Global structure of stationary zonal flow in rotating tokamak plasmas

N.A. Marusov[1−3], V.P. Lakhin[1,2], E.A. Sorokina[1,2], A.I. Smolyakov[2,4]

[1] *NRC "Kurchatov Institute", Moscow, Russia*

[2] *RUDN University, Moscow, Russia*

[3] *Moscow Institute of Physics and Technology, Moscow, Russia*

[4] *University of Saskatchewan, Saskatoon, Canada*

Zonal flows are low-frequency, predominantly electrostatic plasma oscillations $m = 0$, $n = 0$ (m is the poloidal wave-number, and n is the toroidal wave-number) widely observed in modern toroidal magnetic plasma confinement systems, such as tokamaks and stellarators [1]. It is believed that zonal flows are able to regulate the level of anomalous transport in plasma through nonlinear interaction with small-scale drift-wave turbulence [2].

Earlier, in the framework of ideal MHD, the local dispersion relation demonstrating coupling of the so-called stationary zonal flow (ZF) and geodesic acoustic mode (GAM) in rotating plasma was obtained [3, 4]. Under the assumptions of the adiabatic equation of state and the equilibrium with p/ρ^{α} constant on magnetic surfaces (p is the plasma pressure, ρ is the mass density, and α is an arbitrary function of the magnetic surface) the continuous spectrum of ZFs and GAMs in the presence of toroidal plasma rotation is described by the expression [4]:

$$\omega_{\pm}^2 = \frac{\omega_s^2}{2}\left\{2+\frac{1}{q^2}+4M^2+\frac{\gamma M^4}{2\alpha} \pm \left[\left(2+\frac{1}{q^2}+4M^2+\frac{\gamma M^4}{2\alpha}\right)^2 - \frac{\gamma-\alpha}{\alpha}\frac{2M^4}{q^2}\right]^{1/2}\right\}. \quad (1)$$

Here $\omega_s = c_s/R_0$, c_s is the speed of sound, R_0 is the tokamak major radius, q is the safety factor, $M = \Omega R_0/c_s$ is the sound Mach number, Ω is the angular frequency of toroidal plasma rotation, and γ is is the specific heat ratio. The high-frequency branch of (1), ω_+, corresponds to GAM, and the low-frequency branch, ω_-, corresponds to ZF. Equilibrium plasma rotation ($M \neq 0$) results in finite ω_-, which is strongly determined by the type of plasma equilibrium, i. e. by the parameter α. The oscillations appear to be stable if $\alpha < \gamma$ and aperiodically unstable in the opposite case. The mechanism of this instability related to the poloidal plasma stratification on the magnetic surface was considered in Ref. [3]. For isentropic magnetic surfaces ($\alpha = \gamma$) the frequency of ZF is always zero.

In this paper we consider the possibility of the existence of global mode of stationary zonal flow in tokamak with toroidal plasma rotation. The corresponding eigenvalue problem was formulated in Ref. [5] taking into account small magnetic perturbations. For low-frequency zonal flows in large-aspect-ratio axisymmetric tokamaks perturbed plasma displacement, ξ, can be described by the expression

$$\xi = \frac{1}{B^2}\mathbf{B}\times\nabla\phi + \frac{\zeta}{B^2}\mathbf{B}.$$

Here ζ/B is the longitudinal plasma displacement, and ϕ is proportional to the perturbed electric potential. In general, $\phi = \phi(r,\theta)$, where r labels the radius of magnetic surface, and θ is the poloidal angle on magnetic surface, which is assumed to be circular in what follows. Purely electrostatic perturbations are described by $\phi = \phi(r)$, and the poloidal dependence of the potential corresponds to electromagnetic effects: $\partial\phi/\partial\theta \sim \partial A_{||}/\partial t$, where $A_{||}$ is the longitudinal component of vector potential. In the finite-mode approximation: $\phi = \phi_0(r) + \phi_{2c}\cos 2\theta + i\phi_{2s}\sin 2\theta$ and $\zeta = \zeta_c\cos\theta + \zeta_s\sin\theta$, which is justified for small $\beta = 2p/B^2$, the resulting set of equations describing global structure of axisymmetric zonal flows has the form

$$D\frac{d\phi_0}{dr} + \frac{G_c}{r^2}\frac{d\Phi_{2c}}{dr} + \frac{\Phi_{2c}}{\rho r^2 f}\left(\omega^2 - \frac{\omega_s^2}{q^2}\right)\frac{df^2}{dr} + \frac{G_s}{r^2}\frac{d\Phi_{2s}}{dr} = 0, \tag{2}$$

