

3D Equilibrium Reconstruction with Islands¹

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Reconstruction is an inverse method to determine unknown information from known sources. Using a known model, parameter space is searched until an optimal point is reached where synthetic signals match observations. This technique is widely applied to fusion in the form of equilibrium reconstruction. From an equilibrium model, unknown parameters such as current and pressure profiles are determined by matching synthetic signals to measured observations. Equilibrium reconstruction has its origins in axisymmetric reconstruction of tokamak equilibria using solutions of the Grad-Shafranov equation[1].

Symmetry breaking 3D effects in tokamaks[2] and stellarators[3] drove a need to move beyond the axisymmetric limit. By assuming closed nested flux surfaces, VMEC[4] can solve for a 3D equilibrium provided current and pressure profiles, and a boundary. V3FIT[5] searches the VMEC parameter space to reconstruct nested 3D equilibria[6]. Topology breaking effects caused by error fields, edge localized modes, disruptions, or even man-made island divertors[3], break the nested flux surface assumptions of VMEC motivating a need to move beyond the nested flux surface limit. SIESTA[7] is an equilibrium solver that allows for non-nested and stochastic fields. When coupled to V3FIT[8], island and non-nested equilibria can now be reconstructed by searching the SIESTA parameter space. The remainder of this paper will discuss the coupling of SIESTA to V3FIT and its application to reconstruct an island equilibrium in the DIII-D tokamak.

The forward model in equilibrium reconstruction is a coupling of an equilibrium model and synthetic signal models (Fig. 1). The equilibrium is solved then the result of the equilibrium model is used to model the synthetic signals. SIESTA uses an existing VMEC equilibrium as an initial state and a background computational coordinate system. SIESTA applies a helical perturbation to break the nested flux surfaces. The size of this perturbation parameter controls the size of the

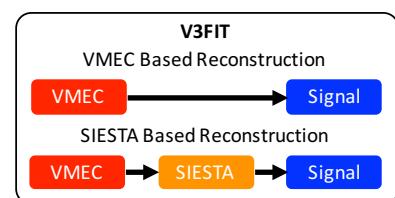


Fig 1. Model coupling of VMEC and SIESTA in V3FIT.

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resulting islands. For island equilibrium reconstruction, the complete forward model is a coupling of VMEC, SIESTA, and synthetic signal models (Fig. 1).

Certain signals like magnetic diagnostics, can be modeled using quantities provided solely by the equilibrium. Other diagnostic signals that measure density, temperature, and soft x-ray emission require additional parameterized profiles of density, temperature and soft x-ray emission. As a result for effective island equilibrium reconstructions, the unknown parameters at all levels of the forward model from the underlying VMEC, to SIESTA parameters, to the model profiles are reconstructed.

For nested flux surface equilibria, density, temperature and soft x-ray emission profiles can be mapped to the normalized flux as a natural radial coordinate. However for island equilibria, flux surfaces are no longer guaranteed or may not align with the radial coordinate of the computational grid. SIESTA solves for a new equilibrium pressure to ensure force balance with perturbed fields. In the presence of a magnetic island, the pressure flattens providing a way to locate magnetic islands. For island equilibria, profiles of density, temperature and soft x-ray emission can be mapped to the normalized pressure (Fig. 2). This provides a simple efficient way to map quantities to the equilibrium.

A reconstruction of DIII-D shot number 154921 at 2530ms was used to demonstrate island based reconstruction. This inner wall limited L-mode shot with low torque had a $n=1$ radial error field applied with a 5° phase by the C-coil set. Temperature diagnostic measurements show flattening across the channels indicating the presence of an island structure. To reconstruct this equilibrium, all available diagnostics modeled in V3FIT were employed including 191 magnetic diagnostics, 15 MSE channels, 67 SXR, 4 interferometry cords, 30 Thomson n_e channels, 30 Thomson T_e channels, 38 ECE channels, a Pseudo signal ensuring edge consistency, and a limiter function.

