Three-dimensional features in burning plasmas

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Abstract

Due to the marked anisotropy of magnetically confined plasmas, three-dimensional effects are expected to play an important role in the path toward fusion burning plasmas. In particular, the drastic change in magnetic topology associated with reconnecting modes on rational magnetic surfaces [1] may decrease the thermal electron conductivity parallel to the magnetic field lines, with consequent impact on fusion heating. We describe this new scenario, and present analytical and numerical calculations aimed at verifying the influence of reconnection on the thermonuclear instability.

Introduction

At the simplest level of description, the approach to ignition in tokamak plasmas is analyzed using 0-D fluid models, in which any form of heating is delivered to the entire plasma volume. Numerical studies with 1+1/2 D transport codes introduce radial dependence, and reveal the importance of controlling plasma profiles and auxiliary heating deposition [2]. No distinction is made between rational and irrational magnetic surfaces. In principle, however, this distinction might be relevant. Any form of heating localized on a irrational magnetic surface is rapidly spread over the entire surface, due to the ergodic character of the magnetic field lines. On a rational magnetic surface, on the contrary, heating is rapidly spread over the finite length of the closed magnetic field lines, with the formation of hot helical filaments weakly thermally connected to one another. It is therefore plausible that if the longitudinal thermal conductivity is reduced by any physical reason, e.g., by the presence of magnetic structures, localized plasma heating can become much more effective.

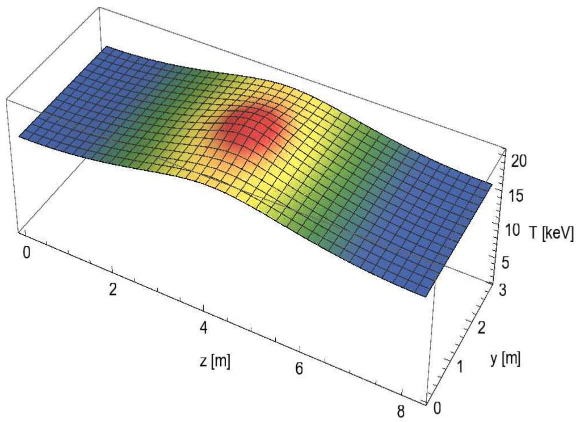
Numerical simulations

To provide an intuitive, although oversimplified, picture of the influence of rational surfaces and parallel heat diffusion on the heating to ignition of a magnetically confined thermonuclear plasma, we have numerically solved the heat diffusion equation on a two-dimensional Cartesian plane, with coordinates y and z:

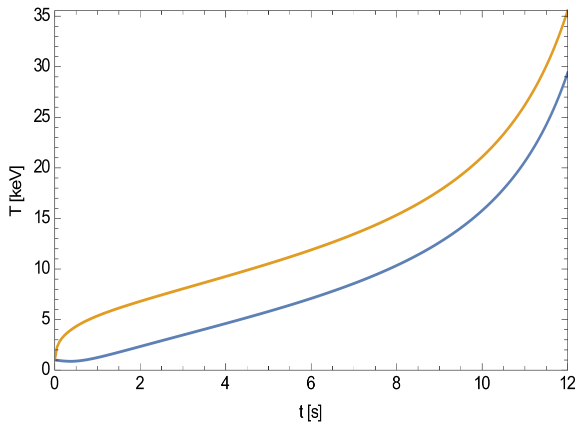
$$\frac{\partial T(y,z,t)}{\partial t} = \frac{K_{yy}}{3n}\frac{\partial^2 T}{\partial y^2} + \frac{K_{zz}}{3n}\frac{\partial^2 T}{\partial z^2} - \frac{L_x}{3n} + \frac{S}{3n},$$

where L_x models heat losses along the third dimension (x, or "radial", coordinate), and the source term S includes fusion power, Bremsstrahlung radiative losses, and auxiliary heating. The domain is a rectangular region of sides $2\pi a$ (the "poloidal" direction) and $2\pi R_0$ (the "toroidal" direction), with $R_0/a \sim 2.8$. Periodic boundary condition are adopted.

We have performed three types of simulation. In the first case (case 1), we consider a steady-state auxiliary heat source heating an irrational magnetic surface, the heat being deposited to a localized narrow central region of the surface. The irrational character of the surface is modelled by simply setting $K_{yy} = K_{zz}$ so that heat diffuses to the entire plane, mocking up the effect of ergodic magnetic field lines. The initial temperature of the surface is set equal to 1 keV. In Fig. 1 we present the temperature distribution at the final simulation time, and the time evolution of



