First-principles simulation of plasma fuelling in a tokamak

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The overall performance of a magnetic confinement fusion device depends critically on the phenomena taking place in the Scrape-Off-Layer (SOL), as they set the boundary conditions for particle and energy confinement, regulate the heat exhaust and control the impurity level. Furthermore, the SOL plays a crucial role in the fueling and removal of fusion ashes. Therefore, understanding the physical processes taking place in the tokamak periphery is of the utmost importance. Theoretical research based on first principles simulations of SOL turbulence may play a determinant role to advance our SOL physics understanding. The simulations can take advantage of the low plasma temperature and high collisionality observed in the tokamak periphery by using a fluid model. This is the strategy followed by the Global Braginskii Solver (GBS) code, a 3D fluid code that solves the drift-reduced Braginskii equations [1, 10], using a flux-driven approach (no separation between fluctuations and background quantities) [3, 6, 7].

Understanding the full picture of SOL physics requires taking into account neutral species. In fact, neutral atoms or molecules are present in the SOL, as they are generated by electron-ion volumetric recombination or plasma recycling at the vessel wall. External injection of neutrals can also be used with the purpose of fueling the plasma. In general, neutral particles play a crucial role in the context of SOL physics and affect the properties of turbulence.

GBS simulates the interaction of the neutrals with the plasma, by solving a kinetic equation for the neutral species with the method of characteristics. Neutral and plasma dynamics are coupled by means of the ionization, charge exchange and recombination frequencies [9]. GBS can be used to address some open questions regarding the neutral-plasma interaction, starting by understanding the mechanism behind fueling.

In order to understand the physical processes behind tokamak fueling, quantitative studies of particle flows have to be carried out within a mass conserving model. In the first part of this work we present the recent progress in GBS. Indeed, several changes were recently implemented in GBS. These include:

- The plasma continuity equation exactly satisfied in a toroidal device geometry;
- Writing the vorticity equation in an exact form, removing the Boussinesq approximation;

- Making sure that toroidal geometry is taken into account when coupling the neutral and plasma dynamics;
- Setting proper boundary conditions so that plasma recycling satisfies mass conservation.

The exact verification of the continuity equation allows one to write the plasma density balance in terms of the divergence of the plasma flux $\boldsymbol{\Gamma}$, which is the starting point for quantitative studies of particle flows,

$$\frac{\partial n}{\partial t} = -\boldsymbol{\nabla}\cdot\boldsymbol{\Gamma_e} + n_{\mathrm{n}}\nu_{\mathrm{iz}} + \mathscr{D}_{\mathrm{n}}(n), \tag{1}$$

where $n_{\mathrm{n}}\nu_{\mathrm{iz}}$ represents the ionization term and $D_n(n)$ particle diffusion and the particle flux $\boldsymbol{\Gamma_e}$ is given by the sum of the $E\times B$, diamagnetic and parallel velocity contributions as

$$\boldsymbol{\Gamma_e} = n(\boldsymbol{v}_{E\times B} + \boldsymbol{v}_{\mathrm{de}} + \boldsymbol{v}_{\|_e})\ ,\ \boldsymbol{v}_{\mathrm{de}} = \frac{1}{B^2}\boldsymbol{\nabla} p_e \times \boldsymbol{B}\ ,\ \boldsymbol{v}_{E\times B} = -\frac{n}{B^2}\boldsymbol{\nabla}\phi\times\boldsymbol{B}. \tag{2}$$

This was made possible by removing two approximations from the GBS code, namely:

- the local inverse aspect ratio $\varepsilon = \frac{r}{R_0}$ taken constant over the whole domain as $\varepsilon_0 = \frac{a_0}{R_0}$;
- the parallel components of Poisson brackets and curvature operators neglected.

In addition, to ensure ion mass conservation, the vorticity equation should state that the electric current density is divergence-free, $\boldsymbol{\nabla}\cdot\boldsymbol{j} = 0$, thus making sure that the divergence of the ion and electron fluxes is the same, apart from diffusion terms added for numerical stability, $\boldsymbol{\nabla}\cdot\boldsymbol{\Gamma_e} = \boldsymbol{\nabla}\cdot\boldsymbol{\Gamma_i} + D_\Omega(\Omega)$. This required removing the Boussinesq approximation which was previously taken into account in the vorticity equation.

As for the neutrals, particle conservation is ensured by the adiabaticity assumption used to solve the neutral kinetic equation ($\frac{\partial n_n}{\partial t} = 0$) and the ionization sink matches the divergence of the neutral flux $\boldsymbol{\Gamma_n}$, yielding $n_{\mathrm{n}}\nu_{\mathrm{iz}} = -\nabla\cdot\boldsymbol{\Gamma_n}$. However, for equation (3) to be satisfied in the context of the 3D particle balance, the effect of toroidicity had to be considered. This was done by taking into account the correct geometry when taking the plasma fluxes as inputs for the neutral solver and when using the neutral moments as sources for the plasma equations. Hence, the continuity equation can now be written in a mass-conserving form as

$$\frac{\partial n}{\partial t} = -\boldsymbol{\nabla}\cdot\boldsymbol{\Gamma_i} - \boldsymbol{\nabla}\cdot\boldsymbol{\Gamma_n} + \mathscr{D}(n,\Omega), \tag{3}$$

where $D_{n,\Omega}$ accounts for the sum of the density and vorticity diffusive terms.

