Application of Laplacian Eigen Functions for the tomographic reconstruction of tokamak

S. Purohit[1], Y. Suzuki[1, 2], S. Ohdachi[2]

[1] *SOKENDAI, The Graduate University for Advanced Studies, Toki, Gifu 509-5292, Japan*

[2] *National Institute for Fusion Science, Toki, Gifu 509-5292, Japan*

Introduction The tomographic reconstruction via global orthogonal pattern, such as Fourier–Bessel functions, efficiently recovers the emission profile and hence employed extensively [1,2]. One of the key features of these global orthogonal patterns is that the emission profile is recovered successfully even with the limited information, unlike the other methods like Regularizations [3], especially for the tangential viewing imaging diagnostic. These orthogonal patterns satisfy the boundary conditions such as the Dirichlet's boundary condition, patterns are zero at the boundary.

The reconstruction by orthogonal patterns is performed via Equation 1, where the 2D emission profile, **E**, is reconstructed by the linear combination of different orthogonal patterns, $\mathbf{P}_i$ where χ_i expansion coefficients. The χ_i values are estimated by regularization. See equation 2, Where $\mathbf{C}_i = A\mathbf{P}_i$, A is the geometric matrix, **B** is the line integrated data and γ is the regularization parameter. The α decides regularization type, in case of α=2, the regularization is referred to as L2/ rigid type and α=1,corresponds to L1, Lasso type [4].

$$\mathbf{E} = \sum_i \chi_i \mathbf{P}_i \qquad (1)$$

$$\underset{\chi}{\text{agr min}} \left\{ \sum_i (B_i - \chi_i \mathbf{C}_i)^2 + \gamma \sum_i |\chi_i|^\alpha \right\} \qquad (2)$$

The patterns aligned to the magnetic surfaces, in other words calculated on the magnetic surfaces, the orthogonality is retained almost the same, near orthogonal nature, even for non-circular plasma cross-sections. The flux surface aligned pattern are important while expressing the localized features like magnetic islands in the tomographic reconstruction smoothly [4]. However, flux surfaces are then always required, which is a practical limitation. The situation is more critical with the near real-time tomographic reconstructions. This article proposes a new tomographic reconstruction process based on the Laplacian's Eigen function (LEF) patterns, as the orthogonal patterns, for tangential viewing imaging diagnostic, where flux-surface information is not required.

Laplacian's Eigen function The Laplacian's Eigen function, equation 3, shows that for any nontrivial solution u = ϕ of such a boundary value problem is termed the Laplacian Eigen-function corresponding to the eigenvalue λ [4,5]. The solution is calculated conveniently via kernel/Green's function, equation 4.

$$\nabla^2 u = \lambda u \qquad (3)$$

$$k(r, r') = -\frac{1}{2\pi} ln|r - r'| \qquad (4)$$

It is clear from equation 4 the kernel is the sole function of the distance, term $|r - r'|$, between the two points r and r'. Hence, LEF patterns calculated via equation 4 are independent of the shape of the domain, in our case plasma shape. LEF based tomographic reconstruction only requires the last closed flux surface (LCFS) information to estimate the patterns, in contrast to the FB where flux surface information is required [4,5]. LEF patterns if aligned to the flux surfaces, hold orthogonal character which comforts the reconstruction. LEF patterns employed for the reconstructions, are global patterns and thus capable of recovering the emission profile with limited information from diagnostic. Therefore it is expected from LEF reconstruction to recover the emission profile even with restricted plasma viewing geometry, imaging diagnostic is viewing only a part of plasma cross-section. Such impressive qualities drive the motivation to present this work.

Reconstruction procedure Proposed LEF based tomographic reconstruction procedure is defined for the circular cross-section tokamak plasma, R= 0.75*m* and a = 0.25 *m*, with tangential viewing imaging diagnostic (30x30 detector pixels, 900 lines of sight), as shown in figure 1(A). The image plane is at $\phi = 270^o$ the location, shown by a red circle, figure 1(A). The present viewing geometry allows the diagnostic to view full plasma cross-section. The first step in the reconstruction procedure is the calculation of Eigen-function patterns, $\mathbf{P}_i$. The patterns are calculated over the domain considering the flux surfaces, for convenience via the Green's function defined in equation 4. The pectoral representation is given in figure 1(B). The Eigen-function patterns, $\mathbf{P}_i$, corresponding to the first 9 Eigen-values are shown in figure 1(C). The second step is the determination of the expansion coefficients χ_i . This determination is carried out by equation 2 while considering $\alpha = 1$, Lasso regularizations. The parameter γ in equation 2 is optimized by the cross-validation method [4]. Once both the information, orthogonal patterns, $\mathbf{P}_i$ and χ_i are achieved, the 2D emission profile is recovered from the linear combination of $\mathbf{P}_i$ and χ_i.

