Study of cavitation in liquid water under the action of inhomogeneous pulsed electric fields: application to sub-nanosecond electrical breakdown

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A sub-nanosecond electrical breakdown in dielectric liquids is of vital interest, e.g. for applications in high-voltage insulation and high-current switching. Liquid dielectrics in strong nonuniform electric fields are under influence of an electrostrictive force that tends to move the fluid into the regions with a higher electric field. If the voltage rise is fast enough, the liquid does not have enough time to be set into motion in order to reduce internal stress. In this case, the ponderomotive force induces significant stress in the bulk of the liquid that is manifested as a negative pressure. At the certain threshold, the negative pressure causes cavitation ruptures of the fluid. Then free electrons can be produced by emission from the surface inside the cavity and accelerated to energies exceeding the energy for ionization of water and contribute to the ultrafast electrical breakdown of water.

In this work we use hydrodynamic model for motion of dielectric fluid to study the dynamics of water in a pulsed strongly inhomogeneous electric fields in the approximation of compressible flow described by equation of continuity for mass and momentum [1, 2]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \tag{1}$$

$$\rho \left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} \right] = -\nabla p + \vec{F} + \eta \left[\nabla^2 \vec{u} + \frac{1}{3} \nabla (\nabla \cdot \vec{u}) \right], \tag{2}$$

and Tait equation of state for compressible water [3, 4, 5]:

$$p = (p_0 + B) \left(\frac{\rho}{\rho_0} \right)^{\gamma} - B \tag{3}$$

where ρ is the fluid density, p is the pressure, $\vec{u}$ is the velocity, η is the dynamic viscosity, and

$$\rho_0 = 1000\,\mathrm{kg/m^3}, \quad p_0 = 10^5\,\mathrm{Pa},$$

$$B = 3.07 \times 10^8\,\mathrm{Pa}, \quad \gamma = 7.5.$$

The volumetric force $\vec{F}$ acting on dielectric fluid, in absence of free charges and inhomogeneity in the fluid, is simply

$$\vec{F} = \frac{\varepsilon_0}{2} \left(\frac{\partial \varepsilon}{\partial \rho} \rho \right) \nabla E^2 \approx \frac{\varepsilon_0 \varepsilon \alpha}{2} \nabla E^2,$$

with $\alpha \approx 1.5$ is an empirical factor for polar dielectrics [6].

Thanks to ∇E^2 dependence, the force can be interpreted as negative pressure acting on the fluid:

$$-\nabla p + \vec{F} = -\nabla \left(p - \frac{\varepsilon_0 \varepsilon \alpha}{2} E^2 \right).$$

So the total pressure in the fluid is then

$$p_{\text{tot}} = p - \frac{\varepsilon_0 \varepsilon \alpha}{2} E^2.$$

Experimental data show that in water the critical tension for the fluid rupture is in range of 6 to 50 MPa. In this work we use critical negative pressure of $p_{\text{c}} = -30\,\text{MPa}$, in accordance with [1, 7].

As shown in [1], changes in the fluid density will not exceed a few percent. Also, the resulting flow rates will be slow compared to the speed of sound, and on nanosecond timescales, viscosity terms can be neglected. So we assume first order variations in the background fluid density:

$$\rho(t) \sim \rho_0 + \tilde{\rho}(t). \tag{4}$$

Then after neglecting viscosity and all second order terms,

$$\nabla \cdot (\tilde{\rho} \vec{u}) \approx 0,$$

we get the continuity equations for density fluctuation

$$\frac{\partial \tilde{\rho}}{\partial t} + \rho_0 \nabla \cdot \vec{u} = 0. \tag{5}$$

Similarly, neglecting second and higher order terms in the momentum equation (2), .i.e.

$$(\vec{u} \cdot \nabla)\vec{u} \approx 0, \quad \tilde{\rho} \frac{\partial \vec{u}}{\partial t} \approx 0,$$

we get linearized momentum equation in the form:

$$\frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho_0} \nabla \left(p - \frac{\varepsilon_0 \varepsilon \alpha}{2} E^2 \right). \tag{6}$$

Both equations (5) and (6) are coupled by equation of state (3), which linearized has a form:

$$p(\rho_0) = p_0 + \frac{(p_0 + B)}{\rho_0} \gamma (\rho - \rho_0) + \dots. \tag{7}$$

Then

$$\frac{\partial p}{\partial t} = \frac{(p_0 + B)}{\rho_0} \gamma \frac{\partial \tilde{\rho}}{\partial t} \approx \frac{B\gamma}{\rho_0} \frac{\partial \tilde{\rho}}{\partial t} = c_{\text{s}}^2 \frac{\partial \tilde{\rho}}{\partial t}, \tag{8}$$

with speed of sound:

$$c_{\text{s}} = \sqrt{\frac{\partial p}{\partial \rho}} \approx \sqrt{\frac{B\gamma}{\rho_0}} \approx 1500\,\text{m/s}.$$

Finally continuity equation can be expressed in terms of pressure p:

$$\frac{\partial p}{\partial t} + \rho_0 c_\mathrm{s}^2 \nabla \cdot \vec{u} = 0. \tag{9}$$

