A new system of gyro-fluid equations with Onsager symmetry

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Abstract: Gyro-fluid equations are velocity space moments of the gyrokinetic equation. The damping due to kinetic resonances is included through a closure scheme chosen to match the collisionless density response functions. This damping allows for accurate linear eigenmodes to be computed, even in the collisionless limit, with a relatively low number of velocity space moments compared to gyrokinetic codes. The standard methods [1, 2] use the truncated moments to close the system of equations. An analysis of the gyro-fluid closure schemes will be presented that demonstrates a number of problems with the standard method. In particular, the Onsager symmetries [3] of the resulting quasilinear fluxes are not preserved. Onsager symmetry guarantees that the matrix of diffusivities is positive definite, an important property for a transport model. The constraints on the closure due to Onsager symmetry and other considerations are shown to be very restrictive. A new, simpler scheme for including the kinetic damping is found that preserves the Onsager symmetry and is scalable to higher velocity space moments without change of the damping model. Linear eigenmodes from the new system of equations are compared with gyrokinetic results, with and without collisions, including parallel and perpendicular electromagnetic fluctuations at high beta. The new system of gyro-fluid equations will be used to extend the TGLF quasilinear transport model [4] so that it can compute the energy and momentum fluxes due to parallel magnetic fluctuations, completing the transport matrix. The Onsager symmetries will enable faster transport solvers since the matrix of convection and diffusion coefficients will all be computed by a single call to the quasilinear transport model.

Advantage of the response matrix flux form

Consider the following "response matrix" form of the heat flux $Q = -n\left(\chi^{neo} + \chi^{turb}\right) dT/dr$.

The first term is the neoclassical thermal diffusivity χ^{neo} which is independent of the gradients. The second term is the due to turbulence and is a non-linear function of the gradients. For example, the turbulence terms could be $\chi^{turb} = \chi_0^{turb} \Big/ \left(1 + c_1\left(\gamma_{ExB}/\gamma_0\right)^2\right)$ where the ExB velocity shear is taken to be $\gamma_{ExB} = \left(c/eBL_n\right) dT/dr$.

This illustrative model is plotted in Fig. 1 for specific parameters chosen make a multi-valued curve. In a transport code, the model heat flux could be at the level of the red line labeled Q_{model} in Fig. 1 for three different values of the temperture gradient. The blue Newton method arrows show the

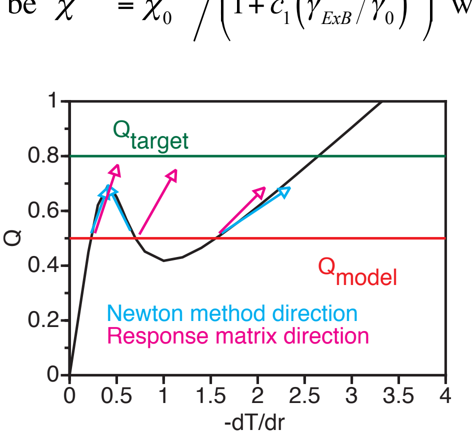

Fig. 1 Illustration of the transport solution problem

direction that would be taken if the gradient of the heat flux with respect to temperature gradient was used to determine the direction to the target flux. The Newton method will stall at the local maximum in the model flux and not be able to reach the target unless the starting point is on the high gradient branch. On the other hand, the response matrix form of the flux always has a positive coefficient $n\left(\chi^{neo}+\chi^{turb}\right)$ of the gradient. Taking the direction to the target just from the gradient gives the pink arrows in Fig.1 . This direction will always lead to the target flux but each iteration does not always reduce the difference between Q_{model} and Q_{target}. In general, the relation between the fluxes and gradients is a postivie definite matrix. This positive definite property is guaranteed by the Onsager symmetry of the response matrix for both neoclassical and turbulent fluxes [3].

