Effect of applied resonant magnetic perturbations on local plasma current density gradient and stability of m/n=2/1 magnetic island

Q. Yu, S. Günter and K. Lackner

Max-Planck-Institut für Plasmaphysik, 85748 Garching, Germany

1. Introduction

Externally applied resonant magnetic perturbations (RMPs) of sufficiently large amplitude are known to cause mode locking or field penetration to generate locked magnetic islands. RMPs of moderate amplitude, however, are found to stabilize rotating islands in some experiments [1,2]. Theoretical results based on a single-fluid model have shown that moderate RMPs cause a non-uniform island rotation and a corresponding net stabilizing effect [1-3]. Based on Ref. [4], the effect of $m/n=4/2$ and $6/3$ RMPs on the local plasma current density gradient around the $q=2$ surface and on the $2/1$ island stability are studied in this paper (q is the safety factor, and m and n the poloidal and toroidal mode numbers).

Our studies are based on the four-field equations, the mass conservation equation, the generalized Ohm's law, the plasma vorticity equation, and the equation of motion in the parallel (to magnetic field) direction [4,5]. The magnetic field is defined as $\boldsymbol{B}=B_t\boldsymbol{e}_t - (k_t/k_\theta)B_t\boldsymbol{e}_\theta + \nabla\psi\times\boldsymbol{e}_t$, where ψ is the helical flux function, $k_\theta=m/r$ and $k_t=n/R$ are the wave vector in $\boldsymbol{e}_\theta$ (poloidal) and $\boldsymbol{e}_t$ (toroidal) direction, and r and R are the minor and major radius. The ion velocity is given by $v=\boldsymbol{v}_{||}+\boldsymbol{v}_\perp$, including the parallel and perpendicular directions. A constant electron temperature T_e and cold ions are assumed. Normalizing the length to the plasma minor radius a, the time t to the resistive time $\tau_R=a^2\mu_0/\eta$, ψ to aB_t, $\boldsymbol{v}$ to a/τ_R, and the electron density n_e to its value at r=0, the four-field equations become [5]

$$\frac{dn_e}{dt}=d_1\nabla_{||}j-\nabla_{||}(n_e v_{||})+\nabla_\perp(D_\perp\nabla_\perp n_e)+S_n, \tag{1}$$

$$\frac{d\psi}{dt}=E_0-\eta_N(j-j_b)-\frac{\eta_N m_e}{n_e e^2}\frac{dj}{dt}+\Omega\nabla_{||}n_e, \tag{2}$$

$$\frac{dU}{dt}=S^2\nabla_{||}j+\mu\nabla_\perp^2 U+S_m, \tag{3}$$

$$\frac{dv_{||}}{dt}=-C_s^2\nabla_{||}P/n_e+\mu\nabla_\perp^2 v_{||}, \tag{4}$$

in cylinder geometry, where $d/dt=\partial/\partial t+\boldsymbol{v}_\perp\cdot\nabla_\perp$, j is the parallel plasma current density, η_N ($=1$) the normalized resistivity, $j_b=-c_b\sqrt{\varepsilon}(\partial p_e/\partial r)/B_p$ the bootstrap current density, c_b a constant of order of unity, $\varepsilon=r/R$, $P_e=n_eT_e$, B_p the poloidal magnetic field, and m_e the electron mass. E_0 is the equilibrium electric field, $U=-\nabla_\perp^2\phi$ the plasma vorticity, ϕ the stream function, S_n the particle source, and S_m the poloidal momentum source leading to an equilibrium poloidal plasma rotation frequency ω_{E0}. $\Omega=\beta d_1$, $d_1=\omega_{ce}/\nu_e$, $\beta=4\pi P_e/B_t^2$, ω_{ce} and ν_e are the electron cyclotron and the collisional frequency. $S=\tau_R/\tau_A$, $\tau_A=a/V_A$, and V_A is the Alfven velocity defined using B_t. C_s, μ and $D_\perp$ are the normalized ion sound velocity, plasma viscosity and perpendicular particle diffusivity. In tokamak experiments the plasma rotation is essentially toroidal [1], while in Eqs. (1)-(4) only the poloidal rotation is included, so that a larger plasma viscosity, by a factor 10^2, is used for the $m/n=0/0$ component to guarantee a reasonable balance between the electromagnetic and viscous force [1,4]. The effect of RMP with an m/n component is taken into account by the boundary condition

$$\psi_{m/n}|_{r=a} = \psi_{a,m/n}\, aB_{0t}\cos(m\theta+n\phi), \qquad (5)$$

where $\psi_{a,m/n}$ is the normalized amplitude of the m/n component helical flux at $r=a$.

