Global MHD stability of plasma in Galatea traps

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Magnetic plasma confinement systems with conductors embedded into plasma is an important class of magnetic traps with high beta, known as Galateas [1], alternative to the mainstream toroidal system designs. As summarized in [2], Galateas are widely diversified, and it provides additional reason to consider them, unlike low-beta traps, as the promising systems for many plasma technologies and for advanced fuel reactors while the technical difficulties arising from the magnetic suspension of the embedded conductors ("myxines") and their operation in reactor conditions can be overcome with present-day technologies.

The studies of geometric and other parameters of axisymmetric plasma configurations maintained in an equilibrium by the magnetic field of both plasma current and toroidal currents in the myxines at zero toroidal field is based on the solution of the Grad-Shafranov equation (for recent results see [3,4]). Various equilibria with complex magnetic field surface topology can be realized in Galateas. The use of the unstructured grid ideal MHD stability code MHD_NX [5] makes possible plasma stability studies in Galatea traps [6]. Apart from the localized convective mode stability criteria (like Rosenbluth-Longmire-Kadomtsev [2]), global mode stability calculations in multiply connected plasma domain can be performed taking into account a gap between the plasma and the vacuum vessel (external modes). For equilibrium configurations in the Galatea magnetic trap "Trimyx" [7,8], the dependence of the growth rates of ideal MHD modes with different toroidal wave numbers on the pressure magnitude is investigated in this paper.

1. Plasma equilibrium in Trimyx magnetic trap. The Grad–Shafranov (GS) equation $\nabla\cdot\left(\nabla\psi / R^2\right) = -\mu_0 j_\varphi / R$ is solved for the axisymmetric equilibrium field $\mathbf{B} = \nabla\psi\times\nabla\varphi + F\nabla\varphi$ without toroidal component $F = 0$. The external magnetic field in the trap is created by currents in the circular coils (Fig. 1) placed in a conductive cylinder of radius R = 0.7 m, -0.4 m < Z < 0.4 m with its boundaries assumed ideally conducting. The level lines of the function ψ for the vacuum magnetic field are determined by solving the GS equation with the condition $\psi = 0$ at the computational domain boundary and localized current sources $j_\varphi = \sum_k I_k \delta(R - R_k, Z - Z_k)$ modeling the coil currents. The separatrices and X-points of the vacuum magnetic field are determined, and the plasma is assumed located in the vicinity of the separatrix of the vacuum magnetic field $\psi = \psi_s$, which encloses three coils with the currents flowing in the same direction. The pressure function is prescribed with the plasma localized near the separatrix wrapping around the coils in thin layers called mantles:

$p(\psi) = p_0 \exp(-(\psi - \psi_s)^2 / \delta^2)$ with the maximum near the separatrix and the parameter δ controlling the "thickness" of the plasma configuration. Limiting the computational domain to magnetic surfaces $\psi = \psi_k$ outside the coil locations (R_k, Z_k), it is possible to reduce the equilibrium problem to the boundary value problem in the multiply connected domain with the uniform right hand side representation $j_\varphi = R\, dp / d\psi$ – so-called equilibrium with the fixed boundary. The equilibrium presented in [4] with the maximal value of pressure $p_0 = 2\pi \cdot 16$ Pa calculated on a rectangular grid was used as a reference one to determine the values of ψ_k. Fig. 1b shows the corresponding magnetic surfaces approximated by circles around the coils in the (*R*,*Z*) plane. The main reason for introducing the internal boundaries is to provide a convenient way to set boundary conditions for the stability problem at the internal conductors. The nonlinear Dirichlet boundary value problem is solved with the standard PDE Toolbox package from Matlab (Fig. 1c) on triangular unstructured grids. For the reference equilibrium with the total current in the coils 66 kA = (25+20+20-21) · 1.5 kA, the values $\psi_s = 3\text{e-}3\text{Wb}/(2\pi)$, $\delta = 1.5\text{e-}3\text{Wb}/(2\pi)$ and the maximal pressure $p_0 = 2\pi \cdot 16$ Pa, the magnitude of the plasma current is 2.67 kA, which corresponds to the reference equilibrium. The values of ψ_k are kept fixed in the series of equilibria with increasing pressure. Wherein the iterations to solve the nonlinear problem converge up to the values of p_0 ten times higher than in the reference equilibrium.

