Hierarchical approach to first principle based reduced transport models

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Introduction

The derivation and validation of reduced transport models describing the evolution of macroscopic plasma profiles on long time scales, comparable with the energy confinement time, which preserve all the necessary physics ingredients is one of the most important areas in fusion theory and code development. In fact, on the one hand extending first-principle-based gyrokinetic simulations to such long time scales is extremely demanding from a computational resource point of view and on the other hand the relevant physics processes can be isolated and extracted from complex simulations. Using the nonlinear dynamic description of PSZS [1], i.e. those corrugations of the distribution function which are not rapidly damped by collisionless processes such as Landau damping, some of the authors have shown that it is possible to describe particle and energy transport up to the energy confinement time as moments of a slowly evolving non-linear plasma *reference state* which is obtained from the toroidal symmetric part of the distribution function with an averaging procedure [2]. The importance of characterizing the evolution of the plasma *reference state* on long time scales instead of radial profiles only is crucial. In particular, we aim at describing a non-Maxwellian *reference state* to study the unique role of energetic particles as mediators of cross-scale couplings in fusion plasmas [1].

In the first part of the present work, we derive the governing equation for the plasma *reference state* (i.e. the PZSZ transport equations) from gyrokinetic theory in conservative form. In the second part of the work we propose a hierarchical approach to solve this equation with different levels of approximation and a simple application to the beam plasma system.

Trasport in the phase space

Following [3], we can write the gyrokinetic equation in conservative form:

$$\frac{\partial}{\partial t}(D_a F_a) + \frac{\partial}{\partial \mathbf{Z}} \cdot \left(D_a F_a \dot{\mathbf{Z}}_a\right) = D_a \left(\sum_b C^g_{ab}[F_a, F_b](\mathbf{Z}, t) + \mathscr{S}_a(\mathbf{Z}, t) \right) \tag{1}$$

where D_a is the Jacobian in the velocity space, $\dot{\mathbf{Z}}_a$ is the generalized phase space velocity, $C^g_{ab}[F_a,F_b](\mathbf{Z},t)$ is the collision integral, $\mathscr{S}_a(\mathbf{Z},t)$ describes external sources of particle, momentum or energy and a,b denote particle species. We assume an axisymmetric reference magnetic field that can be described adopting toroidal flux coordinates (ϕ,θ,ψ): $\boldsymbol{B}_0 = F\boldsymbol{\nabla}\phi + \boldsymbol{\nabla}\phi\times\boldsymbol{\nabla}\psi$. Particle velocity can be decomposed as the sum of two contributions, i.e. $\dot{\mathbf{Z}}_a = \dot{\mathbf{Z}}_{a0} + \delta\dot{\mathbf{Z}}_a$, representing, respectively, integrable motion in the reference state and the effect of fluctuations. The collision-less kinetic equation can be writtens as:

$$\frac{\partial}{\partial t}(DF) + \frac{\partial}{\partial \mathbf{Z}}\cdot\left(D\dot{\mathbf{Z}}_0 F\right) + \frac{\partial}{\partial \mathbf{Z}}\cdot\left(D\delta\dot{\mathbf{Z}}F\right) = 0 \tag{2}$$

where, for the sake of clarity, we have suppressed species index. In the absence of fluctuations, collisions and sources, the particle motion is integrable and characterized by three invariants, i.e. the particle energy (per unit of mass) $\mathscr{E}_0$, the magnetic moment μ and the toroidal angolar momentum $P_{\phi 0}$. For this reason, we will use $(\zeta,\theta,P_\phi,\mu,\mathscr{E})$ as phase space coordinates, where ζ is introduced instead of the toroidal angle ϕ in order to work with straight magnetic field line flux coordinates.. We now consider the zonal (toroidal mode number $n=0$) component of the distribution function to derive PSZS evolution equation. We have:

$$\frac{\partial}{\partial \mathbf{Z}}\cdot\left(D\dot{\mathbf{Z}}_0 F\right)_z = \nabla\cdot\left(D\dot{\mathbf{Z}}_0 F\right)_z = \frac{1}{J_{P_\phi}}\frac{\partial}{\partial\theta}(DJ_{P_\phi}F\dot{\boldsymbol{X}}_0\cdot\boldsymbol{\nabla}\theta)_z \tag{3}$$

where the z subscript indicates the zonal component, $J_{P_\phi} = J(\partial P_\phi/\partial\psi)^{-1}$ and $J = (\boldsymbol{\nabla}\zeta\cdot\boldsymbol{\nabla}\psi\times\boldsymbol{\nabla}\theta)^{-1}$. The toroidal symmetry of reference state, the fact that $\dot{\boldsymbol{X}}_0\cdot\boldsymbol{\nabla}P_\phi = 0$ and that $\dot{\mathscr{E}}_0 = 0$ have been used. Now, we introduce an operator annihilating the remaining term in Eq. (3) which consists in an average along θ weighted by J_{P_ϕ}. Applying this operator to the kinetic equation and assuming that the reference state magnetic equilibrium is slowly evolving on the transport time scale, we obtain:

$$\partial_t \oint d\theta J_{P_\phi} DF_z + \oint d\theta J_{P_\phi}\frac{\partial}{\partial \mathbf{Z}}\cdot(D\delta\dot{\mathbf{Z}}F)_z = 0 \tag{4}$$