$$\frac{1}{q}\frac{d}{dr}\left[\frac{1}{r^3}\frac{d}{dr}\left(\frac{\Phi_{2c}}{q}\right)\right] - \frac{d}{dr}\left(\frac{1}{2\omega_A^2 r}\frac{G_c}{(\omega^2-\omega_s^2/q^2)}\frac{d\phi_0}{dr}\right) = 0, \tag{3}$$

$$\frac{1}{q}\frac{d}{dr}\left[\frac{1}{r^3}\frac{d}{dr}\left(\frac{\Phi_{2s}}{q}\right)\right] - \frac{d}{dr}\left(\frac{1}{2\omega_A^2 r}\frac{G_s}{(\omega^2-\omega_s^2/q^2)}\frac{d\phi_0}{dr}\right) = 0, \tag{4}$$

where

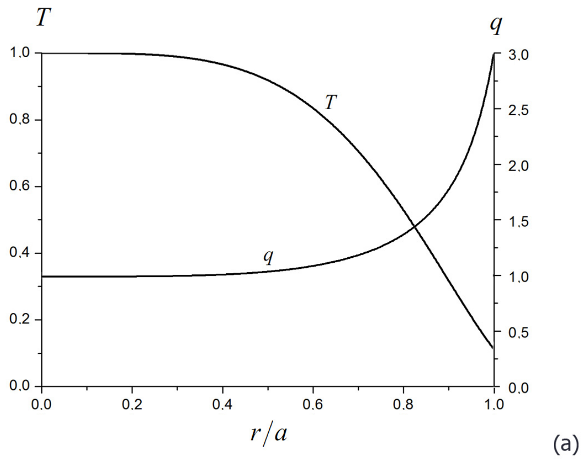
$$D = \omega^4 - \omega_s^2\omega^2\left(2 + \frac{1}{q^2} + 4M^2 + \frac{\gamma M^4}{2\alpha}\right) + \omega_s^4\frac{\gamma-\alpha}{\alpha}\frac{M^4}{2q^2},$$

$$G_c = \frac{1}{2}\left(\omega^2\left(\omega^2 - \frac{\omega_s^2}{q^2}\right) - D\right), \qquad G_s = \frac{2M\omega\omega_s^3}{q}\left(1 + \frac{M^2}{2}\right),$$

$$f = \rho\omega_s^2\frac{M^2}{2}\left(1 + \frac{\gamma M^2}{2\alpha}\right), \qquad \rho = \rho_0(r)\left(1 + \frac{\gamma M^2}{\alpha}\frac{r}{R_0}\cos\theta\right).$$

Here $\Phi_{2c} = r^2\phi_{2c}$, $\Phi_{2s} = r^2\phi_{2s}$, $\omega_A = v_A/R_0$, v_A is the Alfven velocity, $\rho_0(r)$ is the plasma density without rotation, and B_0 is the equilibrium magnetic field on tokamak magnetic axis. Neglecting the poloidal harmonics of the potential, $\phi_{2c} = 0, \phi_{2s} = 0$, Eq. (2) describes the continuous spectrum of GAM and ZF (1), which is defined by the equation $D = 0$. Finite β couples the perturbations of electrostatic potential with magnetic perturbations that gives the possibility for global eigenmode occurrence. In a static case ($M = 0$) with $\phi_{2s} = 0$ Eqs. (2), (3) describe the global GAMs, which were previously studied in Refs. [6, 7].

Results of the numerical solution of Eqs. (2)-(4), where we looked for global solution with eigenfrequency near the frequency of ZF spectrum, ω_-, are presented below. We impose zero boundary conditions for Φ_{2c} and Φ_{2s}: $\Phi_{2c,s}(r=0) = 0$, $\Phi_{2c,s}(r=a) = 0$, where a is tokamak minor radius. The first condition follows from the regularity of the solution on the magnetic



