Forced nonlinear vertical oscillations of a single dust particle trapped in a stratified glow discharge

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Introduction. In dusty plasmas [1,2], which has been intensively investigated over the past 20 years, various types of oscillation phenomena can be investigated at the kinetic level. The investigation of dust particle oscillations make it possible to determine the key parameter of the dusty plasma – dust particle charge. For example, in [3,4] this value was obtained under the conditions of DC discharge with the help of investigations of the relaxation and forced linear vertical oscillations of a single dust particle.

.Obviously, the excitation of the large-amplitude oscillations and the appearance of nonlinear effects are most easily realized near the eigenfrequency and resonant frequency of the systems. The features peculiar to anharmonic oscillator such as hysteresis of the frequency response curve, secondary resonances were investigated under the conditions of RF discharge dusty plasma in [5-7].

Experimental setup. The nonlinear oscillations of the dust particle were caused by the imposition of the external large-amplitude force. The dust particle was placed in the potential trap created by standing striation. The low-frequency square-wave discharge current modulation led to the displacement of the striation. The nonlinear forced oscillations were caused by periodic movement of the dust particle equilibrium position.

The current discharge modulation was realized with the help of the experimental technique suggested in [3,4]. A DC stratified glow discharge was produced in neon at pressure p=0.16 torr and currents 2-3 mA. We used the calibrated spherical melamine formaldehyde particles with diameters $d = 4.10 \pm 0.14 \mu m$. Levitating dust particle was visualized by illumination of a 30 mW diode laser. The light scattered by dust grain was detected by a CCD video camera with the temporal resolution of 40 ms and the resolution of 960×720 pixels, located on the side of the tube. The error in measurement was about 12% as estimated in [3].

The discharge current modulator provided square wave output signals with on/off ratio of $\theta = 1/2$ with different values of modulation depth μ. Under the experimental conditions μ was determined as follows $\mu = 1 - \frac{i_{\min}}{i_{\max}}$. The current switching from $i_{\min} = (1-\mu) i_{\max}$ to $i_{\max}$ =2.6 mA leads to the rigid shift of all striations by the order ΔZ =1.5-3 mm. When the current switches back from $i_{\max}$ to $i_{\min}$ the striations return to the initial positions. The illustration of the periodical movement of the striations caused by square-wave discharge current modulation is presented in figure 1a.

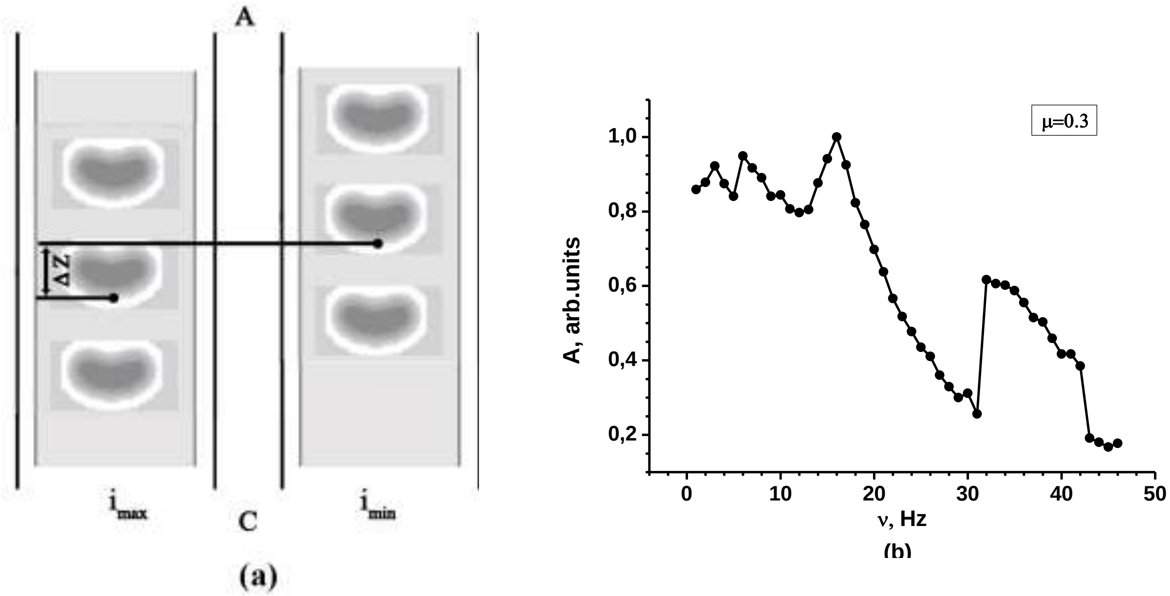

Figure 1 – (a) Illustration of the dust particle displacement caused by modulation of the discharge current. The double ended arrow shows the periodical displacement of the dust particle by an order ΔZ from the cathode (C) to the anode (A) and back. (b) AFC of forced oscillations of the single dust particle.

