

## Statistical modeling of heavy ions quasicontinuum in thermonuclear plasmas

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Optimization of the quasi-stationary mode of operation of a magnetic confinement facility is one of the important reasons to study the tungsten impurities. There are several methods for monitoring the tungsten transport in plasma. One of them is measurement of emission in the 4-8 nm spectral range [1-7]. This kind of spectra is known as the heavy ions quasicontinuum, which presently was observed for various heavy plasma impurities on many modern thermonuclear installations. Here the simulation of these heavy ions characteristic spectra is presented based on the statistical model of atom [8-10]. It considers the ions excitations in terms of the collective oscillations with plasma frequencies  $\omega(r) \equiv \omega_p(r) = \sqrt{4\pi(e^2/m)n(r)}$ , determined by the local atomic electron density  $n(r)$  [11], where “ $r$ ” is the distance from the ion nucleus. “ $e$ ” is the electron charge and “ $m$ ” is its mass. The general formula for the line intensity of the transition  $i \rightarrow j$  is [12]:

$$dQ(\omega) = N_i \cdot \hbar \omega \cdot a(\omega) d\omega, \quad (1)$$

where  $N_i$  is an excited level population,  $\omega$  is a transition frequency,  $a(\omega)d\omega$  is the spontaneous emission rate in the narrow frequency interval  $d\omega$ . The spectral distribution of spontaneous emission  $a(\omega)$  could be expressed via the photoabsorption cross-section [12]:

$$a(\omega) = \frac{\omega^2}{\pi^2 c^2} \sigma(\omega) \quad (2)$$

In the statistical model of atom [8-11] the photoabsorption cross-section has the form:

$$\sigma(\omega) = \frac{2\pi^2 e^2}{m_e c} \int d^3 r n(r) \delta(\omega - \omega_p(r)) = \frac{2\pi^2 e^2}{m_e c} 4\pi \sum_s r_{\omega_s}^2 \left| \frac{d\omega_p(r)}{dr} \right|_{r=r_{\omega_s}}, \quad \omega_s = \omega_p(r_{\omega_s}) \quad (3)$$

Here the electron density distribution of outer ion electron shells is approximated by the Slater-type functions [13-14]  $n(r) \equiv n_{Sl}(r) = Ar^{2\kappa} e^{-2\gamma r}$  (in atomic units), where  $A$  is a normalization constant for the number of electrons in the considered shell,  $\kappa$  is a parameter determined from experimental data,  $\gamma = \sqrt{2mI}/\hbar$ ,  $I$  is an ionization potential. Near the maximum  $r_{max} = \kappa/\gamma$  the density could be approximated by the Gauss distribution

$$n_G(r) = \frac{N_G}{(2\pi)^{3/2} \Delta r^3} e^{-\frac{(r-r_{max})^2}{2\Delta r^2}} = n_{Sl}(r_{max}) \cdot e^{-\frac{(r-r_{max})^2}{2\Delta r^2}}, \quad \text{where } N_G \text{ is a normalization constant, } \Delta r \text{ is}$$

the distribution width of Gaussian. The last one is connected with the parameters of the Slater distribution as  $\Delta r = \sqrt{\frac{n_{Sl}(r_{\max})}{|n_{Sl}''(r_{\max})|}} = \frac{1}{\gamma} \sqrt{\frac{k}{2}}$ . Therefore, the plasma frequency with the help of the Gauss approximation could be expressed as:

$$\omega_p = \sqrt{\frac{4\pi e^2 n_G(r_\omega)}{m}} = \left( \frac{4\pi e^2 N_G}{(2\pi)^{3/2} m \Delta r^3} \cdot \exp \left[ -\frac{(r_\omega - r_{\max})^2}{2\Delta r^2} \right] \right)^{1/2} \quad (4)$$

From formula (4) one can obtain:

$$r_\omega^\pm = r_{\max} \pm 2\Delta r \sqrt{|\ln x|}, \quad x = \omega / \omega^*, \quad \omega^* = \sqrt{\frac{4\pi e^2 N_G}{(2\pi)^{3/2} m \Delta r^3}} \quad (5)$$

The value of  $\omega^*$  is the maximum transition frequency that could be emitted by the ion within the statistical model. As it follows from (5)  $x \leq 1$ , that results in a sharp cut of the spectra at the specific short wavelength for a given shell of a given ion within the local plasma frequency model. For example, the dependence of the plasma frequency versus the distance from the nucleus for the Gaussian distribution (3), corresponding to the  $W^{45+}$  ion, and the scheme of its resonance with the observed frequency  $\omega$  is presented in the figure 1.

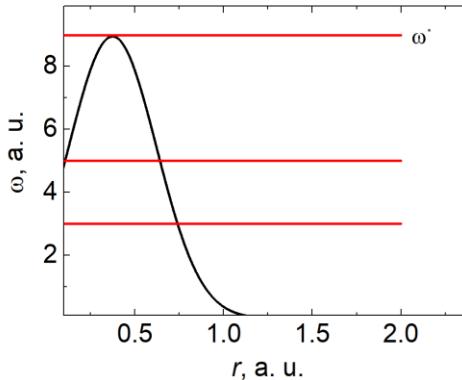


Fig.1. The scheme of resonances between an observed frequency  $\omega$  (horizontal lines) and the atomic transition local plasma frequency, described by the statistical model of atom (solid curve). The existence of the maximum atomic plasma frequency corresponds to a sharp spectral termination of the frequency range above the possible maximum value of  $\omega^*$ .