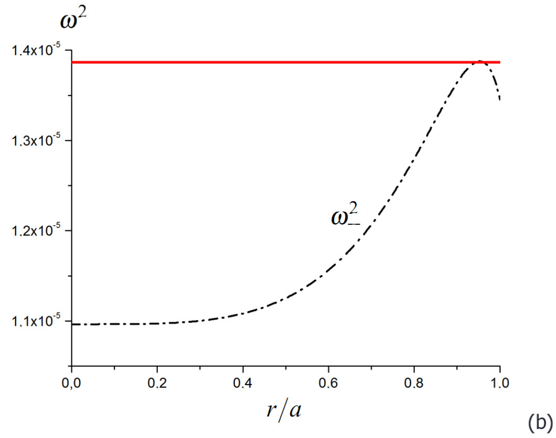


Figure 1: *(a) – radial profiles of the normalized temperature (equals to unity at* $r = 0$*) and of the safety factor; (b) – continuous spectrum (black dashed line) and eigenfrequency (red solid line) of ZF. Here* $M(r = 0) = 0.1$*,* $\alpha = 1$*,* $\beta = 0.04$ *and the frequency is normalized on the frequency of sound at magnetic axis*

axis, and the second one provides zero radial component of the perturbed velocity on plasma boundary. As an example we consider monotonic profiles of safety factor, q, and temperature, T, shown in Fig. 1a, and rigid-body profile of equilibrium toroidal rotation, $\Omega(r) = \text{const}$. In Fig. 1b continuous spectrum of ZF, ω_-^2, calculated from (1) for relatively small toroidal rotation with $M(r = 0) = 0.1$ and isothermal magnetic surfaces ($\alpha = 0$), is shown. It has the characteristic maximum on the periphery of plasma due to the increase of $M \sim 1/T$ with radius.

For isothermal magnetic surfaces ($\alpha = 1$) the eigenmode of stable ZF with frequency $\omega^2 \approx 1.4 \cdot 10^{-5}\, \omega_s^2(r = 0)$, which is independent on radius, exists near the maximum of the local ZF frequency, ω_-, and this is quite similar to the case of the global GAM [8, 9]. The radial structure of the eigenmode is shown in Fig. 2. The electric field of the global ZF is localized at the periphery of plasma column – see Fig. 2a, meanwhile the associated small magnetic perturbations, correspond to ϕ_{2c} and ϕ_{2s}, are radially extended – see Fig. 2b. The estimation of the mode frequency for $R_0 \sim 1\,\text{m}$ and temperature at plasma center $\sim 1\,\text{keV}$ gives $\omega_{ZF}/2\pi$ of the order of several kHz.

The calculation for the same equilibrium plasma profiles, but for isodense magnetic surfaces ($\alpha \to \infty$) reveals the existence of the aperiodically unstable ZF near the minimum of $\omega_-(r)$ with increment of instability $\gamma = \sqrt{-\omega^2} = 5 \cdot 10^{-3}\, \omega_s(r = 0)$. The radial structure of its eigenfunction is similar to one shown in Fig. 2, i.e., this mode excites on tokamak plasma edge.

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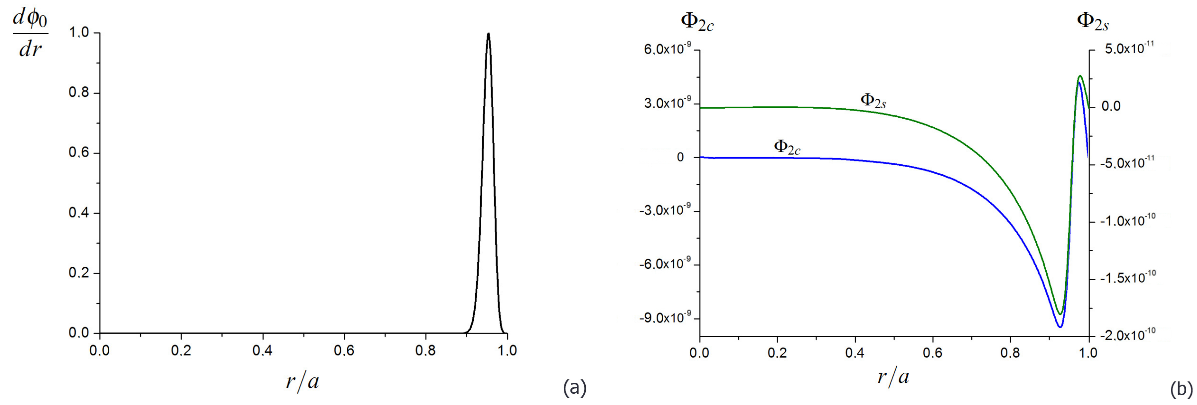

Figure 2: *Radial profiles of normalized eigenfunctions: (a) $d\phi_0/dr$; (b) Φ_{2c} (blue line) and Φ_{2s} (green line), corresponding to stable global ZF with $\omega^2 \approx 1.4 \cdot 10^{-5}\,\omega_s^2(r=0)$. Here $M(r=0) = 0.1$, $\alpha = 1$, $\beta = 0.04$*

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