2. Numerical results

A monotonic equilibrium q-profile is assumed with the $q=2$ surface located at $r_{2/1}=0.628a$. The $m/n=2/1$ magnetic island is unstable with this q-profile for $\psi_a=0$ and grows to a width of $0.2a$ in the nonlinear phase, even for zero local bootstrap current density fraction, f_b=0.

We first study the case with the following input parameters: $T_e=300eV$, $n_e=10^{19}m^{-3}$, $B_t=2T$, $a=0.25m$ and $R=1.0m$, leading to $S=3.76\times10^7$, $C_s=8.4\times10^5(a/\tau_R)$, $d_1=5.4\times10^6$, and $\Omega=8.2\times10^2$. A parabolic profile for the equilibrium electron density is taken, and the local electron diamagnetic drift frequency is $\omega_{*e0}=6.43\times10^3/\tau_R$ at $r=r_{2/1}$. Furthermore, f_b=0, $m_e=0$ and $\mu=D_\perp=0.2m^2/s$ are assumed. The time evolution of the normalized $m/n=2/1$ island width, calculated from $W=4[\psi_{2/1}/(B_p\, q/q')]^{1/2}$ at $r_{2/1}$, is shown in Fig. 1 for $\psi_{a,4/2}=0,\ 1.8\times10^{-4}$ and 2.5×10^{-4} with $\omega_n=-11.5$, where $\omega_n\equiv(1+\omega_{E0}/\omega_{*e0})$ is the normalized (to ω_{*e0}) equilibrium electron fluid frequency at $r_{2/1}$. The electron fluid velocity is in the ion drift direction for $\omega_n<0$. The $2/1$ island grows for zero or a small $\psi_{a,4/2}$ but is suppressed for $\psi_{a,4/2}=2.5\times10^{-4}$, indicating a threshold in the $4/2$ RMP amplitude for suppressing the $2/1$ island growth. A too large $4/2$ RMP amplitude is however found to cause field penetration. The blue curve in Fig. 1 corresponds to a case with $\psi_{a,6/3}=5\times10^{-4}$ and $\omega_{E0}=9.81\times10^4/\tau_R$, obtained from single fluid equation by only solving equations (2)-(3) and taking $\Omega=m_e=0$. The $2/1$ island growth is also stabilized in this case, being not due to the non-uniform mode rotation caused by RMPs.

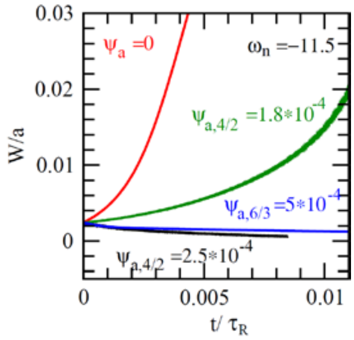

Figure 1 Time evolution of normalized m/n=2/1 island width for $\psi_{a,4/2}$*=0, 1.8× and 2.5×10*$^{-4}$ *with* ω_n*=-11.5. The blue curve is for* $\psi_{a,6/3}$*= 5×10*$^{-4}$ *and* ω_{E0}*=9.81×10*^{4}*/*τ_R*, obtained from single fluid equation .*

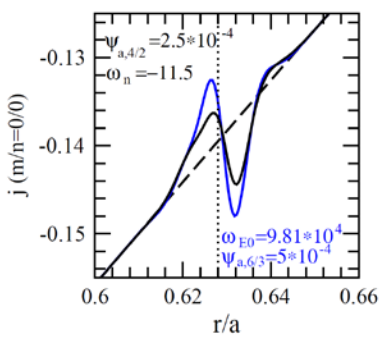

Figure 2 Radial m/n=0/0 component plasma current density profiles for $\psi_{a,4/2}$*=2.5×10*$^{-4}$ *and* $\psi_{a,6/3}$*=5×10*$^{-4}$ *corresponding to Fig. 1. The dashed curve is the equilibrium one. The q=2 surface is marked by the vertical dotted line.*

Corresponding to the cases with $\psi_{a,4/2}=2.5\times10^{-4}$ and $\psi_{a,6/3}=5\times10^{-4}$ in Fig. 1, radial profiles of the (normalized) *m/n=0/0* component plasma current density in steady state are shown in Fig. 2. The dashed curve is the original equilibrium plasma current density. The local current density gradient is significantly changed by the RMPs for both cases, being reversed at the *q=2* surface and causing the stabilization of the *2/1* mode seen from Fig. 1.