Using formulas (3)-(5), one obtains

$$\sigma(\omega) = \frac{2\pi^2}{c} \omega r_{\max} \left[ \frac{r_{\max}^2 + 4\Delta r^2 |\ln x|}{\gamma \Delta r \sqrt{|\ln x|}} \right] \quad (6)$$

Then the relative line intensity per one ion with charge  $z$  could be evaluated by the formula:

$$dQ(\omega)_z = \frac{2\hbar}{c^3} \omega^3 \int d\omega' \omega' r_{\max} \left[ \frac{r_{\max}^2 + 4\Delta r^2 |\ln(\omega'/\omega^*)|}{\gamma \Delta r \sqrt{|\ln(\omega'/\omega^*)|}} \right] \frac{1}{\sqrt{2\pi\delta_w}} e^{-\frac{(\omega-\omega')^2}{2\delta_w^2}} d\omega \quad (7)$$

where  $\delta_W$  is a squared halfwidth of the spectrometer instrumental function. In the wavelength scale the above expression (7) could be presented in the form:

$$dQ(\lambda)_z = \frac{64\pi^5 \hbar c^2}{\lambda^{*4}} \left( \frac{\lambda^*}{\lambda} \right)^5 \left( \int_{\lambda_{\min}}^{\lambda_{\max}} \frac{d\lambda'}{\lambda'^2} \cdot r_{\max} \left[ \frac{r_{\max}^2 + 4\Delta r^2 |\ln(\lambda^* / \lambda')|}{\gamma \Delta r \sqrt{|\ln(\lambda^* / \lambda')|}} \right] \cdot \frac{1}{\sqrt{2\pi\delta_{\lambda}}} e^{-\frac{(\lambda' - \lambda)^2}{2\delta_{\lambda}^2}} \right) d\lambda \quad (8)$$

The formula (8) should be averaged over the charge states distribution function for represented ions. As the mentioned above sharp cuts were regularly observed in the experimental spectra on different installations [1-7], the modeling of the measured peak position for the particular tungsten ions from [6] allowed to determine the parameters  $\kappa$  and  $\gamma$ . Assuming that in the narrow spectral range the population distribution over excited levels of the different ions is proportional to the distribution of ionization states one could identify the final results with the relative spectral distribution of measured quasicontinuum. Figures 2-4 demonstrate the correspondence of the observed quasicontinuum of tungsten, gold, and lead with the statistical modeling. So, as it is seen from the figures, the statistical approach provides the specific form of a quasicontinuum with the sharp cuts from the side of short wavelengths, which satisfactorily coincides with the experimental observations.

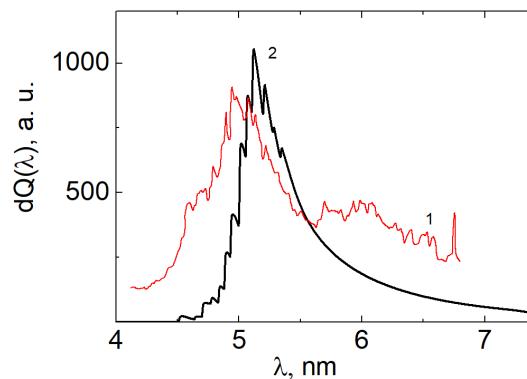


Fig.2 Comparison of the experimental quasicontinuum (1) from tungsten ions on LHD installation [6] with the theoretical calculations (2).

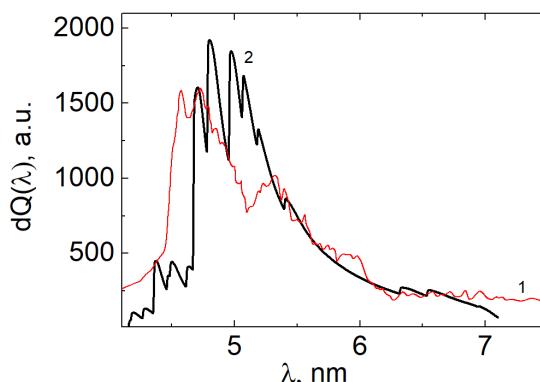


Fig. 3 Comparison of the experimental quasicontinuum (1) from gold ions on TEXT installation [4] with the theoretical calculations (2).

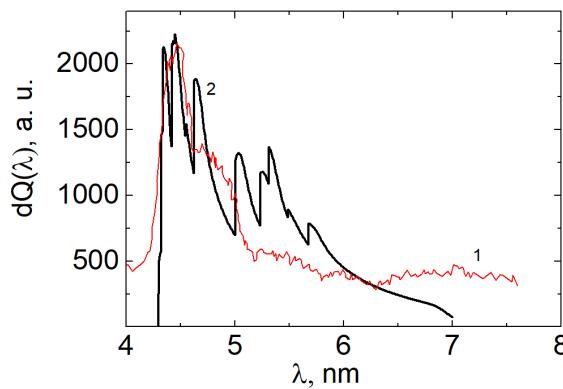


Fig. 4 Comparison of the experimental quasicontinuum (1) from lead ions on TEXT installation [4] with the theoretical calculations (2).

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