Advanced homogenization techniques in a tokamak plasma medium with ellipsoidal blobs: Mathematical treatment

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Abstract

Homogenization of a dielectric mixture is not a new concept, as it dates back to 1996 [2] for plasma mediums. All previous models that have been made have significant limitations. The most basic existing model ignores the shape of the blobs [1],[2],, while the more advanced ones fall short of correct predictions if the wavelength of the incoming beam is not much greater than the radius of the insert dielectric. We present a new formalism, tailored for magnetized plasmas which makes use of quantities that are known under Fourier transformations, such as Green's function and electric fields. This leads to a valid equation for any wavelength and blob (insert) size, which will need no modification if the filling ratio exceeds 50% (current formalisms need to invert the definitions of ambient plasma medium and blob in order to be valid). Finally, the equation can in turn be integrated in the Fourier space and then solved numerically to give the components of the dielectric tensor of the composite plasma medium. Results are only dependent upon (but not limited by) blob size, and there are no restrictions on wavelength magnitude.

Introduction

Radio frequency (RF) waves are extensively used for plasma heating, current drive and suppression of neo-classical tearing modes. RF waves have to propagate through frequently turbulent plasma edge before they reach their target. The edge is populated by filamentary and blob-like structures of different shapes and density contrast. As a result the plasma permittivity of these structures differs from the permittivity of the background plasma. This leads to a modification of the propagation characteristics of the incident RF beam. Thus, an estimate of an effective permittivity that can successfully characterize the turbulent edge as a whole is necessary. Furthermore, due to the statistical nature of the turbulent structures (shape, size, contrast), the estimation method must be such as to be easily amenable to statistical treatment. Briefly, the overall endeavor consists of the following three steps. The first step is the subject matter of the present work. In the second step the probability distribution of the geometrical characteristics, the contrast and the respective filling ratios are suitably provided (as close to experimental observations as possible) and a statistically averaged homogenized permittivity is estimated as general as possible. In the last step the latter is used for a turbulent layer populated by blobs. In a scheme which involves the so-called 4×4 matrix technique for a beam. The beam can easily be constructed as a superposition of as many planar waves as necessary. In the present work we develop a homogenization method capable of encompassing the geometrical characteristics of an aggregate of identical structures (as far as their shape, size and contrast are concerned) immersed in the ambient magnetized plasma of the edge. The blobs are ellipsoidal structures of given size of their three independent semi-axes and density contrast , aligned along the ambient magnetic field and immersed in the ambient magnetized plasma that is considered cold. The filling ratio at this stage is a free parameter. In this paper we will briefly discuss a new method that is proposed to overcome the long wavelength limitations of previous formalisms, while also "smoothing out" the behaviour of the tensor components with respect to the filling ratio.

Proposed method

The electric field solution of the dyadic Maxwell equations for a dielectric medium with a single embedded blob is [1]:

$$e = [e]_P + \int \mathbf{G}_P(\rho - \rho')\mathbf{Q}_{equiv}(\rho')d^3\rho' \tag{1}$$

The above equation has been the basis form many previous homogenization methods. However, no other formalism tackles the problem without making approximations from the onset of the method, thus limiting its effectiveness and area of application. The main approximation was that previous formalisms only took into account the contribution of (1) in the center of the blob $\rho = 0$. This hides both the long wavelength approximation (small blob size compared to wavelength of incoming beams) and disregard for blob shape (they are considered very small spheres in previous methods). This limits the application of the results severely, making the methods inapplicable in the "grey area" of blob ratio 35%-65%. Our proposed method tackles all of these problems by not making approximations, instead engineering an equation based on 1 and utilizing Fourier space properties. This leads to smooth behaviour of the solution over all filling ratios, as will be illustrated below.