Using the equation of motion in the absence of fluctuations, i.e. $\dot{\psi} = -\frac{v_\parallel}{JB^*_\parallel}\frac{\partial G}{\partial\theta}$, $\dot{\theta} = \frac{v_\parallel}{JB^*_\parallel}\frac{\partial G}{\partial\psi}$, $G = \psi - F(\psi)v_\parallel/\Omega \simeq -(c/e)P_{\phi 0}$, we can re-write the average on the LHS as the time average along the integrable particle orbit, denoted in the following by $\overline{(...)} = \tau_b^{-1}\oint d\theta(...)/\dot{\theta}$, with $\tau_b = \oint d\theta/\dot{\theta}$. Proceeding as above and noting that $\partial_{\mathscr{E}} J_{P_\phi} = 0$, we can re-write the orbit averaged kinetic equation as:

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b\delta\dot{P}_\phi\delta F\right)_z} + \frac{\partial}{\partial\mathscr{E}}\overline{\left(\tau_b\delta\dot{\mathscr{E}}\delta F\right)_z}\right] = \overline{\left(\sum_b C^g_b[F,F_b] + \mathscr{S}\right)_z} \tag{5}$$

where $\delta \dot{P}_\phi = \delta \dot{X} \cdot \boldsymbol{\nabla} P_\phi$. Here, we have let $F_z = \bar{F}_{z0} + \delta F_z$ and, more generally, $F = \bar{F}_{z0} + \delta F$. With this choice, orbit averaged polarization response due to zonal structures, $\delta \bar{F}_z$, is extracted from the banana center PSZS definition, $\bar{F}_{z0}$ and is formally treated as fluctuation [2]. The explicit expression for the fluxes is obtained using the equation of motion: $\delta \dot{X} = \frac{\boldsymbol{b}}{B^*_\parallel} \times \nabla \langle \delta \psi_{gc} \rangle - \frac{q}{m} \left\langle \delta A_{gc\parallel} \right\rangle \frac{\boldsymbol{B}}{B^*_\parallel}$, $\delta \dot{\mathscr{E}} = -\frac{q}{m} \dot{X}_0 \cdot \nabla \langle \delta \psi_{gc} \rangle - \frac{q}{m} \left\langle \delta A_{gc\parallel} \right\rangle \mu \frac{\boldsymbol{B}}{B^*_\parallel} \cdot \nabla B$ where $B^*_\parallel \equiv \mathbf{B}^* \cdot \mathbf{b}$, $\mathbf{B}^* \equiv \nabla \times \mathbf{A}^*$, $\frac{e}{c} \mathbf{A}^* \equiv \frac{e}{c} \mathbf{A}_0 + m \left(v_\parallel \mathbf{b} \right)$. Consistently with Ref. [2], the banana center PSZS is described by the low frequency component of this expression, remains undamped by collisionless processes and evolves according to the transport time scale ordering. The ratio between the third and the second terms of Eq. (5) can be estimated as $\delta \mathscr{E} / \Delta \mathscr{E}$, where $\Delta \mathscr{E}$ is the width of the nonlinear distortion in phase space. This is consistent with the fact that phase space transport generated by broad band turbulence on a relatively short time-scale is mainly a $1D$ process in the radial direction (negligible parallel non-linearity). Meanwhile, for the quasi-coherent spectrum typical of Alfvénic fluctuations [1], $\delta \mathscr{E} / \Delta \mathscr{E} \sim |\omega / \omega_{*E}| \sim r/R \ll 1$. Thus, phase-space transport is still nearly $1D$ due to the presence of an additional integral of motion, although transport in energy direction is less slower due to dominant role of resonant wave-particle interactions.

Noting that the unperturbed particle motion, thus characterized by a given value of P_ϕ, can be parameterized as $\psi = \bar{\psi} + \delta \tilde{\psi}(\theta)$, where $\bar{\psi}$ is the average value of ψ, we can re-write orbit averaging using $\bar{\psi}$ as radial coordinate:

$$\oint \frac{d\theta}{\dot{\theta}} F(P_\phi, \theta) = \oint \frac{d\theta}{\dot{\theta}} F(P_\phi(\bar{\psi} + \delta \tilde{\psi}(\theta)), \theta) = \oint \frac{d\theta}{\dot{\theta}} e^{iQ} F(\overline{\psi}, \theta) \tag{6}$$

In this expression a shift operator e^{iQ} has been introduced, consistent with previous work [2], where its leading order expression is calculated for describing particle and energy transport in an axisymmetric magnetized plasma. Noting the connection of the gyrocenter particle distribution with the nonadiabatic particle response [1, 4], $\delta F = \delta G + (q/m) \left\langle \delta \psi_{gc} \right\rangle \partial \bar{F}_{z0} / \partial \mathscr{E}$, Eq. (5) reduces to the limiting case discussed in Ref. [2] by dropping parallel nonlinearity in addition to source and collision terms on the RHS.