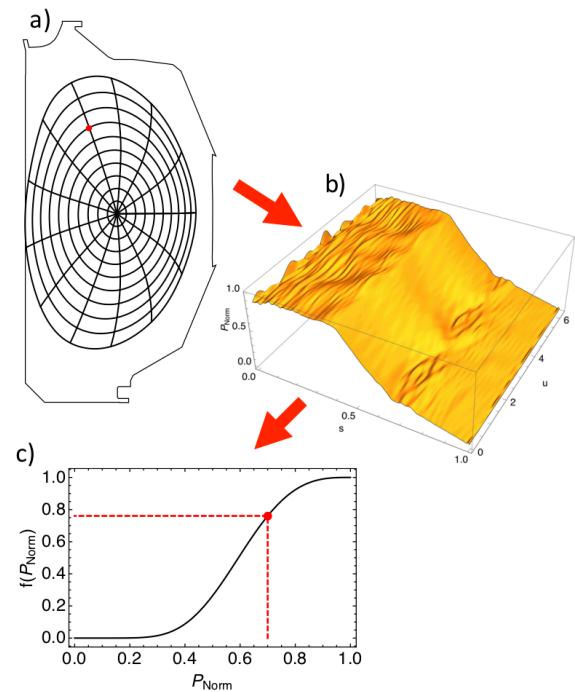


Fig 2. a) A position in laboratory space is converted to the SIESTA coordinate system. b) The SIESTA coordinates are used to evaluate the normalized pressure. c) Profile quantity is evaluated from the normalized pressure map.

