Scattering of radio frequency waves by plasma turbulence

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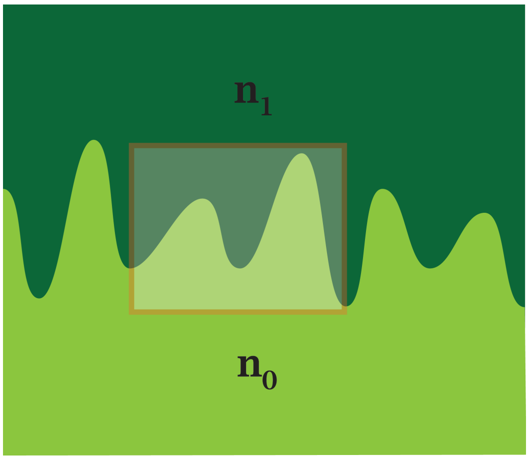
Introduction

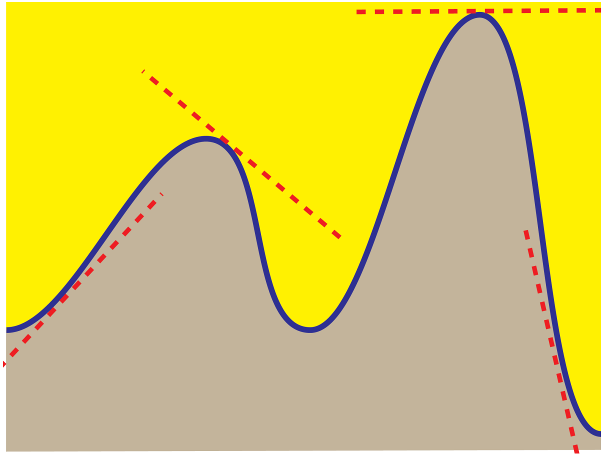
The scrape-off layer and the edge region in fusion plasmas are replete with turbulence induced incoherent fluctuations, and coherent fluctuations, such as blobs and filaments. Radio frequency (RF) electromagnetic waves, excited by antenna structures placed near the wall of a fusion device, encounter this turbulent region as they propagate towards the core. In order to optimize the heating of plasmas, or the generation of non-inductive plasma currents, it is necessary to properly assess the effect of this turbulence on RF waves. We have undertaken a set of theoretical and computational studies that model the propagation of RF waves through turbulent plasma. The theoretical models are mathematically tractable, and provide physical and intuitive insight into the effect of turbulence on RF waves. The computational studies provide support for these theoretical models. We use two complementary theoretical approaches – geometrical optics and physical optics – for magnetized plasmas with a tensor permittivity. The former, an approximation to the latter full-wave approach, is useful for incoherent fluctuations and leads to Snell's law and the Fresnel equations in plasmas. This is the basis of the Kirchhoff's approximation for scattering off density fluctuations [1, 2]. The physical optics method is the basis for studying scattering from coherent fluctuations [3, 4]. The two complementary analyses reveal important physical insights into the scattering of RF waves. Besides refraction and reflection, the spatial uniformity of power flow into the plasma is affected by side-scattering, diffraction, shadowing, and interference. Significantly, the incident RF wave power can couple to other plasma waves as a result of fluctuations. Within the framework of the COMSOL software, we have built a numerical code to study scattering of RF waves by fluctuations [5]. The code has been benchmarked against theoretical results, and is being used to study scattering from complex representations of density fluctuations.

Kirchhoff tangent plane approximation

The theoretical model we use for studying the scattering of electromagnetic waves from turbulent plasma is based on the Kirchhoff approximation that is commonly used to study the scattering of waves from rough surfaces [1, 2]. The basis of the Kirchhoff approximation, also



(a)



(b)

Figure 1: *(a) a surface separating two different plasma densities n_0 and n_1; (b) a closeup of the shaded region in (a) with examples of local tangent planes (red dashes).*

referred to as the tangent plane approximation, is built upon the theory of geometrical optics. In this approximation, the scattered field, due to planar turbulence, is determined by the wave fields on the surface of the turbulence separating two different plasma densities. Each point on this surface is assumed to be part of an infinitely homogeneous plane that is along the local tangent at that point. Consequently, the Kirchhoff approximation leads to the study of reflection and refraction of plane waves by planar surfaces using the theory of geometrical optics. This simplifies the modeling of RF scattering by turbulence and, importantly, gives insight into several, practically significant, aspects of scattering. Through this approach, we can obtain results for scattering in magnetized plasmas, described by an anisotropic permittivity tensor, that are equivalent to Snell's laws and the Fresnel's equations which are traditionally obtained for plane electromagnetic waves in isotropic scalar dielectrics.

