Toroidal momentum maintenance and transport in simulations of nonlinear turbulent convection in tokamak core plasmas

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The paper continues our previous theoretical study and simulations of low-frequency turbulence and the associated cross-field anomalous transport processes in tokamak core plasmas [1-3]. The main goal of the paper is the analysis and simulations of toroidal momentum transport in the presence of self-consistent nonlinear turbulent plasma convection. Contrary to many other studies our simulations are based on adiabatically-reduced MHD-like plasma dynamic model (ARD-model) in which the toroidal plasma flows are the inherent parts of the nonlinear turbulent plasma convection [1, 2]. Code CONTRA-C developed in a frame of simplified cylindrical model for tokamaks was used for this series of simulations.

We have to remind that all physical functions such as electron and ion pressures $p_{e,i}$, plasma density n, electric potential ϕ, dynamic vorticity w in the ARD-model [1, 2] consist of averaged over magnetic surface and fluctuating parts ($\phi = \bar{\phi} + \phi_f$). Correspondingly, the equation set consists of transport equations for the surface averaged values and equations those determine temporal and 2D spatial behavior of the fluctuations. Transport equations include both background (typically neoclassical) fluxes and non-diffusive (anomalous) fluxes those depend on fluctuations. All equations are written in terms of the following flux coordinates: minor plasma radius $\rho = \sqrt{\Psi(r,z,t)/\pi B_0}$, where $\Psi(r,z,t)$ is toroidal magnetic flux and B_0 toroidal field at the magnetic axis, φ is toroidal angle, and θ is poloidal angle.

The equations of motion in our model [1-3] have the form of equations for the dynamic vorticity $w = \bar{w} + w_f$, which is a generalization of the conventional vorticity $\Delta\phi$ in the case of non-uniform density and magnetic field. We assume that the magnetic field in tokamaks is axi-symmetric one with nested magnetic flux surfaces. As a result, there are inherent conservation laws for the toroidal momentum and the dynamic vorticity in this model. Keeping in mind the toroidal momentum conservation, we introduce the specific toroidal momentum of the plasma sheath of thickness $d\rho$:

$$M'(t,\rho) = m_i \bar{n} V' \langle r^2 \rangle \Omega(t,\rho) = m_i \bar{n} V' \langle r^2 \rangle c \frac{h}{\rho}\left(\partial_\rho \bar{\phi}(t,\rho) + \frac{1}{e\bar{n}} \partial_\rho \bar{p}_i(t,\rho) \right), \quad (1)$$

where m_i is ion mass, c is speed of light, prime denotes derivative ∂_ρ, $V' = \oint \sqrt{g} d\theta d\varphi$ is the specific volume of the plasma sheath, $\langle r^2 \rangle = (1/V')\oint r^2 \sqrt{g} d\theta d\varphi$ is the averaged square of the major radius, $\Omega(t,\rho)$ is the frequency of the toroidal plasma rotation, $h = q(\rho)/B_0$, $q(\rho)$ is the safety factor. The definition (1) for $M'(t,\rho)$ accounts the major two-fluids effect that is the contribution of the ion diamagnetic drift velocity (which is proportional to $\partial_\rho \bar{p}_i$) to the specific toroidal momentum. Then, instead of equation for the averaged vorticity $\overline{w}(t,\rho)$ we can write the equation for the specific toroidal momentum:

$$\partial_t M' - \partial_\rho \left(m_i \bar{n} V' \langle r^2 \rangle \left(c \frac{h}{\rho} \right)^2 \left(\overline{\partial_\varphi \phi_f \partial_\rho \phi_f} \right) \right) = \partial_\rho \left(m_i \bar{n} V' \langle r^4 \rangle \left(X \frac{\bar{n}}{\sqrt{T_i}} \right) \frac{h^2}{\rho^2 + \rho_0^2} \partial_\rho \Omega \right) + Q_M . \quad (2)$$