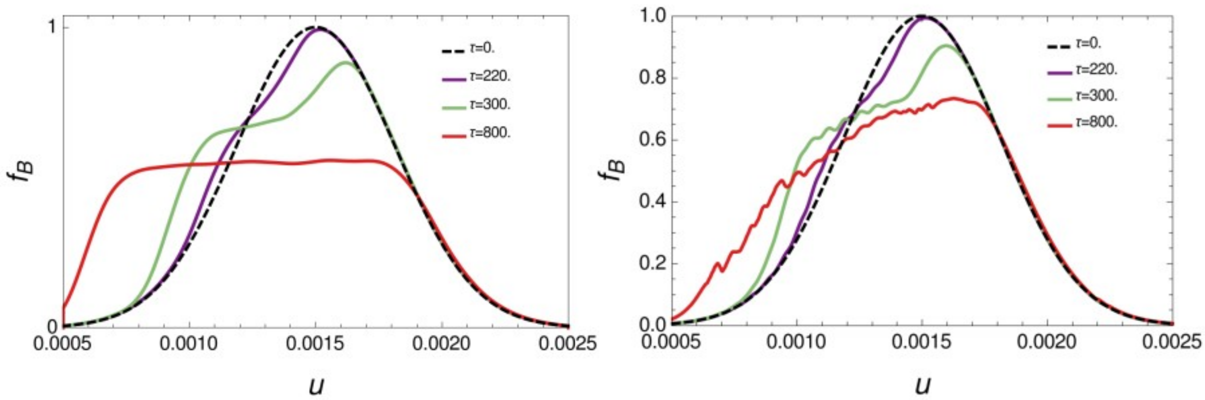
Hierarchical approach

The fluxes appearing in Eq. (5) can be calculated within different levels of increasing simplification of plasma dynamics. As an example, using the framework introduced in [1], we can retain only the effect of non linear diagonal interactions obtaining the following result:

$$\partial_t \overline{F}_{z0} \sim \frac{n^2 c^2}{2} \Im\mathrm{m} \sum_k \frac{\partial}{\partial \psi} \left[\overline{e^{-iQ} \left\langle \delta \psi_{gc} \right\rangle_k e^{iQ}} \Big|^*_\psi \mathscr{L}_k^{-1} \, \overline{e^{iQ} \left\langle \delta \psi_{gc} \right\rangle_k e^{-iQ}} \Big|_\psi \frac{\partial}{\partial \psi} \overline{F}_{z0} \right] \tag{7}$$

where $\overline{(\ldots)}|_\psi$ denotes orbit averaging using $\bar{\psi}$ as radial coordinates as in Eq. (6), $\mathscr{L}_k^{-1}$ is the inverse of the operator $\mathscr{L}_k \equiv \overline{\omega}_{dk} - \omega_k - i\partial_t + i\Delta \ldots$, $i\Delta \ldots$ formally denotes resonance broadening

[5], $v_\parallel \nabla Q - (\omega_d - \bar{\omega}_d) = 0$ and $\boldsymbol{k} \cdot \boldsymbol{v}_d \equiv \omega_d$. Furthermore, this expression could be calculated in the quasi-linear limit increasing the level of simplification. This expression describes only the phase space fluxes in the radial direction (dropping parallel nonlinearity) for low frequency fluctuations (fishbone paradigm [1]). Different model expressions for the fluxes will allow to calculate and compare the evolution of the plasma reference state, validating the reduced model and illuminating the essential underlying physics. As a simple example, we have applied this methodology to study the beam-plasma system [6]as a toy model describing wave-particle interactions in fusion experiments: in particular, the velocity coordinate in the beam plasma system can be mapped into the radial flux variable. In the following figure, we plot the beam velocity distribution function at different instants obtained from an N-body approach (left-hand panel), in comparison with the reduced model description (right-hand panel), where we have retained only diagonal interactions. This elucidates the hierarchical approach already discussed. It clearly emerges how the distribution function flattening is well described by the reduced model on relatively short time scales but some discrepancies emerge in the late evolution. This is due to the reduction to diagonal interactions and having neglected mode-mode coupling.



References

[1] Chen, Liu, and F. Zonca Reviews of Modern Physics 88.1 (2016): 015008.

[2] Falessi, M. V., and F. Zonca. Physics of Plasmas 26.2 (2019): 022305.

[3] Sugama, H. Reviews of Modern Plasma Physics 1.1 (2017): 9.

[4] Frieman, E. A., and L. Chen. The Physics of Fluids 25.3 (1982): 502-508.

[5] Dupree, T. H. The Physics of fluids 9.9 (1966): 1773-1782.

[6] Carlevaro, N., et al. Entropy 18.4 (2016): 143.