Mass conservation in GBS requires that proper boundary conditions are provided to the neutral solver to ensure that the ion outflow to the limiter/walls matches the resulting inflow of neutrals. Therefore, the exact ion fluxes have to be used, taking into account all components ($E \times B$, ion diamagnetic and polarization fluxes). The poloidal flux to the limiter and radial flux to the walls thus yield

$$(n\vec{v})^{\theta^*}_{lim} = nv_{||i}b^{\theta^*} + (\mathbf{n}\vec{\mathbf{v}}_{\mathrm{di}})^{\theta^*} + (\mathbf{n}\vec{\mathbf{v}}_{E\times B})^{\theta^*} + (n\vec{\mathbf{v}}_{pol,i})^{\theta^*}, \qquad (n\vec{v})^{r}_{wall} = (\mathbf{n}\vec{\mathbf{v}}_{\mathrm{di}})^{r} + (\mathbf{n}\vec{\mathbf{v}}_{E\times B})^{r} + (n\vec{\mathbf{v}}_{pol,i})^{r}. \quad (4)$$

Integrating Eq. (1) and Eq. (3) over the poloidal and toroidal directions and performing the time averaging, a 1D radial model was obtained for particle balance in GBS. Therefore, the RHS and LHS of Eq. (1) are plotted radially in Fig. 1, showing that the continuity equation is satisfied in GBS within the numerical approximations used by the code. The curves match each other very well except at the LCFS, where gradients are too large for the grid resolution, and near the core, where a variable particle source is implemented to mimic the plasma inflow from the core, thus perturbing local gradients.

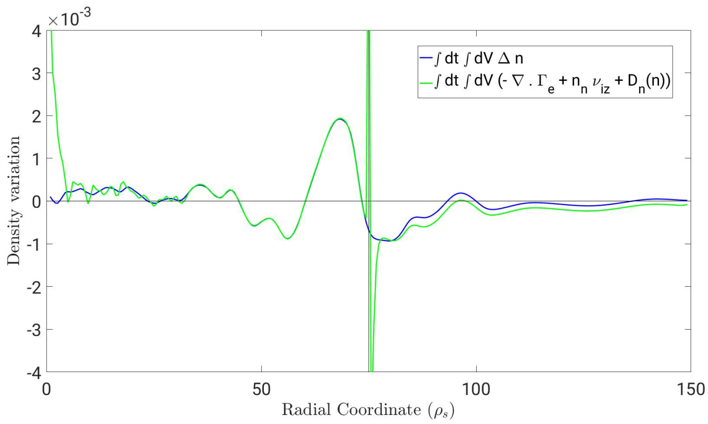

Figure 1: *Radial particle balance following Eq. (1) averaged over* $\Delta t = 1.0R_0/c_{s0}$. *The volume-integrated density variation (blue) can be seen to match the divergence of the electron flux added by volumetric ionization (green).*

On the other hand, the RHS and LHS of Eq. (3) are also plotted radially in Fig. 2, accounting for conservation of the number of ions + neutrals in the GBS. One can see that the two curves follow the same trend, which accounts for mass conservation, but the matching is much worse than in Fig. 1. This is so partly because of the coarseness of the neutral grid (8 times coarser than the plasma grid), which becomes more important at the regions where gradients are steeper). Another important source of numerical error comes from the vorticity equation, with the terms coming from the ion polarization flux and the diffusion contributions giving rise to numerical noise over the main trend exhibited by the RHS.

Such a model can now be used to study the steady state regime, where the plasma profiles remain constant, thus allowing for plasma density to be conserved both globally and locally. The radial components of neutral and ion fluxes are evaluated, splitting the contributions of $E \times B$, diamagnetic and polarization fluxes, as well as the ion and neutral density radial profiles.

A quantitative analysis of particle flows based on these results is now being undertaken as a starting point for studying tokamak fueling.

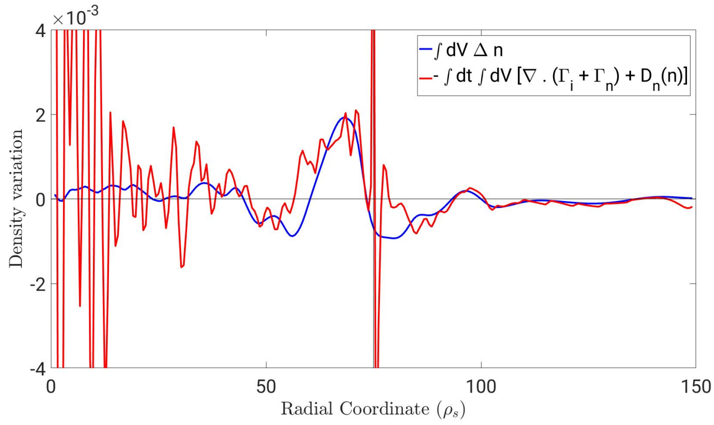

Figure 2: *Radial particle balance following Eq. (1) averaged over* $\Delta t = 1.0 R_0/c_{s0}$. *The volume-integrated density variation (blue) can be seen to match the sum of the divergence of the ion and neutral fluxes (red).*

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