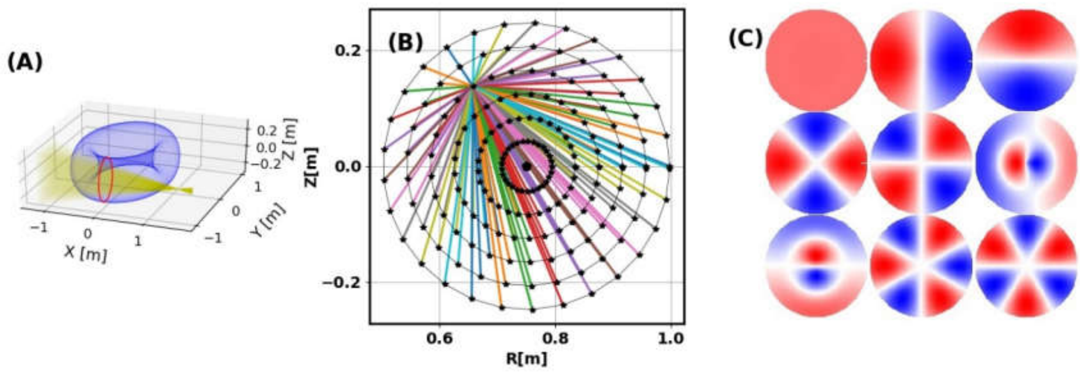

Figure 1: The viewing geometry (A), the LEF pattern calculation (B) and the final LEF patterns corresponding to first 9 Eigen values (C) are presented, respectively.

Results The tomographic reconstruction for different emission profile and viewing condition are shown in figure 2. The first column (*a*) shows the assumed Gaussian profile and the outer white circle represents the circular poloidal limiter, the second column (*b*) represents the synthetic image seen by the detector (30x30 pixels). The reconstructed image by standard Phillips-Tikhonov (PT) [6] , for reference, and LEF are shown in (*c*) and (*d*), respectively. The recovered 1D emission profiles, from reconstructed images are compared with the assumed profile, in column (*e*). The first row corresponds to the Gaussian soft X-ray profile. The PT 1(*c*) and LEF 1(*d*) successfully recovers the 2D emission profile. While comparing the 1D profiles from these two images both LEF and PT matches with the assumed profile and determines the magnetic axis location efficiently, see 1(*e*). The second row corresponds to a VUV / visible like emission profile. The PT 2(*c*) and LEF 2(*d*) able to recover the typical assumed profile, 2(*a*). The LEF is able to estimate the VUV / visible like emission location in accordance with the assumed profile, see 2(*e*). The third row in figure 2 is a very special case and is termed as the restricted viewing case. The motivation behind taking up such a situation is that in future fusion devices it is highly expected that the imaging diagnostic may not able to view the full plasma cross-section. Referring to the synthetic image 3(*b*), in figure 2, exhibits the restricted view case where only 50% of the plasma is visible to the diagnostic. The reconstruction by the PT 3(*c*) fails to recover the assumed 2D profile. This is most likely associated with the least-square approximation, core analogy of the PT. In absence of the data the least-square approximation is not efficient enough to recover the emission. In contrast to the PT the LEF 3(*d*) is able to recover the 2D emission profile as well as the recovered 1D emission profile matches well with the assumed profile 3(*e*). The success of the LEF is attributed to the fact that the LEF patterns, $\mathbf{P}_i$, are global patterns, calculated over the complete domain.

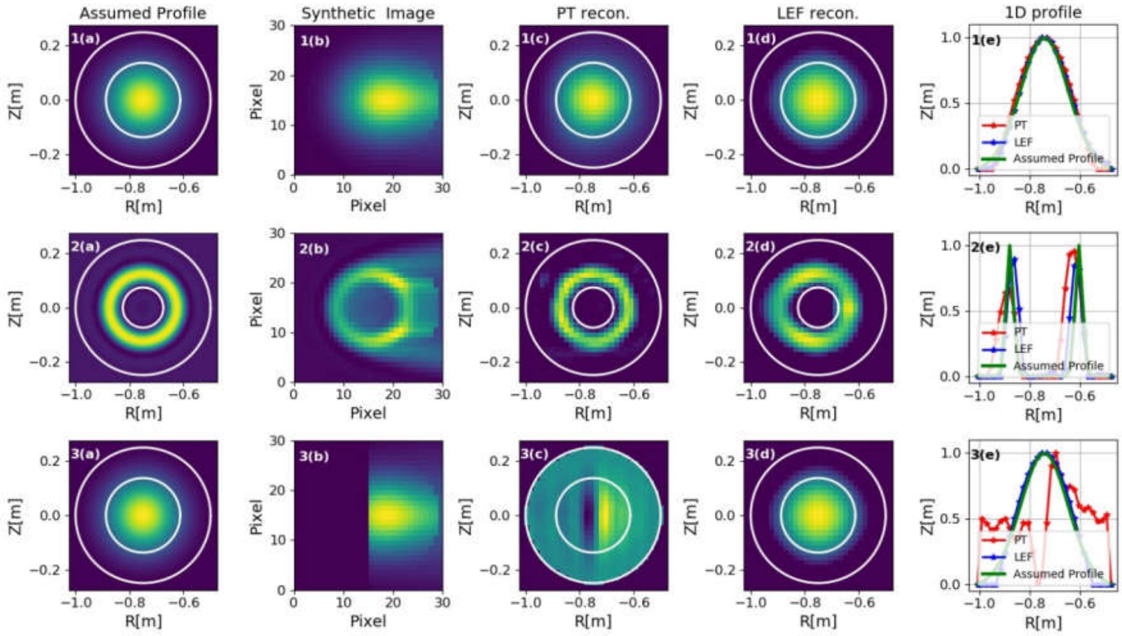

Figure 2. The Assumed Profile (*a*), synthetic image (*b*), PT reconstruction (*c*), LEF reconstruction (*d*) and the recovered 1D profiles (*e*) shown respectively.

Conclusions The LEF based tomographic reconstructions for circular cross-section tokamak is performed and results are compared with standard Phillips-Tikhonov method. The LEF performance is comparable with the PT for the SX/VUV type of emissions. However for the restricted viewing case the LEF performance is significantly better than the Phillips-Tikhonov method in determining the magnetic axis location.

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