For a higher temperature plasma with $T_e=2keV$, $n_e=3\times10^{19}m^{-3}$, $B_t=2T$, $a=0.5m$ and $R=1.7m$, one has $S=2.6\times10^8$, $C_s=2\times10^7(a/\tau_R)$, $d_1=3.1\times10^7$, and $\omega_{*e0}=1.5\times10^5/\tau_R$ ($f_{*e0}=1kHz$) for $\Omega=2\times10^4$. Assuming $f_b=0.35$, $\mu=0.2m^2/s$ and $D_\perp=\mu/5$, and excluding the *2/1* component in the calculations, radial profiles of the *m/n=0/0* component of plasma current density in steady state are shown in Fig. 3 for $\omega_n=1$, *2.1*, *5*, and *-1.3* with $\psi_{a,4/2}=10^{-4}$. The *4/2* RMP has not penetrated in for all cases. The dashed curve is the original equilibrium plasma current density. In this case the local *m/n=0/0* component plasma current density gradient is changed more significantly by the *4/2* RMPs than that shown in Fig. 2 for $T_e=300eV$. A negative value of ω_n, ω_n *=-1.3* (plasma rotation in the ion drift direction), results in an increased (reversed) local current density gradient outside (inside) the *q=2* surface, being stabilizing for the *2/1* mode. With a positive value of ω_n, the current density profiles become more unstable to the *2/1* mode for ω_n *=1* or *2.1* but stable for ω_n *=5*.

Including the *2/1* component in calculations, the stability diagram in the ($\omega_n - \psi_{a,4/2}$) plane obtained from four-field equations is shown in Fig.4. The black circles (red squares) are for the cases in which the *2/1* island growth is (not) stabilized by the *4/2* RMP with

T_e=2keV. The stabilization region is asymmetrical on the two sides of $\omega_n=0$, being wider for the plasma rotation in the ion drift direction ($-\omega_n>-1$) and can be explained by the results in Fig.3 and the effect of RMPs on the density profile and accordingly the diamagnetic drift frequency. Such an asymmetry was also found in field penetration experiments [6]. The results for the case with T_e=300eV (and $m_e=0$) are shown by blue diamonds (stabilized) and green triangles (not stabilized). In this case the stabilization region is much smaller and exists only for sufficiently large $|\omega_n|$ values.

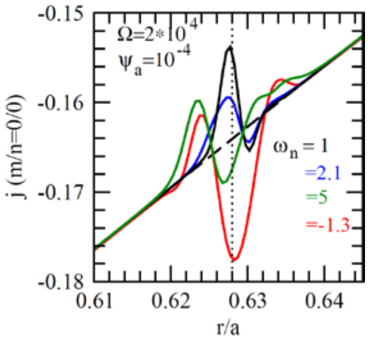

Figure 3 Radial profiles of the m/n=0/0 component current density for ω_n=1, 2.1, 5, and -1.3 with $\psi_{a,4/2}=10^{-4}$, without including the 2/1 component in calculations. The dashed curve is the original equilibrium one. The q=2 surface is marked by the vertical dotted line.

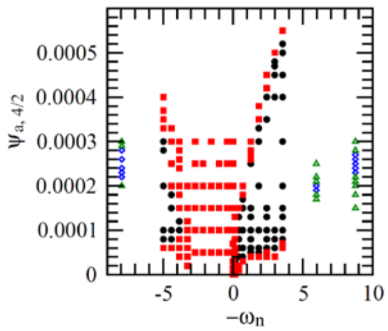

Figure 4 (left) Stability diagram in ($\omega_n - \psi_{a,4/2}$) plane. The black circles (red squares) are the cases that the 2/1 island growth is (not) stabilized by the 4/2 RMP for T_e=2keV. The case for T_e=300ev (m_e=0) is shown by blue diamonds (stabilized) and green triangles (not stabilized).

3. Summary

Static *m/n=4/2* or *6/3* RMPs of moderate amplitude are found to cause significant changes in the local *m/n=0/0* component of the plasma current density gradient around the *q=2* surface, which suppresses the growth of the *2/1* mode if the local electron fluid velocity is sufficiently large. For higher temperature plasmas, RMPs generate a larger change in the local plasma current density gradient and a larger stabilization region for the 2/1 mode, suggesting the larger role of RMPs in affecting the 2/1 mode stability in ITER.

[1] Hender T.C. *et al* 1992 *Nucl. Fusion* **32**, 2091

[2] Hu Q., Yu Q., Rao B., Ding Y. H. *et al* 2012 *Nucl. Fusion* **52**, 083011

[3] Fitzpatrick R. 1993 *Nucl. Fusion* **33**, 1049

[4] Yu Q., Günter S. and Lackner K. 2018 *Nucl. Fusion* **58** 054003

[5] Hazeltine R. D. *et al* 1985 *Phys Fluids* **28** 2466.

[6] Koslowski H.R., Liang Y., Krämer-Flecken A. *et al* 2006 *Nucl. Fusion* **46**, L1.