2. Ideal MHD stability of Galateas. The equilibria on the unstructured triangular grids are used as inputs for the ideal stability MHD_NX code [5]. The code has been upgraded to include the plasma compressibility term $1/2\int_{V_p} \Gamma p \,|\nabla \cdot \boldsymbol{\xi}|^2 dV$ into the perturbed potential energy functional W_F, where Γ is the adiabatic index, $\boldsymbol{\xi}$ is plasma displacement vector. The boundary conditions at the ideally conducting surfaces are set in terms of perturbed electric field: $\mathbf{e} \times \mathbf{n} = 0$, $\mathbf{n}$ is the normal vector to the boundary, $\mathbf{E} = i\omega\mathbf{e}$, $\mathbf{e} = -\boldsymbol{\xi} \times \mathbf{B}$ assuming time dependence $\exp(i\omega \mathrm{t})$. The plasma may either extend up to the conducting surfaces or to be separated from the conductors by a vacuum layer outside some magnetic surface. In the latter free plasma boundary case, the natural boundary condition at the plasma-vacuum interface for the energy principle formulation of the ideal MHD stability problem $\delta(W_F + W_V - \omega^2 K) = 0$ corresponds to the linearized total pressure continuity across the perturbed plasma boundary. Here $K = 1/2\int_{V_p} \rho \,|\boldsymbol{\xi}|^2 dV$ is the kinetic energy functional, ρ is the mass plasma density assumed to be constant in the plasma volume V_p .

In the absence of the toroidal equilibrium field, there are only two projections of the perturbed electric field – normal to magnetic surfaces and along the toroidal direction – due to the ideal MHD condition $\mathbf{e}\cdot\mathbf{B}=\mathbf{e}\cdot\nabla\psi\times\nabla\varphi=0$. The projections of plasma displacement $\boldsymbol{\xi}_\perp=\mathbf{B}\times\boldsymbol{\xi}\times\mathbf{B}/B^2=\xi_\psi\nabla\psi/|\nabla\psi|+\xi_\varphi\nabla\varphi/|\nabla\varphi|$ normal to the equilibrium magnetic field are related to the electric field projections onto the orthogonal directions as follows: $\xi_\psi=e_\varphi/B$, $\xi_\varphi=-e_\psi/B$. It can be also shown that for complex amplitudes of the toroidal harmonics $\mathbf{e}_n\exp(in\varphi)$ and $\boldsymbol{\xi}_n\exp(in\varphi)$, the projections $\xi_{n\psi}$, $\mu_n=\boldsymbol{\xi}_n\cdot\mathbf{B}/B^2$, $e_{n\varphi}$ can always be set real which corresponds to imaginary $\xi_{n\varphi}$, $e_{n\psi}$.

As discussed in the review [2], the most dangerous MHD modes are interchange-type "flute" instabilities. For low values of $\beta=2\mu_0 p/B^2$, the electric field perturbations are potential, i.e. magnetic field is not perturbed, and the Rosenbluth-Longmire-Kadomtsev (RLK) stability criterion for such modes takes the form: $(U\mathbf{n}\cdot\nabla p)/(p\mathbf{n}\cdot\nabla U)<\Gamma$, where $\mathbf{n}$ is the normal to magnetic surface, $U=\int 1/B dl$ is the volume between magnetic surfaces with the integral taken over closed magnetic line.

3. Pressure stability limits. The "min B" condition $(U\mathbf{n}\cdot\nabla p)/(p\mathbf{n}\cdot\nabla U)<0$ is not satisfied everywhere in Trimyx. The Ohkawa surface where U reaches its minimal value approximately corresponds to the magnetic surface with the pressure $p=0.96p_0$ (see the inner contour in Fig.1c). So for any toroidal mode number $n>0$ the plasma is unstable for $\Gamma=0$ as shown in [6]. But the instability can be suppressed by taking into account the plasma compressibility. The growth rate dependence on the maximal pressure magnitude for different toroidal mode numbers for $\Gamma=5/3$ is shown in Fig. 2a. Due to the localized nature of flute modes, the higher toroidal mode number n the higher is the growth rate. There is a well defined stability limit on the pressure p_0<450 Pa which is not described by the RLK criterion, where only logarithmic derivative of pressure enters, and magnetic field deviation from the vacuum one and other finite-β effects are not taken into account. The limiting pressure value changes with the plasma boundary approaching the Ohkawa surface (Fig. 2b). The mode structure is shown in Fig. 2c. Vacuum is assumed outside the magnetic surface with pressure $p=0.5p_0$ (the outer contour in Fig.1c) which is close to the plasma boundary in experiment and is also close to the mode localization. Resulting pressure limit p_0<600 Pa ($\beta_0=2\mu_0 p_0/B^2=0.15$) is compatible with the experimental pressure estimate p=750 Pa in [8] for the same value of the barrier magnetic field near the Ohkawa surface B=0.1 T. It is remarkable that the pressure limit increases for lower toroidal mode numbers with the $p=0.5p_0$ plasma boundary up to p_0<700 Pa ($\beta_0=0.175$) for $n<5$. We can also note that the wall stabilization for the free boundary modes is rather weak even for low toroidal mode numbers due to weak magnetic field perturbations in vacuum for flute-type modes.