Our aim is to construct an equation that readily gives the required results for all volume fractions, as well as in the cases of 0 and 100% blob volume. To this aim, supposing that all blobs are aligned (elongated towards the external magnetic field axis) we propose the modification of 1 to the following one.

$$e = (1-\sigma)([e]_P + \int \mathbf{G}_P(\rho-\rho')\mathbf{Q}_{equiv,P}(\rho')d^3\rho') + \sigma([e]_B + \int \mathbf{G}_B(\rho-\rho')\mathbf{Q}_{equiv,B}(\rho')d^3\rho') \tag{2}$$

Where $\mathbf{Q}_{equiv,P,B}$ are given by

$$[\mathbf{N}(\partial) + i\mathbf{K}_B]\cdot e = \mathbf{Q}_{equiv,B} \tag{3}$$

$$\mathbf{Q}_{equiv,B} = i(\mathbf{K}_P - \mathbf{K}_B)e, \rho \notin V_E$$

$$\mathbf{Q}_{equiv,B} = 0, \rho \in V_E$$

The indices P and B denote, as usual, plasma and blob respectively. Note that the area where $\mathbf{Q}_{equiv,B} = 0$ is the area where $\mathbf{Q}_{equiv,P} \neq 0$ and vice versa. It is useful to discuss derivation of Eq.2 at this point. Assuming all blobs are aligned with the external magnetic field, we approximate the solution of Eq.(1) with the sum of Eq.(1) multiplied by the volume fraction of the plasma, and the solution of its "conjugate" (interchanging plasma and blob indices) multiplied by the volume fraction of the blobs. Equation (2) is engineered to not need modifications for volume fractions greater than 50% , include correct results for the case of one blob of practically zero volume ($\sigma = 0$ and $\sigma = 1$ respectively) and finally , include both the effects of the plasma medium on the blob and the opposite. It is not proposed to be an exact rigorous mathematical solution. Instead, it is the basis upon we attempt to formulate a more exact homogenization method than any preceding it. It is based on (1) , a proven and mathematically rigorous solution. After justification of the proposed equation, it is time to transform it into a more usable form. Utilizing Fourier transform, the equation becomes:

$$(1-\sigma)(\mathbf{K}_H - \mathbf{K}_P)\cdot([e]_P + \int \mathbf{G}_P(\rho-\rho')\mathbf{Q}_{equiv,P}(\rho')d^3\rho') + \sigma(\mathbf{K}_H - \mathbf{K}_B)\cdot([e]_B + \int \mathbf{G}_B(\rho-\rho')\mathbf{Q}_{equiv,B}(\rho')d^3\rho') - i[(1-\sigma)\mathbf{Q}_{equiv,P}(\rho)H(\rho \in V_E) + \sigma\mathbf{Q}_{equiv,B}(\rho)H(\rho \notin V_E)] = 0 \tag{4}$$

Where H denotes the Heaviside step function, valued 1 when the condition in its argument holds, and 0 elsewhere. Fourier transforming the above equation finally gives

$$(1-\sigma)(\mathbf{K}_H - \mathbf{K}_P)\cdot([\hat{e}]_P + i\hat{\mathbf{G}}_P(q)\cdot(\mathbf{K}_P - \mathbf{K}_B)\cdot F(q)) + \sigma(\mathbf{K}_H - \mathbf{K}_B)\cdot([\hat{e}]_B + i\hat{\mathbf{G}}_B(q)\cdot(\mathbf{K}_B - \mathbf{K}_P)\cdot F(q)) + [(1-\sigma)(\mathbf{K}_P - \mathbf{K}_B)\cdot F(q) + \sigma(\mathbf{K}_B - \mathbf{K}_P)\cdot F(q)] = 0 \tag{5}$$

where $F \equiv 4\pi \int \frac{sinc|\mathbf{E}^{-1}\cdot(q-q')| - cos|\mathbf{E}^{-1}\cdot(q-q')|}{|\mathbf{E}^{-1}\cdot(q-q')|^2}\hat{e}(q')d^3q'$.

Utilizing (5) to express the electric fields as functions of wavenumber and dielectric tensor components, and then integrating 5 over the domain of q provides a system of three equation with 3 unknown variables, which are the components of the dielectric tensor $\mathbf{K}_H$. Solving this system provides the necessary solutions for the homogenized tensor.

Conclusion

We have proposed a new method that attempts to tackle the shortcomings of previous homogenization formalisms. Results, which are promising and behave more smoothly for different filling ratio of the blobs, are to be presented in poster session of EPS 2018 in Prague.

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