In figure 1b the typical multiresonance curve of the forced dust particle oscillations is shown. It is observed the main resonance frequency at ν_{res} =16 Hz, the resonance peaks at subharmonic frequencies ν_{res}/k, where $k = 3,5...$, caused by nonharmonic time dependence of the driving force [4], resonance at double frequency $2\nu_{res}$ =32 Hz.

Hysteresis. The occurrence of the asymmetric resonance at double frequency $2\nu_{res}$, see figure 1 b, point to the appearance of the hysteresis character of the AFC of the dust particle oscillations. In order to obtain the hysteresis resonance curves the frequency response was measured in detail with increments of 0.5 Hz for increasing and decreasing frequency of excitation. Figure 2a shows the AFC obtained close to the resonance maximum at frequency ν_{res} =16 Hz under the square wave discharge current modulation with modulation depth of μ =0.36. The hysteresis resonance curve shown makes it possible to determine the width of hysteresis zone, which takes the value of $\Delta\nu$ =1.5 Hz. Figure 2b shows the AFC obtained close to the resonance maximum at frequency $2\nu_{res}$ under the square wave discharge current modulation with modulation depth of μ =0.41. It is observed that for increasing of excitation frequency the amplitude reaches the maximum value at double frequency $2\nu_{res}$ =34 Hz and thereafter decrease monotonically. For decreasing of excitation frequency the amplitude follows the same resonance curve until the doubled frequency $2\nu_{res}$, but thereafter the amplitude increases continuously from the value of 0.9 mm to 1.3 mm.

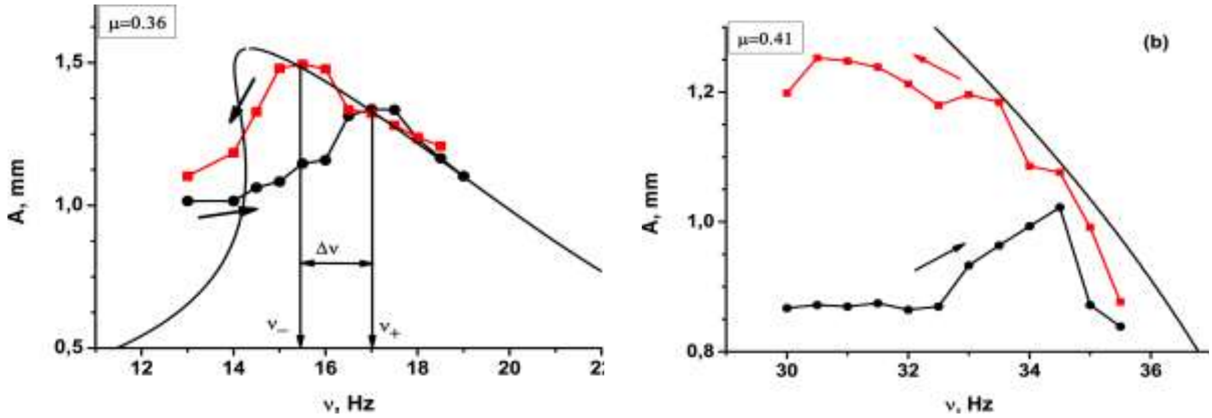

Figure 2 – The fragments of AFC obtained with increasing (black line) and with decreasing (red lines) frequency of excitation. AFC were obtained (a) close to the ν_{res} under the square wave discharge current modulation with μ =0.36, (b) close to the $2\nu_{res}$ under the square wave discharge current modulation with μ =0.41. The vertical arrows show the values of the resonance frequencies ν_{+} for increasing and ν_{-} for decreasing frequency of excitation, the double ended arrows indicates the width of hysteresis zone $\Delta\nu = \nu_{+} - \nu_{-}$. The symbols represent experimental data, the solid line corresponds to approximation curve, described by (a) – equation (2), (b) – equation (3).

