RF beam scattering by cylindrical filaments and interfacial density fluctuations (*)

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Radio Frequency (RF) waves are routinely used in tokamaks for heating, current drive, NTM control, as well as for diagnostics purposes. Frequently, RF waves, aiming towards the plasma core, propagate through a turbulent environment. The latter can exhibit strong coherent density fluctuations as well as filamentary structures mainly (though, not perfectly) aligned along the local magnetic field lines. The scattering process of RF waves by these structures is studied both analytically and numerically. RF waves can be either single plane waves or spatially confined beams. For that purpose, the filaments are considered to have cylindrical shape with infinite length with the cylinder axis not aligned with the local magnetic field and the results are compared to the ones from the study of the aligned case [1, 2]. On the other hand, the interfacial density fluctuations are considered periodic with spatial periods larger, smaller or of the same order of the wavelength of the incident RF waves. The frequency range of the RF waves studied is mainly in the Electron Cyclotron (EC) range of frequencies for ITER-like and Medium Size Tokamak applications. Furthermore, the study covers a variety of density contrasts, filament sizes and fluctuation strengths.

(a) Main assumptions and the geometry

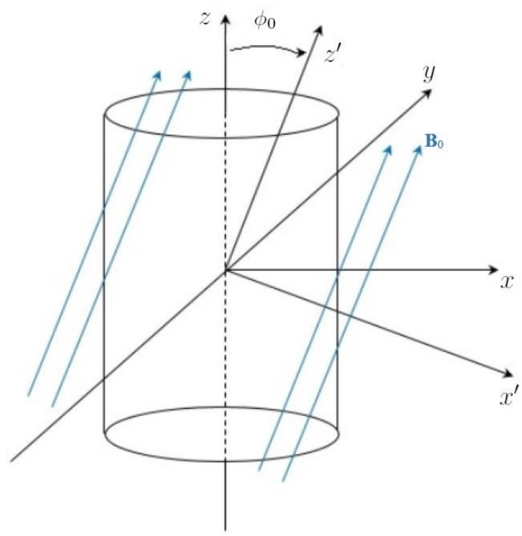

Figure 1: Magnetic field and cylinder coordinate systems

The axis of the cylindrical filament is not necessarily parallel to the externally imposed magnetic field $\mathbf{B}_0$, but in angle ϕ_0 (Figure 1). The magnetic field lines are parallel to the *(z-x)*-plane, with *z* being the axis of the cylinder and *z'* parallel to the magnetic field lines. The cylinder is considered to have infinite length and the plasma is assumed to be homogeneous and cold. Plane waves (or Gaussian beams) propagating in the ambient plasma region (with different electron density from the filament plasma region), are hitting the cylindrical filament and are being scattered. It should be also mentioned, that that the incident propagating waves have their own orientation.

(b) Analytical theory and results for plane wave scattering by a cylindrical filament

After making appropriate rotations and coordinate transformations, Faraday-Ampere equation in the Fourier domain is used, with the electric field being analyzed in the cylindrical vector functions. Dispersion relation is calculated by using the permittivity tensor and then solved. Boundary conditions are also used and finally, for the normalized with respect to the incident fields, dimensionless, independent of z (denoted by a tilde) electric field and magnetic field respectively, in cylindrical coordinates one obtains:

$$\tilde{\mathbf{e}}(\rho,\varphi)_{(FI,SC)} = \sum_{M=O,X} \sum_{m=-\infty}^{m=\infty} i^m \mathrm{e}^{im\varphi} \left[\mathcal{E}_{mr}^{M}(\rho)\hat{\mathbf{r}} + \mathcal{E}_{m\varphi}^{M}(\rho)\boldsymbol{\varphi} + \mathcal{E}_{mz}^{M}(\rho)\hat{\mathbf{z}} \right]_{(FI,SC)} \tag{1}$$

$$\mathbf{h}(\rho,\varphi)_{(FI,SC)} = \frac{E_0}{H_0}\sqrt{\frac{\varepsilon_0}{\mu_0}} \sum_{M=O,X} \sum_{m=-\infty}^{m=\infty} i^m \mathrm{e}^{im\varphi} \left[\mathcal{H}_{mr}^{M}(\rho)\hat{\mathbf{r}} + \mathcal{H}_{m\varphi}^{M}(\rho)\boldsymbol{\varphi} + \mathcal{H}_{mz}^{M}(\rho)\hat{\mathbf{z}} \right]_{(FI,SC)} \tag{2}$$

