]. It is introduced by $\mu_r(L_i) = \int_{L_i(r)} h(r) / \int_L |h(r)|$ where $L_i(r) \subset L$ is a hyerarchy of a disjoint subsets of size r covering the full domain L. The expected value of $|\mu_r(L_i)|$ at scale r defines $\chi(r) = \sum_{L_i(r)} |\mu_r(L_i)|$ for which Ott *et al*. have conjectured that exhibits a scaling $\lim_{r\to 0} \frac{\log \chi(r)}{\log r} = -\kappa$. If $\chi(r)$ increases as $r \to 0$ then $\kappa > 0$ and the measure is said "sign-singular". κ is called "cancellation exponent".

In the case of tokamak turbulence, the spectrum of $\mu_r(L_i)$ results from the wide range of relevant physical scales, between ρ_s and the large radial extension of tilted eddies or the transient coherent structures. We can expect a fast oscillation of helicity's signs in the small scales $\sim \rho_s$ where nonlinear coupling of drift waves generates random transfers of energy and transient structures of the helical lines of flow having left or right helicity. A non-zero cancellation coefficient κ indicates unbalanced cancellations *i.e.* the dominance of some chirality and is equivalent to loss of parity invariance of the fluctuating potential of the turbulent field [4]. There are several possible sources for $\kappa \neq 0$, of which we can retain the tilting instability, where the role of the scalar nonlinearity (distinct from the vectorial, Hasegawa-Mima nonlinearity) becomes manifest. The scalar nonlinearity $\phi \partial \phi / \partial y$ is obviously non-parity-invariant. A correlation between the enhanced role of the scalar nonlinearity and the magnitude of κ should be measured in numerical simulations.

In **Conclusion** the helicity is a part of the "vorticity dynamo" and is present in plasma turbulence. Technically, the random generation of helicity can be described by a scalar field that multiplies the Chern-Simon topological charge, like in *baryogenesis*. It is rather surprising to find purely classical realisation of the axial anomaly.

Aknowledgement. This work is supported by Contracts PN-01-01 and CEA-RO-04.

References

[1] S. S. Moiseev, P. B. Rutkevich, A. V. Tur, and V. V. Yanovskii, Sov. Phys. JETP **67**, 294 (1988) [Zh. Eksp. Teor. Fiz. **94**,144 (1988)]

[2] Chechkin, A. Phys. Plasmas **1**, 2566 (1994).

[3] Ott, E., Du, Y., Sreenivasan, K. R., Juneja, A., & Suri, A. K., Phys. Rev. Lett., **69**, 2654 (1992).

[4] V. Carbone, R. Bruno, The Astrophysical Journal **488**, 482 (1997)