Shaping effects on the interaction of shear Alfvén and slow sonic continua

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Alfvénic instabilities can cause energetic-particle redistribution that could be detrimental for tokamak operation. Understanding the equilibrium conditions that admit Alfvénic modes and which particles can drive them unstable is fundamental to confine and ignite a fusion plasma. Our aim is to investigate low frequency magnetic activity, experimentally observed, whose properties are not yet well established [1].

To achieve this purpose the linearised MHD model is used to address the plasma equilibrium and compute the possible eigenmodes of the plasma configuration (`HELENA` [2], `CASTOR` [3]). A non-perturbative drift-kynetic approach is employed to access the energy transfer occurring between waves and ions in order to analyse the mode stability (`CASTOR-K` [4]).

Assuming a displacement of the kind $\xi = \sum_m \xi_m(r)\exp[i(\omega t + m\theta - n\phi)]$ the linearised MHD equations can be written as a system of differential matrix operators acting on the perturbed pressure and the three components of the displacement as done in [5]. The frequencies that solve

$$\begin{bmatrix} \frac{\omega^2\rho|\nabla\psi|^2}{B^2} + \mathbf{B}\cdot\nabla\big(\frac{|\nabla\psi|^2\mathbf{B}\cdot\nabla}{B^2}\big) & \gamma P \kappa_g \\ \kappa_g & 1+\gamma P[\frac{1}{B^2} + \frac{1}{\omega^2\rho}\mathbf{B}\cdot\nabla\big(\frac{|\nabla\psi|^2\mathbf{B}\cdot\nabla}{B^2}\big)\end{bmatrix}\begin{bmatrix}\xi_s \\ \nabla\cdot\xi\end{bmatrix} = 0 \qquad (1)$$

where $\kappa_g = 2\kappa\cdot\big(\frac{\mathbf{B}\times\nabla\psi}{B^2}\big)$ is the geodesic curvature and $\kappa = (\mathbf{b}\cdot\nabla)\mathbf{b}$, are the ones at which MHD eigenmodes would be singular and thus damped. When the harmonics of the displacement are decoupled all frequencies intersect a singularity line (MHD continua) and modes cannot be present in the plasma. Instead, if branches interact, frequency gaps can be opened and discrete eigenmodes allowed. The geodesic curvature couples Shear Alfvén and Sonic continua in equation (1) and the harmonic content of κ_g allows the interaction among branches with different poloidal mode numbers.

An equilibrium with circular magnetic surfaces couples a m_A branch with $m_S = \pm 1$ ones [6]. The magnetic flux parametrization presented in [7] as

$$\psi(r,\theta) = S_0 r^2[\Theta_0(\theta) + \varepsilon r\Theta_1(\theta) + \varepsilon^2 r^2\Theta_2(\theta)]$$
$$\Theta_0(\theta) = 1 + \hat{\kappa}\cos(2\theta) - \check{\kappa}\sin(2\theta),$$

with Θ_1, Θ_2 other angular functions, permits to compute a κ_g with higher harmonic content with respect to the circular case, that is

$$\kappa_g = A\varepsilon\big(K_g[0] + K_g[1]e^{i\theta} + K_g[2]e^{i2\theta} + K_g[3]e^{i3\theta} + cc.\big).$$

The constant $A\varepsilon$ will disappear when solving the continuum equation and higher terms in κ_g are negligible for usual tokamak parameters. We report the lowest order terms in inverse aspect ratio:

$$K_g[1] \sim i\left(-1+\frac{3}{4}\hat{\kappa}+\frac{1}{16}\hat{\kappa}^2+\frac{3}{4i}\check{\kappa}+\frac{9}{128i}\check{\kappa}\hat{\kappa}^2\right)$$
$$K_g[3] \sim i\left(\frac{1}{4}\hat{\kappa}+\frac{5}{32}\hat{\kappa}^2+\frac{5}{16i}\check{\kappa}\hat{\kappa}+\frac{3}{128i}\check{\kappa}\hat{\kappa}^2\right)$$
$$K_g[0] \propto \varepsilon, K_g[2] \propto \varepsilon, K_g[4] \propto \varepsilon, \cdots$$

The harmonic content, increased by shaping, couples branches with a difference in the poloidal mode number greater than 1. The coupling creates tiny gaps in the continuum where MHD eigenmodes can be found.