Resulting system of equations 5 and 8, for spherically symmetric case can be written in matrix form:

$$\frac{\partial}{\partial t}\begin{pmatrix} v \\ p \end{pmatrix} + \begin{bmatrix} 0 & 1/\rho_0 \\ \rho_0 c_\mathrm{s}^2 & 0 \end{bmatrix} \frac{d}{dr}\begin{pmatrix} v \\ p \end{pmatrix} = \begin{pmatrix} \frac{\varepsilon_0 \varepsilon \alpha}{2\rho_0}\nabla E^2 \\ -\rho_0 c_\mathrm{s}^2 \frac{2v}{r} \end{pmatrix} \tag{10}$$

for radial velocity v and radial coordinate r. This is linear transport equation that can be solved by finite volume method. We use CIR (Courant-Issacson-Rees) approximation for fluxes with dissipation parameter $\xi = 0.2$. Following boundary conditions are applied: no slip condition on the electrode surface Γ:

$$v|_\Gamma = 0 \quad \left|\frac{\partial p}{\partial r}\right|_\Gamma = 0,$$

and open boundary on the outer edge of the computational domain:

$$\left.\frac{\partial v}{\partial r}\right|_\mathrm{out} = 0 \quad \left.\frac{\partial p}{\partial r}\right|_\mathrm{out} = 0.$$

Figure 1 shows results of calculations for spherical electrode of radius $r_0 = 100\,\mu$m and voltage amplitude $U_0 = 54\,$kV. The electric field has a linear ramp with front time t_0:

$$E(r,t) = \frac{U_0 r_0}{r^2}\frac{t}{t_0}.$$

Hydrostatic pressure P, electrostrictive pressure P_E, total pressure P_tot, and velocity of the fluid as a function of distance from the electrode for $t = t_0 = 100\,$ns and $t = t_0 = 3\,$ns are shown in Figure 1. It is clear that for the rapid rise of voltage, the liquid does not have enough time to move and only small changes in hydrostatic pressure are induced, that cannot compensate the electrostrictive force. On the other hand, for slow voltage rise, the fluid is able to move to the electrode, which leads to an increase of the hydrostatic pressure. As a result, the total pressure does not reach the critical value to initiate discontinuity in the fluid.

The hydrodynamic model allows to find time dependence for pressure and velocity in the liquid for given electrode radius and voltage pulse. Thus we are able to identify conditions favourable for cavitation voids generation.

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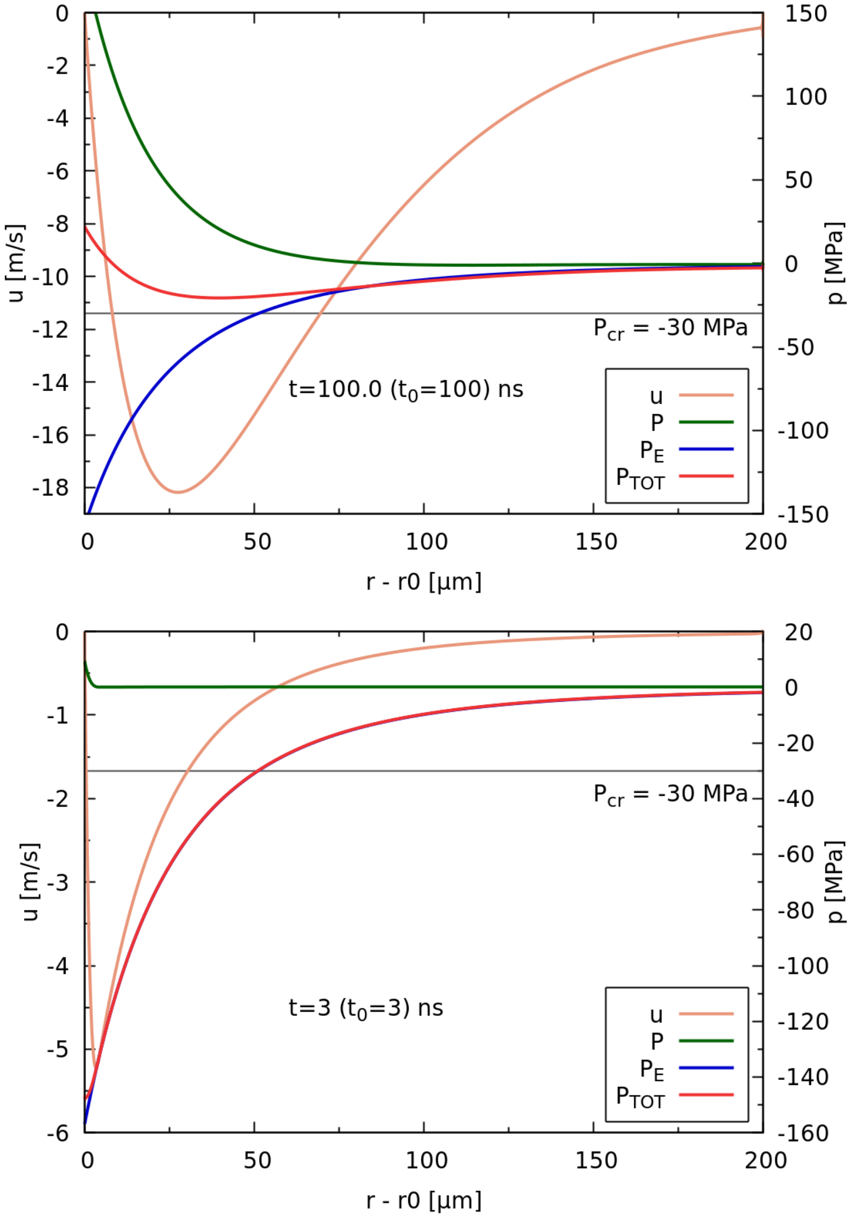

Figure 1: *Hydrostatic pressure P, electrostrictive pressure P_E, total pressure P_{tot}, and velocity of fluid as a function of distance from the electrode at (Top): $t = t_0 = 100$ ns, (Bottom): $t = t_0 = 3$ ns. Horizontal line indicates critical pressure $P_{cr} = -30$ MPa. Negative values of the velocity corresponds to the motion of the fluid towards the electrode.*