Onsager symmetry

The linear electrostatic gyrokinetic equation for the fluctuation of the distribution function $\tilde{f}$ and the electrostatic potential $\tilde{\Phi}$ can be written in the form

$$G\tilde{f}=\sum_n X_n P_n\left(\mathrm{u},\mathrm{v}\right)F_u F_\mathrm{v} J_0^2\tilde{\Phi} \quad \text{where} \quad G=-\omega+\mathrm{u}k_p-\left(2\omega_{du}\mathrm{u}^2+\omega_{d\mathrm{v}}\mathrm{v}^2\right),$$

$$F_u=\frac{1}{\sqrt{\pi}}e^{-\mathrm{u}^2},\ F_\mathrm{v}=2\mathrm{v}e^{-\mathrm{v}^2},\ \int_{-\infty}^{\infty}d\mathrm{u}F_\mathrm{u}\int_0^{\infty}d\mathrm{v}F_\mathrm{v}P_n P_m=\delta_{n,m},$$

$$\sum_n X_n P_n=\left(\omega z/\tau+\omega_n\right)+\omega_p\mathrm{u}+\omega_{T_\parallel}\left(\mathrm{u}^2-1/2\right)+\omega_{T_\perp}\left(\mathrm{v}^2-1\right). \tag{1}$$

The parallel (u) and perpendicular (v) velocities are normalized to the species thermal velocity. The normalized equilibrium drift frequencies are given by: density gradient ω_n, parallel velocity gradient ω_p, parallel temperature gradeint $\omega_{T_\parallel}$ and perpendicular temperature gradient $\omega_{T_\perp}$. The parallel gradient operator is k_p and the curvture and grad-B drift terms are ω_{du}, ω_{dv}. The equilibrium parallel flow is neglected for simplicity. The electrostatic transport fluxes can be written as a product of velocity moments of the fluctuating distribution function and the fluctuating ExB drift.

$$Q_n=\int_{-\infty}^{\infty}d\mathrm{u}\int_0^{\infty}d\mathrm{v}P_n\tilde{f}\left(-ik_y\tilde{\Phi}^*\right)=\sum_m X_m\int_{-\infty}^{\infty}d\mathrm{u}\int_0^{\infty}d\mathrm{v}P_n P_m\left(\mathrm{u},\mathrm{v}\right)F_u F_\mathrm{v}G^{-1}J_0^2\tilde{\Phi}\left(-ik_y\tilde{\Phi}^*\right)=\sum_m X_m R_{n,m} \tag{2}$$

The first form in Eq. 2 is the definition of the flux, the second form has the linear solution for Eq. 1 substituted for the fluctuating distribution function. The last form defines the response matrix R which relates the fluxes to the linear gradient driven drift vector X. The real part of Eq. 2 is the physical transport flux. Clearly the response matrix is symmetric $R_{m,n}=R_{n,m}$. This is the essence of Onsager symmetry. It can be shown that this symmetry ensures that the entropy production is postitive [3].