The `HELENA` code calculates numerically equilibria with JET-like parameters and the `CASTOR` code provides the continuum spectrum of the frequencies normalized as $\tilde{\omega} = \frac{\omega}{\omega_A(0)}$ and the mode amplitude. The gap in figure 1 is generated by the interaction between a shear branch

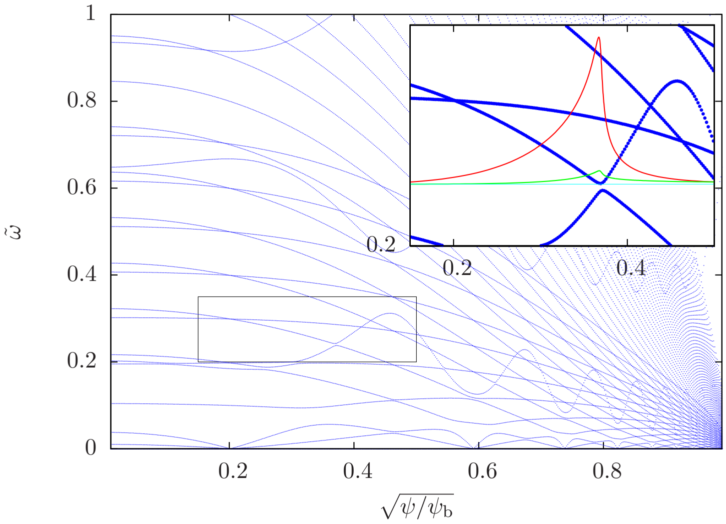


Figure 1: Continuum spectrum for JET-like equilibrium with $n = 3$ and gap eigenmode in the inset.

with $m_A = 4$ and a sonic with $m_S = 7$, so that $\delta = | m_A - m_S | = 3$.

The effect of shaping is illustrated in figure 2, where the parameter $\hat{\kappa}$, weighting $\cos(2\theta)$ in

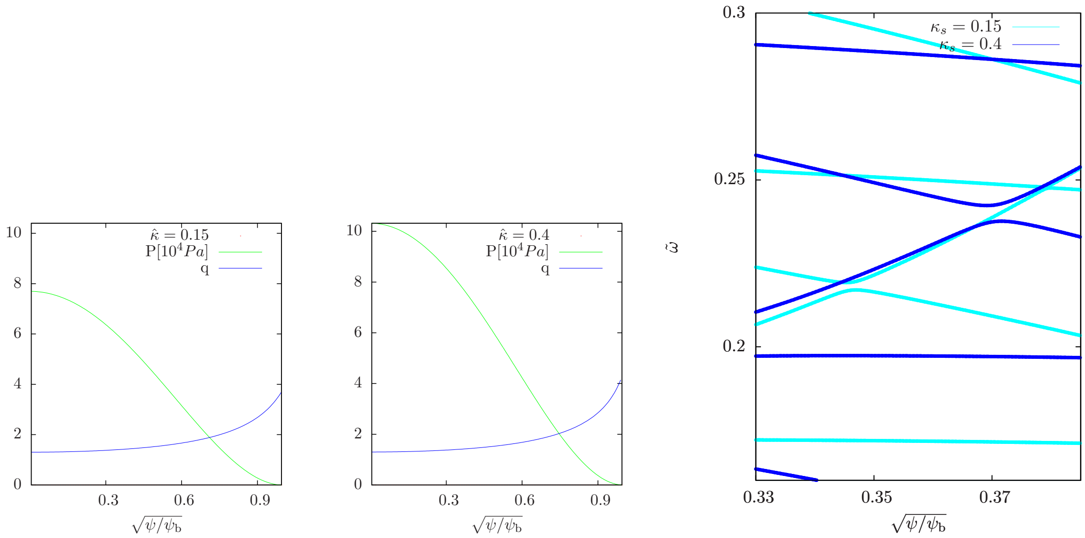


Figure 2: On the left are showed the pressure and q profiles of two equilibria with different $\hat{\kappa}$ values, on the right the produced gaps.

ψ, controls the gaps' width. It is related with plasma ellipticity (κ_e) by:

$$\kappa_e \approx \sqrt{\frac{1+\hat{\kappa}}{1-\hat{\kappa}}}.$$

We conclude that more elongated plasmas produce wider gaps.

We are now interested in the mechanism that can suppress the presented magnetic activity, in case it would reveal problematic for the integrity and the operation of a fusion device. A possibility is to prevent the crossing between the shear and sonic branches differing in poloidal mode number by a δ able to create gaps. The frequency of the shear branch, upshifted due to $\delta = 1$ shear-sonic interaction, at radial locations where the parallel wave vector vanishes, is

$$\tilde{\omega}_{UP}^2 = \frac{\gamma\beta_0\bar{p}}{1+\gamma\beta_0\bar{p}} \frac{1+2q^2K_g[1]K_g[-1]}{\bar{\rho}\, q^2}.$$

In [8] an estimation of $\tilde{\omega}_{UP}$, corrected for non circular magnetic surfaces, was presented.
The sonic frequency of the branch differing by δ, also evaluated at the rational surfaces is

$$\tilde{\omega}_S^2 = \frac{\gamma\beta_0\bar{p}}{1+\gamma\beta_0\bar{p}\bar{\rho}} \frac{\delta^2}{q^2}.$$