(a) *2D final temperature.*



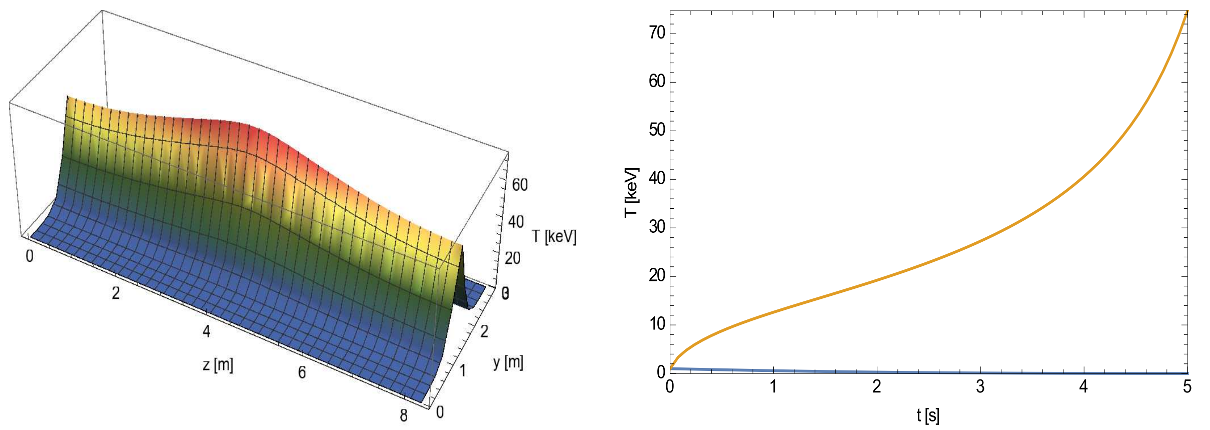
(b) *Time evolution of maximum (upper curve) and minimum temperature.*

Figure 1: *Case 1: modeling heat diffusion on an irrational magnetic surface.*

the minimum and maximum values of the temperature. Ignition is reached at about $t \sim 10$ s.

The second simulation (case 2) replicates case 1 but inhibiting heat diffusion along y (we set $K_{yy} = 0$), thereby modeling a rational surface with magnetic field parallel to the z direction. In Fig. 2 we present the result for the temperature. Ignition is now reached at about $t \sim 4$ s.

As a final case we repeat case 2 but lowering the value of the heat diffusion coefficient along the z direction. With this setting we want to model the possible reduction of parallel heat transport which might be caused by magnetic structures forming at rational surfaces. Ignition is found to occur at $t \sim 3$ s.



(a) *2D final temperature.*

(b) *Time evolution of maximum (upper curve) and minimum temperature.*

Figure 2: *Case 2: modeling heat diffusion on a rational magnetic surface.*

Analytical considerations

At the simplest level of description, we have considered the linearized equation for the total energy of the plasma,

$$\left(\gamma_F^T - \overline{\gamma}_T\right)\tilde{T}_e(x_\perp) - D_e^{\|} k_{\|}'^2 x_\perp^2 \tilde{T}_e(x_\perp) + D_e^{\|} i k_{\|}' x_\perp T_e' \frac{\tilde{B}_x(x_\perp)}{B} + D_e^{\perp} \frac{d^2\tilde{T}_e(x_\perp)}{dx_\perp^2} = 0\,,$$

having in mind long-wavelength drift-tearing perturbations $\hat{T}_e(\mathbf{x},t) = \tilde{T}_e(x_\perp)\exp[\gamma t + ik_y y + ik_x z]$ developing around a rational surface, on which $\mathbf{k}\cdot\mathbf{B} = k_{\|}B = k_{\|}' x_\perp B = 0$. Here, $x_\perp = x - x_0$ is the distance from the rational surface, $k_{\|}' = k_y B_y'/B$, $\overline{\gamma}_T \equiv (3/2)[\gamma(1+\varepsilon_T)+\gamma_B] + \varepsilon_T \nu_i^D$, $\gamma_F^T \equiv \varepsilon_T \gamma_F$, with ν_i^D a quantity related to ion radial diffusion, and ε_T a quantity proportional to the electron-ion equilibration frequency ν_{eq}. A key parameter is the fusion heating rate $\gamma_F \propto d\langle\sigma v\rangle/dT_i$, which for the chosen parameters reaches a maximum of 14.6 1/s at $T_i = 14.4$ keV. The Bremsstrahlung loss rate γ_B is usually small.