New gyro-fluid closure

Gyro-fluid models take velocity space moments of the gyrokinetic equation and then close the system in a way to model the residue of the collisionless kinetic poles (resonances) in velocity space where G = 0. Without a closure, the collisionless gyro-fluid linear response will have singularities at a sequence of values along the real freqency line. Adding more velocity space moments does not remove these resonances it just makes more of them. A few properties of the standard gyro-fluid closures will be stated here without proof. Taking fluid moments of Eq. 1 with ortho-normal polynomials P_m results in a matrix form for the linear gyrokinetic equation. The matrix $G_{n,m}$ must be symmetric in order to satisfy Onsager symmetry (neglecting the mirror force, collisions and FLR terms). The standard gyrofluid method of closure [1, 2, 4] writes velocity space moments, that are above the order of the highest polynomial moment of the time derivative term in the fluid moments, as a linear combination of the lower moments. The coefficeints of this closure relation are then determined by fitting to the linear response for the lowest moments. This can produce a very accurate fit to the low moments (e.g. density) at the expense of the higher moments. This method does not respect Onsager symmetry since it makes $G_{n,m}$ not have any symmetry. A simple closure, that respects the Onsager symmetry, and still gives a reasonable model of the residues of the kinetic resonances, is to add a velocity space independent damping term to G: $G' = G - ic_p\left|k_p\right| - ic_{\mathrm{du}}\left|\omega_{\mathrm{du}}\right| - ic_{\mathrm{dv}}\left|\omega_{\mathrm{dv}}\right|$. The three coefficeints $\left(c_p, c_{du}, c_{dv}\right)$ are chosen to fit the collisionless kinetic response functions. This model has the advantage that it does not depend on the number of velocity space moments so the velocity space resolution of the gyro-fluid equations can be changed as needed. This was not possbile with the standard closure. One of the goals of this new model is to be able to accurately resolve the energy and momentum fluxes due to parallel magnetic field fluctuation response which requires a higher number of perpendicular velocity space moments than any of the previous gyrofluid models. The spatial direction parallel to the magnetic field is represented by the same system of Hermite polynomials as in Ref. 4. The parallel space resolution can also be changed without changing the new closure. A gyro-fluid system of equations using this closure has been coded in Mathemetica for verification with gyrokinetic linear results. This version is for the s-alpha model geometry (shifted circle, large aspect ratio). The mirror force terms coming from the conversion of the parallel gradient operator for constant energy and magnetic moment to a parallel gradient at constant u and v are included. This eliminates the need for bounce averaging of trapped particles that greatly added to the compexity of the TGLF equations [4]. The full Bessel function gyro-averging term in Eq. 1 is included for electrons and ions. The agreement between the linear growth rates for the new gyrofluid model and the gyrokinetic code GKS [5] is good for for both cases in Fig. 2 and 3. The mirror force term in

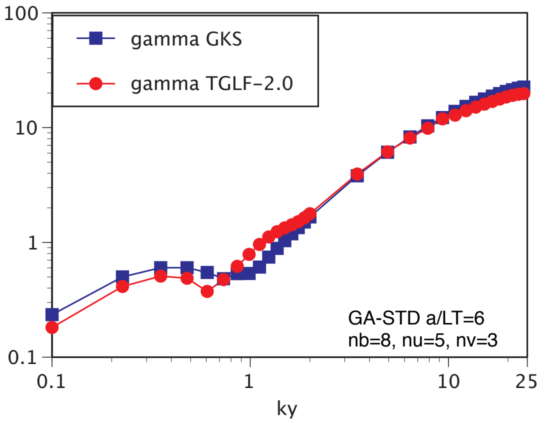

Fig.2 Linear growth rates for GA-STD case with a/LT=6.0 comparing the new gyro-fluid model (TGLF-2.0) to the GKS code [5]. Compare to Fig. 11 of Ref. 4.

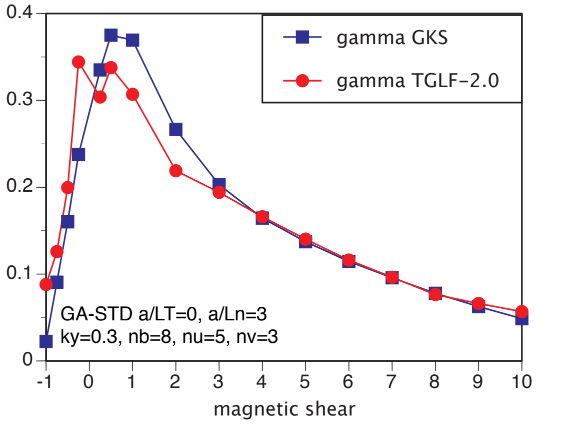

Fig. 3 linear growth rates for a trapped electron mode case (GA-STD with a/LT=0, a/Ln=3, ky=0.3) comparing the new gyro-fluid model (TGLF-2.0) to the GKS code [5]. Compare to Fig. 9 of Ref. 4.

the new model does not require the additional model tuning required for the bounce average TGLF [4] for the trapped electron mode case in Fig. 3. This is the same case as in Ref. 4. A large database of gyrokinetic linear stability calculations will be used to determine the optimum value of the closure coefficients for different velocity space and parallel moment resolutions. This is a promising first verification test of the new linear gyrofluid model. Electromagnetic and collision tests will be shown in the poster presentation.

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