As a consequence of the Kirchhoff tangent plane approximation, the scattering of RF waves by a turbulent plasma is treated locally in space. We follow a local Cartesian coordinate representation of a toroidal device in (x,y,z) space, with the unit vector $\hat{x}$ representing the radial direction, $\hat{y}$ being along the poloidal direction, and $\hat{z}$ being the direction of the ambient, homogeneous, magnetic field. Consider a three-dimensional surface in this geometry that separates plasmas with two different densities,

$$z = f(x,y), \tag{1}$$

as shown in Fig. 1(a). This surface is a representation of local turbulent region. At any point (x_0,y_0,z_0) on this surface, with $z_0 = f(x_0,y_0)$, for $z < z_0$ the electron density is n_0 and for $z > z_0$ the electron density is n_1. The RF wave is assumed to be incident on this surface from the region with density n_0. Figure 1(b) is a magnification of the shaded region in Fig. 1(a), and shows the local tangent lines (in red) that are indicative of the Kirchhoff approximation. The tangent plane is defined by the local normal. From,

$$\xi = x_0 - f(y_0, z_0) = 0, \tag{2}$$

the local unit normal vector is given by,

$$\hat{n} = \frac{\nabla \xi}{||\nabla \xi||}. \tag{3}$$

Let us define a new coordinate system $(\hat{x}', \hat{y}', \hat{z}')$, where $\hat{x}' = \hat{n}$. In general, $(\hat{x}', \hat{y}', \hat{z}')$ can be constructed from $(\hat{x}, \hat{y}, \hat{z})$ through two Euler rotations,

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix}, \tag{4}$$

where,

$$\tan\phi = -\frac{z_0}{x_0} \quad \text{and} \quad \tan\theta = \frac{y_0}{x_0 \cos\phi - z_0 \sin\phi}. \tag{5}$$

An important consequence of the Kirchhoff approximation follows in a straight forward fashion. In the $(\hat{x}, \hat{y}, \hat{z})$ system, let the incoming RF plane wave have the wave vector $\vec{k}_0 = k_{0\perp}\hat{x} + k_{0\|}\hat{z}$. In order to satisfy the electromagnetic boundary conditions at the plane separating two different plasma densities, all waves – incident, reflected, and transmitted – need to have common components of the wave vector in the tangent plane. This is a kinematic property that, for example, in conventional electrodynamics, leads to Snell's law for reflection and refraction of electromagnetic waves at an interface [4, 6] The dynamic properties, such as amplitudes and polarizations of the electromagnetic fields, also follow from the boundary conditions, but will not be discussed in this paper. If in the $(\hat{x}', \hat{y}', \hat{z}')$ frame, the wave vector of either the reflected or transmitted wave is given by $\vec{k}' = k'_x\hat{x}' + k'_y\hat{y}' + k'_z\hat{z}'$, then from the kinematic property,

$$k'_y = k_{0\perp} \sin\theta \cos\phi + k_{0\|} \sin\theta \sin\phi, \tag{6}$$

$$k'_z = k_{0\perp} \sin\phi + k_{0\|} \cos\phi, \tag{7}$$

for all waves. The k'_x for any wave is determined from the appropriately transformed dispersion relation for the magnetized plasma with the given density – for reflected wave the density is n_0 and for the transmitted wave it is n_1. If we inverse transform Eqs. (6) and (7) to the $(\hat{x}, \hat{y}, \hat{z})$ system, then it is clear that for the reflected wave having the same polarization as the incident wave, $k_y^R = 0$ and $k_z^R = k_{0\|}$. However, if $\phi \neq 0$ and $\theta \neq 0$, then, for the transmitted wave, $k_y^T \neq 0$

and $k_z^T \neq k_{0\|}$. This implies that, due to the fluctuations, part of the transmitted power is side-scattered, and the component of its wave vector along the magnetic field is different from that of the incident wave. The spectrum of the electromagnetic wave propagating past the fluctuation toward the core of the plasma is modified.

Another aspect of scattering that arises in plasmas is related to the dynamical property of the boundary conditions [4, 6]. At the interface separating two different densities, Maxwell's equations require the following,

$$\Delta\left(\hat{n}\cdot\overleftrightarrow{K}\cdot\vec{E}\right) = 0, \qquad \Delta\left(\hat{n}\cdot\vec{B}\right) = 0, \tag{8}$$

$$\Delta\left(\hat{n}\times\vec{B}\right) = 0, \qquad \Delta\left(\hat{n}\times\vec{E}\right) = 0, \tag{9}$$

where Δ indicates a jump across the tangent plane, $\overleftrightarrow{K}$ is the cold plasma permittivity [4], and $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, respectively, of the plane waves. These boundary conditions come from Gauss' laws for the electric and magnetic fields, Faraday's equation, and Ampere's equation, respectively. Of these six boundary conditions only four are independent for a cold plasma permittivity [4]. Consequently, apart from the incident wave, there have to be four waves involved in the scattering process. Otherwise, the boundary conditions cannot all be satisfied. Since the cold plasma dispersion relation has two independent wave modes, each of the reflected and transmitted fields have to be a sum of these two independent modes in the two, different, density regions. This implies that waves with polarizations different from the incident wave can be excited by turbulence induced scattering. For example, suppose an ordinary wave, in the electron cyclotron range of frequencies, is incident on the planar surface. Then the reflected and transmitted fields will be a sum of ordinary and extraordinary waves.

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