LHS in Eq.(2) includes radial flux of momentum caused by the “Reynolds stress”, while RHS accounts momentum flux due to collisional ion viscosity with factor $X = 0.4\sqrt{\pi m_i} e^2 c^2 \Lambda$ and torque source Q_M, where Λ is Coulomb logarithm, $\rho_0^2 = 4 q_0 \rho_{Li} R$, R is major plasma radius, ρ_{Li} is ion gyro radius at the magnetic axis. The convective momentum flux in LHS of Eq. (2) vanishes at the both boundaries. Boundary condition $\partial_\rho \Omega = 0$ at $\rho = 0$ provides zero viscous fluxes of momentum, dynamic vorticity, and kinetic energy at the axis. If the viscous fluxes of the momentum and the dynamic vorticity also vanish at the external boundary $\rho = \rho_L$ and torque source $Q_M = 0$, the Eq. (2) provides conservation of the total momentum and the integral vorticity:

$$M(t) = \int_0^{\rho_L} M' d\rho = const \ , \qquad W(t) = \frac{h}{\rho} M' \Big|_0^{\rho_L} \equiv m_i \bar{n} \frac{h}{\rho} V' \langle r^2 \rangle \Omega \ \Big|_0^{\rho_L} = const . \quad (3)$$

Thus, the intrinsic toroidal rotation essentially depends on the external boundary condition for the viscous flux of momentum (at $\rho = \rho_L$). Assuming that $\phi = 0$ at the external boundary ($\rho = \rho_L$), plasma potential can be restored by the following expression:

$$\bar{\phi}(t,\rho) = \int_{\rho_L}^{\rho} \left(\frac{\rho}{ch} \Omega(t,\rho) - \frac{1}{e\bar{n}} \partial_\rho \bar{p}_i(t,\rho) \right) d\rho . \quad (4)$$

Temporal evolution of self-sustained plasma turbulence and the resulting anomalous transport processes was simulated for parameters of quasi-steady OH stage in shot #61203 in T-10 tokamak: central plasma density $n_0 \approx 3.1 \cdot 10^{19} m^{-3}$, central temperatures of electrons $T_{e0} \approx 1.1 keV$ and ions $T_{i0} \approx 0.5 keV$, plasma energy confinement time $\tau_E \approx 32$ms.

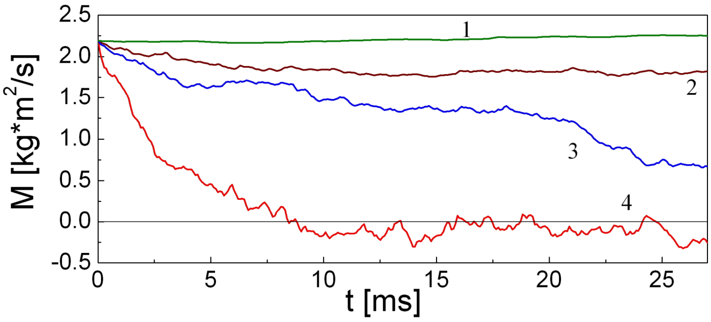


Fig. 1. Evolution of the total toroidal momentum *M*(*t*): 1 – low viscosity regime ($\eta|_{\rho=\rho_L} \approx 0.05 m^2/s$); 2 – viscosity enhancement factor is 10; 3 – enhancement factor is 20; 4 – enhancement factor is 50.

We assume also the standard hydrodynamic condition of "boundary layer" $\Omega(t,\rho)|_{\rho=\rho_L} = 0$.

The simulations were performed for various initial values of the total toroidal momentum *M*(*t*=0). Temporal evolution of the total toroidal momentum *M*(*t*) is shown at Fig. 1, which demonstrates that *M*(*t*) is still almost constant (curve 1) in the case of sufficiently low kinematic ion viscosity (which corresponds to the above mentioned plasma parameters), in spite of nonzero momentum flux through the surface $\rho = \rho_L$, while in regimes with enhanced viscosity (curves 2, 3, 4) *M*(*t*) decreases with time and then oscillates near zero level. For another positive and negative initial values of *M*(0), *M*(*t*) demonstrates the similar behavior.