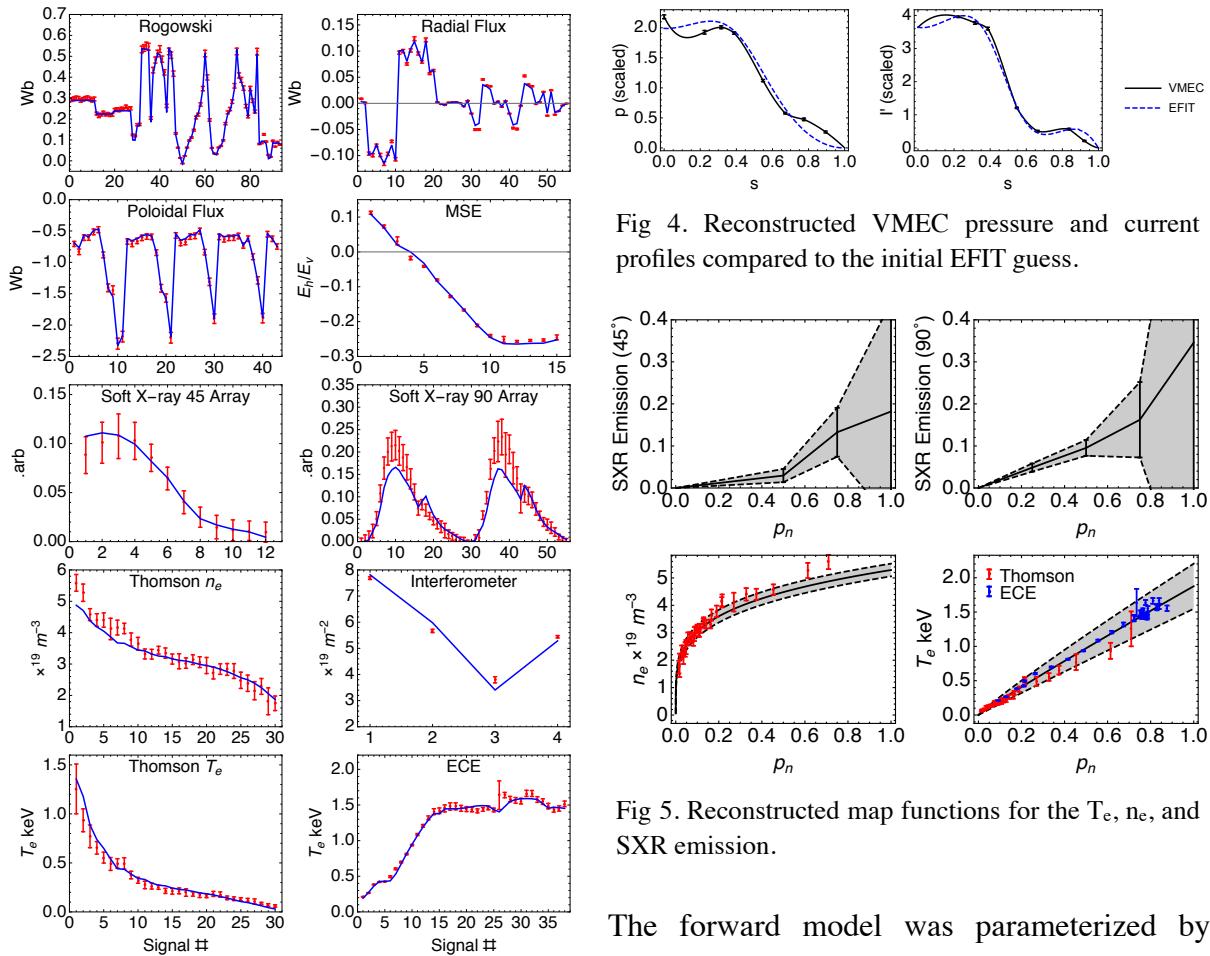


Fig 3. Reconstructed model signal (blue line) vs. the observed signal (red points).

walls. To ensure the best initial conditions for siesta, the pressure and current profiles were parameterized using Akima splines employing 8 free parameters each. The edge toroidal flux ensures the initial VMEC equilibrium remains within the bounds of the limiter. SIESTA was parameterized by the size of $m=2$ helical perturbation. Map functions of density and temperature were parameterized as a power function $f(p_n) = \alpha p_n^\beta$ with two free parameters (α, β) each. Two soft x-ray emission profiles were parameterized by straight line segments using 3 free parameters for the filter with the smaller number of channels and 4 free parameters for the filter with the larger number channels. In total, the 377 signals with 52 free parameters ensures the system is over specified reducing the possibility of multiple minima. Using 52 Cori Haswell Nodes at NERSC, this reconstruction took 7.5 hours to complete.

Figure 3 shows at the optimal position in parameter space, V3FIT is able to match synthetic signal models to their observed quantities. Soft x-ray emission in the core show worse agreement than in the edge indicating either a hollow profile or an island structure not

Fig 4. Reconstructed VMEC pressure and current profiles compared to the initial EFIT guess.

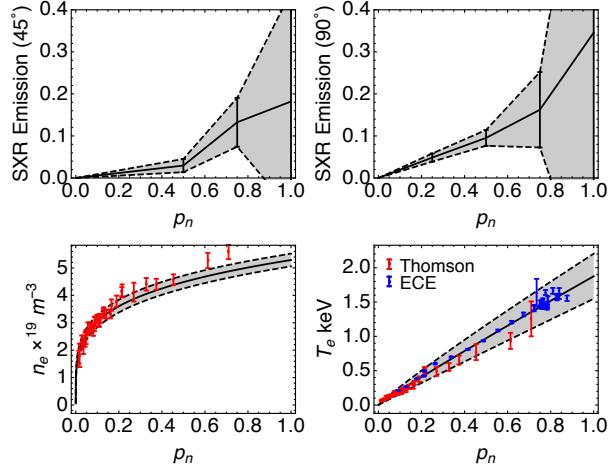


Fig 5. Reconstructed map functions for the T_e , n_e , and SXR emission.