We have considered first the special case $\gamma_F^T = \overline{\gamma}_T$, which leads to instability for $\gamma_F > \nu_i^D$, a condition that can be realistically met. Adopting the constant-B approximation inside the "thermal" layer of width $\delta_T^4 = (D_e^{\perp}/D_e^{\|})(1/k_{\|}'^2)$, the equation for the temperature fluctuation ($\bar{x}_\perp = x_\perp/\delta_T$),

$$\frac{d^2\overline{T}(\bar{x}_\perp)}{d\bar{x}_\perp^2} - \bar{x}_\perp\left[\bar{x}_\perp \overline{T}(\bar{x}_\perp) - 1\right] = 0\,,$$

where $\overline{T} = \tilde{T}_e/\tilde{T}_m$ with $\tilde{T}_m = iT_e'\tilde{B}_x^0/(k_{\|}'\delta_T B)$ (the solution in the hyper-conductivity limit), has a well known solution, and the required stability parameter Δ' can be calculated. The plasma displacement $\tilde{\xi}_x$, defined in terms of the $E\times B$ drift velocity by $\gamma\tilde{\xi}_x = \tilde{u}_{Ex}$, obeys a similar equation, $\gamma^2(d^2\tilde{\xi}_x/dx_\perp^2) - (k_y^2\omega_H^2/d_T^2)x_\perp^2\tilde{\xi}_x = i(k_y/(\rho\mu_0))(B_y'/d_T^2)x_\perp\tilde{B}_x^0$, where $\omega_H^2 = (B_y')^2/(\rho\mu_0)$ and $d_T^2 = d_I^2 + D_m/\gamma$, the latter quantity consisting of the inductive skin depth term, and the magnetic diffusion contribution. Balancing the terms on the right-hand side we find for the reconnection scale-length $\delta_m^4 = \gamma^2 d_T^2/(k_y^2\omega_H^2) < \delta_T^4$.

An interesting feature of the problem is the presence of the "thermonuclear" singularity, the location x_s at which $(x_s - x_0)^2 = (\gamma_F^T - \overline{\gamma}_T)/(D_e^{\|} k_{\|}^{\prime 2}) \sim \delta_T^4/\delta_F^2$, where $\delta_F^2 = D_e^{\perp}/(\gamma_F^T - \overline{\gamma}_T)$ is a characteristic "fusion heating" scale length.

A more complete analysis requires the consideration of the electron longitudinal and total momentum balances. In view of the two main characteristic scale lengths of the problem, we divide the magnetic field into a part varying on the thermal scale length and a second contribution varying on the reconnection scale length: $\tilde{B}_x(x_\perp) = \tilde{B}_{x,m}(x_\perp) + \tilde{B}_{x,\Delta}(x_\perp)$. Likewise for the temperature perturbation, $\tilde{T}_e(x_\perp) = \tilde{T}_{e,m}(x_\perp) + \tilde{T}_{e,\Delta}(x_\perp)$. Both perturbations are assumed to be localized in $\delta_m, \delta_T \ll a$. The analysis leads to the following equation for the rescaled plasma displacement $\overline{Y}(\overline{x}_\perp) \equiv i(k_y\delta_m)B_y'\tilde{\xi}_x^0(\overline{x}_\perp)/\tilde{B}_\Delta^0$ ($\tilde{B}_\Delta^0$ being the value at x_0):

$$\Gamma^2 \frac{d^2\overline{Y}(\overline{\overline{x}}_\perp)}{d\overline{\overline{x}}_\perp^2} = \overline{\overline{x}}_\perp \left[\overline{Y}(\overline{\overline{x}}_\perp)\overline{\overline{x}}_\perp - 1\right] ,$$

where $\overline{\overline{x}}_\perp = x_\perp/\delta_m$, and $\Gamma^2 \equiv (\gamma^2/\omega_H^2)(d_T^2/\delta_m^2)(1/k_y^2\delta_m^2)$. The matching condition leads to the following growth rates valid, respectively, in the $d_I^2 \gg D_m/\gamma$ and $d_I^2 \ll D_m/\gamma$ limits: $\gamma \propto (d_I^2)^{3/2}(D_e^{\|}/D_e^{\perp})^{1/2}(k_{\|}' k_y)\,\omega_H(\tilde{T}_{e,\Delta}^0/\tilde{T}_e^0)^{1/2}$ and $\gamma \propto (D_m)^{3/5}(D_e^{\|}/D_e^{\perp})^{1/5}\,(k_{\|}' k_y)^{2/5}\omega_H^{2/5}\,(\tilde{T}_{e,\Delta}^0/\tilde{T}_e^0)^{1/5}$, where $\tilde{T}_e^0$ and $\tilde{T}_{e,\Delta}^0$ are constants related to the amplitude of the temperature fluctuation.

Conclusion

We have performed numerical simulations to model the time evolution of the temperature on a magnetic surface resulting from a localized deposition of steady-state auxiliary heating, showing that the path to ignition depends crucially on whether the surface is irrational or rational, and in the latter case on the value of the parallel heat diffusivity. We have analyzed a reduced set of equations governing reconnection in the presence of fusion heating. Considering only the total energy balance, we find that in the special case $\gamma_F^T = \overline{\gamma}_T$ the instability condition $\gamma_F > \nu_i^D$ is likely to be met. The more general case requires considering also the electron longitudinal and total momentum balances, and a throughout analysis is still underway. A characteristic feature of the problem is the insurgence of a singularity related to the fusion heating rate.

References

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