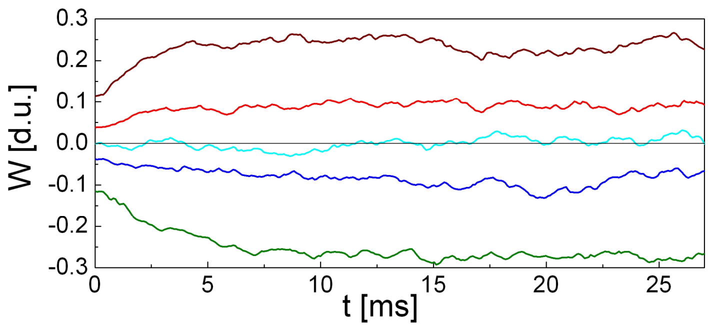
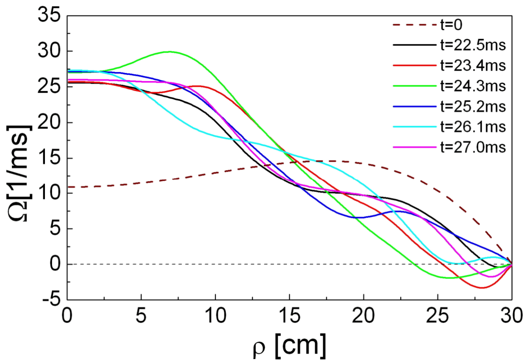


Fig. 2. Evolution of the integral dynamic vorticity *W*(*t*) for 5 different initial values of the toroidal momentum *M*(*t*=0) in the case of low viscosity $\eta|_{\rho=\rho_L} \approx 0.05 m^2/s$.

Fig. 2 shows that in the case of the low ion viscosity the integral vorticity *W*(*t*) tends to a finite quasi-steady level and then stochastically oscillate near this level. Simulations also have shown, that in regimes with the enhanced ion viscosity *W*(*t*) decreases similarly to *M*(*t*) and then oscillates near zero level.

Maintenance of a quasi-steady intrinsic toroidal rotation in the presence of viscous friction at the boundary and without a torque source seems a little bit surprising. The physical reason of the quasi-steady maintenance of the toroidal momentum *M*(*t*) in regimes



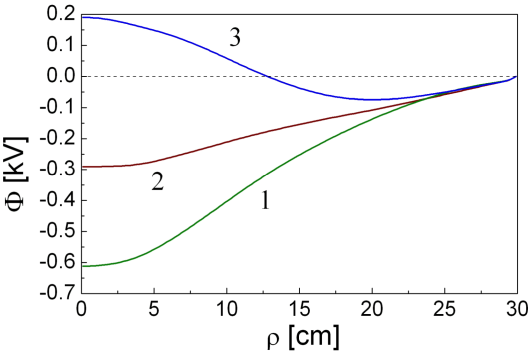


Fig. 3. Profiles of $\Omega(t,\rho)$ in various moments of time in regime with low viscosity ($\eta|_{\rho=\rho_L} \approx 0.05 m^2/s$) and initial value of total momentum $M(t) \approx 2.2\cdot10^{-3} kg\cdot m^2/s$.

Fig. 4. Profiles of the plasma potential $\bar{\phi}(t,\rho)$ for various $M(t=0)$: 1 – $M(t) \approx 2.2\cdot10^{-3} kg\cdot m^2/s$; 2 – $M(t) \approx -2.2\cdot10^{-3} kg\cdot m^2/s$; 3 – $M(t) \approx -8.8\cdot10^{-3} kg\cdot m^2/s$.

with low viscosity is illustrated by Fig. 3, which shows profiles of $\Omega(t,\rho)$ in various moments of time. It is seen that $\partial_\rho\Omega$ oscillates near the external boundary with a characteristic frequency which is higher than the dissipative rate in the low viscosity regimes. As a result, the time-averaged flux of the toroidal momentum at the external boundary can vanish. Thus, the simulations reveal a new possible mechanism of quasi-steady maintenance of the intrinsic toroidal rotation in regimes with finite, but the sufficiently low plasma viscosity.

Profiles of the plasma potential for 3 initial values of toroidal momentum in the case of the low ion viscosity are shown at Fig. 4. It is seen that due to the presence of the diamagnetic ion drift in Eq. (4) there is an asymmetry between profiles, which correspond to positive and negative momentums. Typically, in the most experiments, plasma potential is negative at the axis. Our simulation gives the similar result. However, in the case of the sufficiently high negative momentum the potential can become a slightly positive one (curve 3 at Fig .4).

References

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