(SC: scattered field, FI: field inside the filament and 0: incident field) where $\mathcal{E}_{mr}^{M}(\rho)$, $\mathcal{E}_{m\varphi}^{M}(\rho)$, $\mathcal{E}_{mz}^{M}(\rho)$, $\mathcal{H}_{mr}^{M}(\rho)$, $\mathcal{H}_{m\varphi}^{M}(\rho)$ and $\mathcal{H}_{mz}^{M}(\rho)$ are functions only of ρ and can be calculated by an appropriate mathematical analysis. As expected, since the cylinder blob has infinite length, the results for the electric field and magnetic field are z independent. It should be also noted that the parallel to the cylinder axis wave vector component stays the same for all regions. From equations (1) and (2) the calculated Poynting vector can be easily calculated and the aligned filament's case (with respect to the externally imposed magnetic field) can be compared to the inclined (Figure 2). Furthermore, by Fourier transforming the Poynting vector, one can get a polar diagram of the normalized spectral amplitude as a function of the horizontal and vertical projections of the Fourier mode number, normalized to the incident index of refraction (Figure 3). The peak (dot) in the center of the diagram (point (0, 0)), corresponds to the spectrum of the incident wave. Because of the filament, the spectral amplitude is distributed as a cardioid (instead of a single dot) due to the presence of the filament. Note that the maximum of the spectral density is on the forward direction. However, there is significant scattering at the filament's sides.

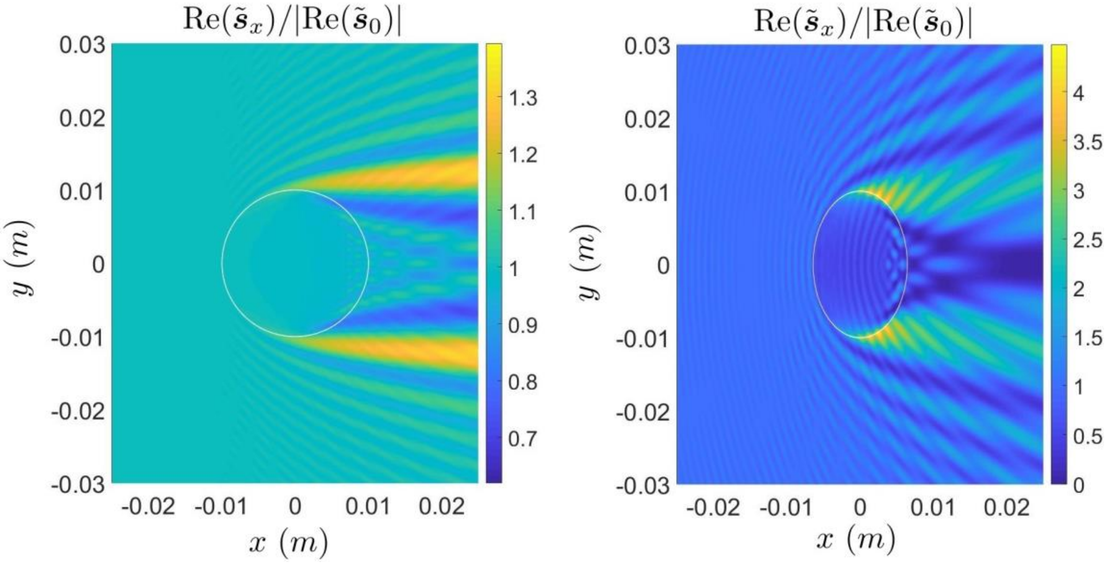

Figure 2: Poynting flux in the forward direction, f = 170 GHz (EC), polarization of incident wave: O-mode, r = 10 mm, ambient density 10^{19} m^{-3}, filament's density 2.0×10^{19} m^{-3}, B = 4.5 T with inclination 0º (left) and 50º (right)