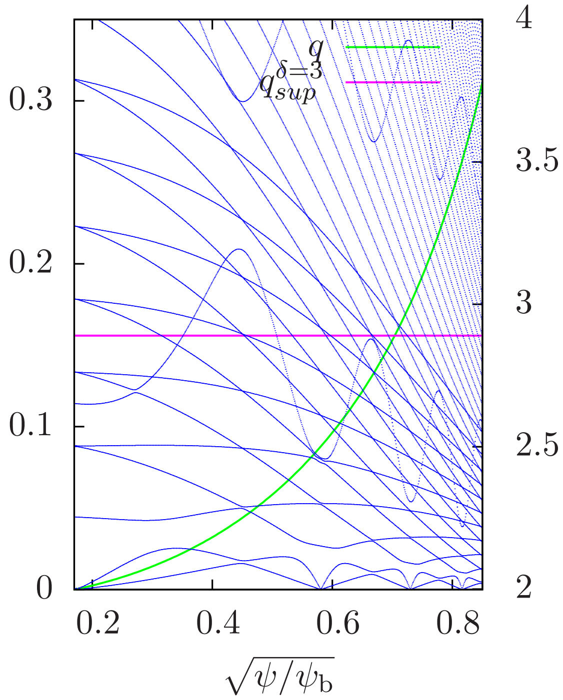
The $\delta > 1$ interaction is prevented if

$$\tilde{\omega}_{UP}^2 > \tilde{\omega}_S^2$$

that is equivalent to require that the safety factor has to exceed a suppressing value:

$$q^2 > \frac{\delta^2 - 1}{2K_g[1]K_g[-1]} = q_{sup}^2$$

In the plot on the right q_{sup}^2 assumes the value of 2.89, above which the gaps are suppressed.



Resonance maps for the particles, that exchange energy in the most efficient way, are displayed in figure 3. Fast ions have a density of 1% of n_0, an on axis temperature of $1MeV$ and a distribution function peaked at $\Lambda = 1$ in order to emulate the ions accelerated by the ICRH heating system. The eigenmode is damped by thermal and driven by fast ions, resulting in a net growth rate of $\sim 4.5\%$.

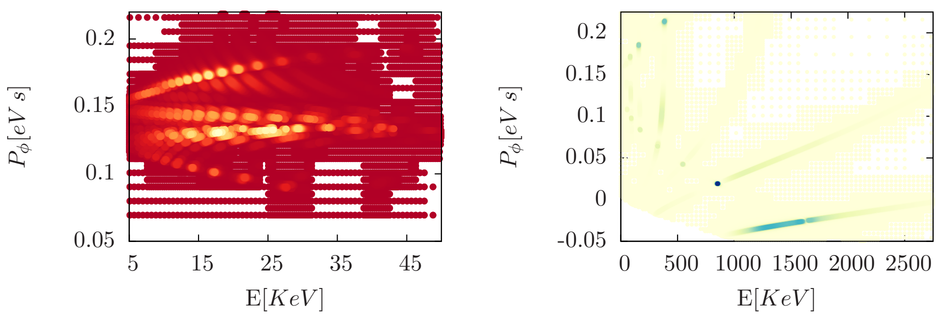


Figure 3: Resonance maps at $\Lambda = 0.905$ $\left(\Lambda = \frac{\mu B}{E}\right)$ for damping thermal ions (left) and for driving fast ones at $\Lambda = 1.012$ (right).

Summarizing, shaping effects are responsible for the opening of gaps in the continuum spectrum, different from the ones allowed by a circular flux surfaces equilibrium. We computed that eigenmodes can be found in the presented gaps and specific particles can drive them unstable. The concern, for this type of activity, is not on the growth of the instability but on the effect of the interaction on particles orbits. It could comport a massive ions redistribution and non linear codes should be employed to evolve orbits and study the losses.

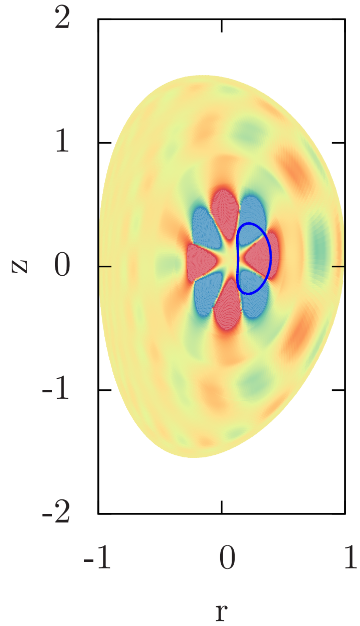


Figure 4: Poloidal mode structure in terms of amplitude of the plasma displacement. It is represented the orbit transferring energy in the most efficient way for fast ions at $\Lambda = 1.012$

Acknowledgments

This work has been carried out within the framework of the EUROfusion Consortium and has received funding through FuseNet from the Euratom research and training programme under Grant Agreement No. 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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