4. Conclusions. The growth rates of the ideal MHD stability of the plasma equilibrium configurations in the Trimyx Galatea trap with current-carrying conductors immersed in plasma are calculated. The unstructured grid ideal MHD stability code MHD_NX was upgraded to take into account plasma compressibility. The stability limits of the pressure magnitude in the Trimyx plasma are computed with plasma boundary cut-off from the vacuum vessel. The computed pressure limits are compatible with the experimental data [8]. The extension of plasma beyond the Ohkawa surface in the experiment can be explained by the compressional stabilization of the flute modes with a possible relation of both the plasma boundary extent and the pressure magnitude to the limiting finite-β MHD modes.

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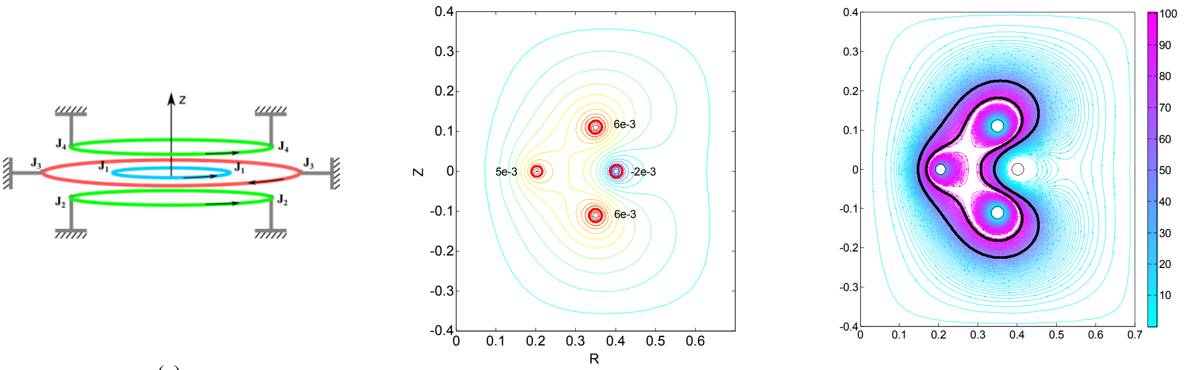
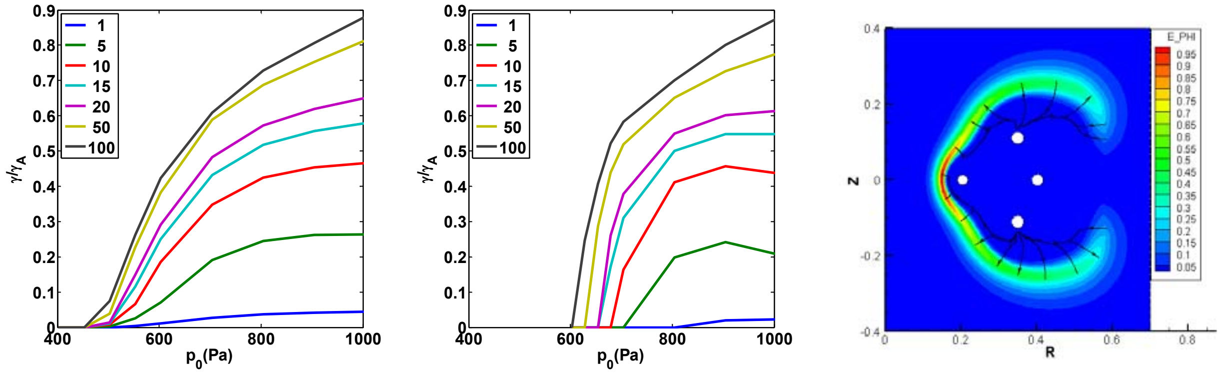


(a) (b) (c)

Figure 1. a) Current-carrying coils layout in Trimyx; b) Level lines of poloidal flux function for the reference equilibrium, the values of ψ_k in $\mathrm{Wb}/(2\pi)$ at the magnetic surfaces around the coils are given; c) Pressure level lines (colorbar in Pa) for the equilibrium recomputed on unstructured triangular grid. Contours of Ohkawa surface $p = 0.96\,p_0$ and $p = 0.5\,p_0$ surface are also shown (thick black lines).



(a) (b) (c)

Figure 2. a) Growth rates of most unstable modes for different toroidal mode numbers (shown in legend) vs. maximal pressure values, the growth rates are normalized by Alfvén frequency $\gamma_A = B/(a\sqrt{\mu_0\rho})$; b) Growth rates of most unstable modes for different toroidal mode numbers with $p = 0.5\,p_0$ boundary; c) Contour plot of toroidal component of electric field and streamlines of plasma displacement for n=10 mode with $p = 0.5\,p_0$ boundary, $p_0 = 704\,Pa$, $\gamma/\gamma_A = 0.16$.