The quantitative description of the experimentally obtained dust particle oscillatory motion was made on the theory of the forced anharmonic oscillator [8]. The equation describing the anharmonic oscillations with damping constant γ under the action of the driving force $f(t)$ has the form $\ddot{z} + 2\gamma\dot{z} + \omega^2{}_0 z = f(t)/\mathrm{M}_d - \alpha z^2 - \beta z^3$. Here ω_0 is the eigenfrequency of dusty plasma oscillatory system, α and β are the anharmonic coefficients. The driving force $f(t)$ takes the form

$$f(t) = \begin{cases} f_{max}\ , for\ 0 < t < \theta T \\ f_{min}, for\ \ \theta T < t < T \\ f\left(t + (j+1)T\right) = f\left(t + jT\right)\ \ j = 0,1,2\ldots \end{cases}, \qquad (1)$$

Here f_{max} is the maximum value of the driving force amplitude corresponding to the value of discharge current $i_{\max}$, $f_{\min}$ is the minimum value of the driving force amplitude corresponding to the value of discharge current $i_{\min} = (1-\mu)i_{\max}$, T is the period.

First, let us consider the main resonance $\omega = \omega_0$, where the quantitative description is made in a narrow region $\varepsilon = \omega - \omega_0$. This approach is normally valid in the limit $\gamma << \omega_0$ and is justified for our case. The value of eigenfrequency obtained under the same experimental conditions in [3] is $\omega_0/2\pi = 21$ Hz, the damping constant calculated with the help of the Epstein formula [9] takes the value of γ=15 s^{-1} at pressure p=0.16 torr. The dependence of the amplitude A on ε and F_0 is given by equation. Here κ is a nonlinear coefficient.

$$A^2\left[\left(\varepsilon-\kappa A^2\right)^2+\gamma^2\right]=\frac{F_0^2}{4\omega_0^2}. \quad (2)$$

The AFC of nonlinear dust particle oscillations measured close to main resonance, see figure 2a, were described by equation (2) with the value of $\kappa = 1130 s^{-1}cm^{-2}$. It observed that the maximum value of amplitude at ν_+ for increasing excitation frequency and the resonance peak at ν_- for decreasing excitation frequency are well described by the theoretical curve.

Let us consider the double resonance $\omega = 2\omega_0$ in a narrow region $\varepsilon = \omega - 2\omega_0$. The amplitude of oscillation is given by the equation

$$A^2\left[\left(\frac{\varepsilon}{2}-\kappa A^2\right)^2+\gamma^2\right]=\frac{\alpha^2 A^2 F_0^2}{36\omega_0^6}. \quad (3)$$

The resonance at double frequency caused by the parametric instability [8] appears in the range $\varepsilon_1 < \varepsilon < \varepsilon_2$, where $\varepsilon_{1,2} = \pm\sqrt{\left(\frac{\alpha F_0}{3\omega_0^3}\right)^2 - 4\gamma^2}$. Figure 2b shows the hysteresis curve measured close to the $2\nu_{res}$ and its theoretical description given by equation with the value of the anharmonic coefficient $\alpha = -5.8*10^5\, rad*s^{-2}/cm$. It is observed that the experimental data coincide with the calculated boundary values $\varepsilon_{1,2}$ and fit quite well on the stable branches of the solution of equation (3).

Acknowledgements. Work was supported by RFBR grant No. 18-32-00685.

References

[1] P. Shukla, Introduction to Dusty Plasma Physics, Institute of Physics Publishing, Bristol 2002.
[2] H. Thomas, G. Morfill, V. Demmel et al., Phys. Rev. Lett.**73**, 5 1994.
[3] Yu. Golubovskii, V. Karasev and A. Kartasheva, Plasma Sources Sci. Technol. **26,** 11 2017.
[4] Yu. Golubovskii, V. Karasev and A. Kartasheva, Plasma Sources Sci. Technol. **27,** 6 2018.
[5] C. Zafiu, A. Melzer and A. Piel Phys. Rev. E **63**, 6 2001.
[6] A. Ivlev, R. Sutterlin, V. Steinberg et al., , Phys. Rev. Lett. **85**, 19 2000.
[7] H. Schollmeyer, A. Melzer, A, Homann et al., Phys. Plasmas **6,** 7 1999.
[8] L. D. Landau, Mechanics (Course of Theoretical Physics vol. 1), Pergamon, Oxford 1980
[9] P. S. Epstein, Phys. Rev. **23,** 6 1924.