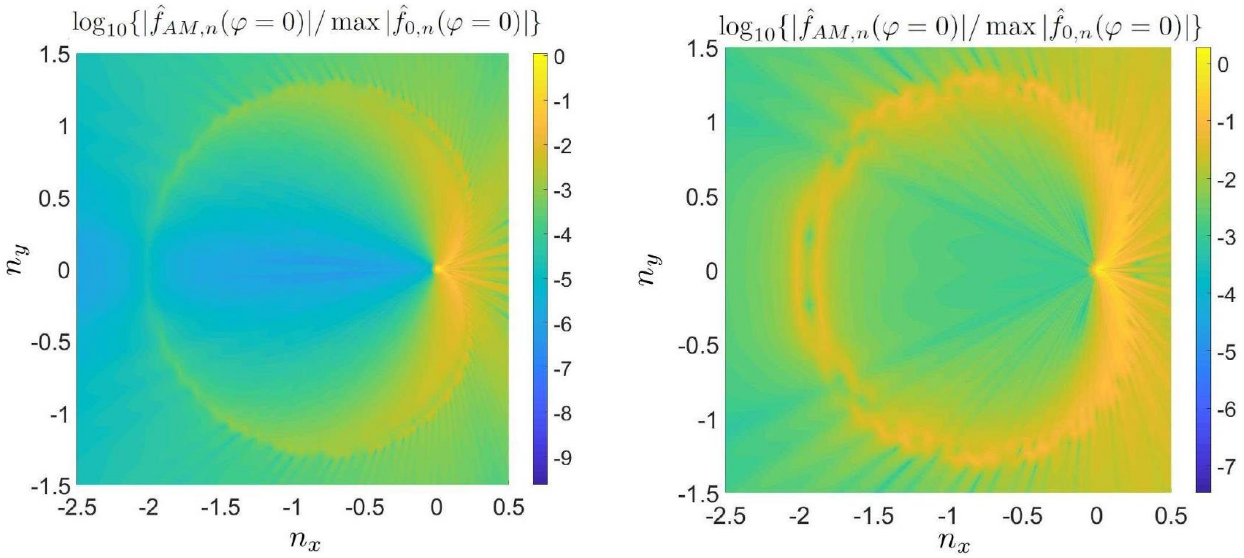

Figure 3: Poynting flux in the Fourier domain, f = 170 GHz (EC), polarization of incident wave: O-mode, r = 10 mm, ambient density 10^{19} m^{-3}, filament's density 2.0×10^{19} m^{-3}, B = 4.5 T with inclination 0º (left) and 50º (right)

(c) Gaussian beam scattering by a cylindrical filament

A Gaussian beam can be approximated by a number of plane wave modes with parallel to the cylinder axis wave vector components and azimuths around a central plane wave mode. In

Figure 4, a case of Gaussian beam that hits the aligned to the externally imposed magnetic field cylindrical filament with non-zero eccentricity scattering appears.

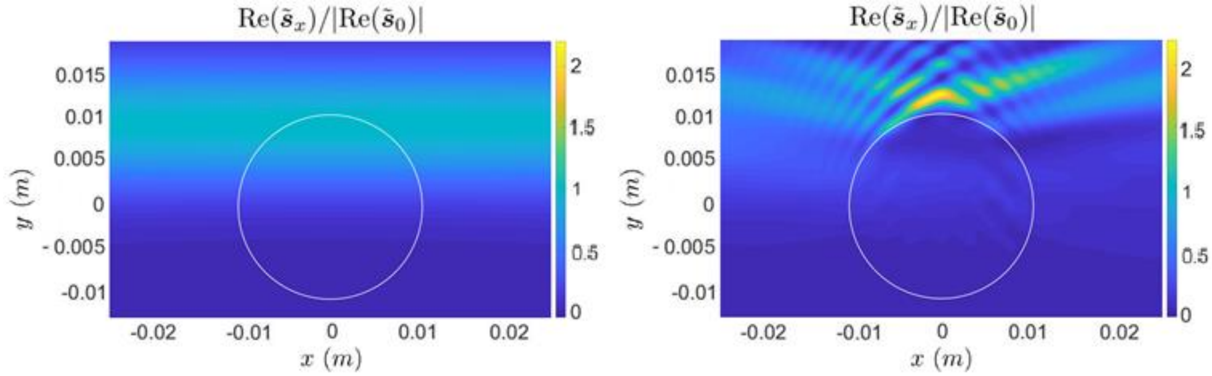

Figure 4: Poynting flux in the forward direction (left: incident beam, right: scattered beam), $f = 170$ *GHz* (EC), O-mode, $r = 10$ *mm*, ambient density 2.0×10^{20} m^{-3}, filament's density 3.0×10^{20} m^{-3}, $B = 4.5$ *T* with inclination 0°

(d) Sinusoidally modulated interfaces

The interfacial density fluctuations on both regions (filament and ambient plasma) can be considered periodic, e.g. sinusoidal. By considering the periodicity along the *y*-axis and independent of *z,* Figure 5 shows the *x*-component of the Poynting flux.

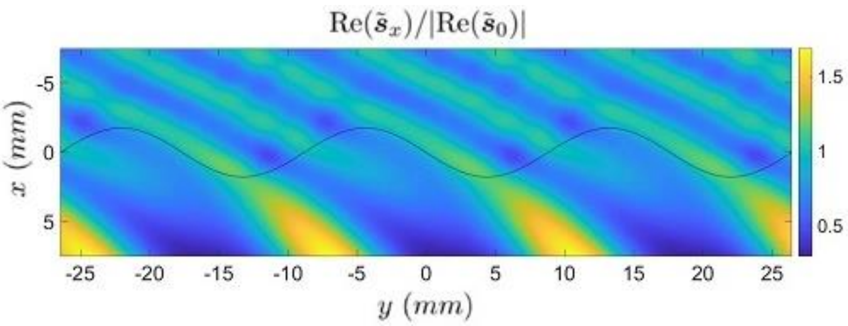

Figure 5: Poynting flux *x*-component for scattering on sinusoidally modulated interface, $f = 170$ *GHz* (EC), O-mode, ambient density 3.0×10^{20} m^{-3}, filament's density 3.2×10^{20} m^{-3}, $B = 4.5$ *T* with inclination 0°

References

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[2] Z. C. Ioannidis, A. K Ram, K. Hizanidis, I. G. Tigelis, "Computational studies on scattering of radio frequency waves by density filaments in fusion plasmas", Physics of Plasmas 24, 102115 (2017)

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