The forward model was parameterized by VMEC parameters for the 18 F-Coil currents, 2 E-Coil Currents, and 3 C-coil currents to account for eddy currents in the vacuum vessel

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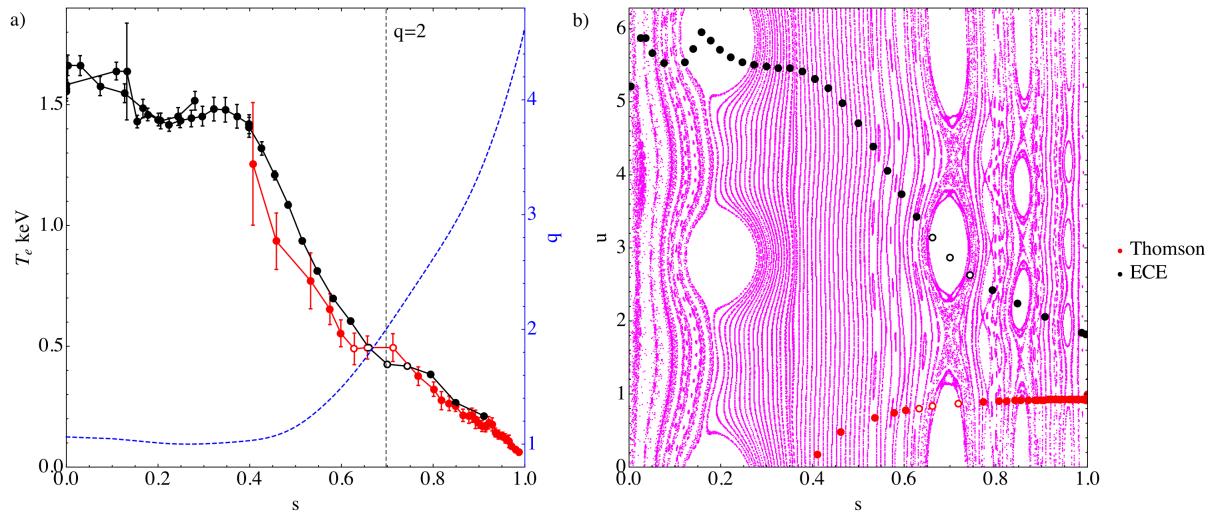


Fig. 6. a) Thomson and ECE electron measurements overlaid on the VMEC q profile. Flat spots in the temperature line up with the $q=2$ surface. b) Poincaré plot of equilibrium fields with T_e diagnostics mapped to the field lines.

achievable with the chosen parameterization. This is also reflected in the large uncertainty of SXR emission profiles in the core, $p_n=1$ (Fig. 5). Figure 4 shows the reconstructed initial VMEC pressure and current profiles. Since SIESTA perturbs the fields and pressure to find a new equilibrium, these profiles only represent the optimal initial SIESTA state. Figure 5 shows the reconstructed map functions for the T_e , n_e , and SXR emission. Figure 6a shows that the flat spots in T_e coincide with location of the $q=2$ surface. Figure 6b shows the island size, location, and phase align with flat T_e profiles.

These results represent the first ever reconstruction of island equilibria and demonstrate the practicability of mapping quantities to the normalized pressure. While SIESTA was only configured to apply a $m=2$ perturbation, mode coupling also opened up 3-1 and 4-1 island chains. In the core a 2-2 island was also opened. The author believes this should really be a 1-1 island. However due to the chosen parameterization of SIESTA a 1-1 helical perturbation was not applied. This may account for the poor results in the core SXR emission.

- [1] L.L.Lao, H.St. John, R.D. Stambaugh, A.G. Kellman and W. Pfeiffer *Nucl. Fusion* **25** 1611 (1985)
- [2] I.T Chapman *et al*, *Nucl. Fusion* **54** 083006 (2014)
- [3] T.S. Pedersen *et al*, *Nature Comm.* **7** 13493 (2016)
- [4] S.P. Hirshman and J.C. Whitson, *Phys. Fluids* **26** 3553 (1983)
- [5] J.D. Hanson, S.P. Hirshman, S.F. Knowlton, L.L. Lao, E.A. Lazarus and J.M. Shields, *Nucl. Fusion* **49** 075031 (2009)
- [6] J.D. Hanson *et al*, *Nucl. Fusion* **53** 083016 (2013)
- [7] S.P. Hirshman, R. Sanchez, and C.R. Cook, *Phys. Plasmas* **18** 062504 (2011)
- [8] M. Cianciosa, S.P. Hirshman, S.K. Seal and M.W. Shafer, *Plasma Phys. Control. Fusion* **60** 044017 (2018)