Kinetic full-wave analysis of electromagnetic waves in tokamak plasmas using an integral-form of dielectric tensor

Atsushi Fukuyama

Kyoto University, Kyoto, Japan

The propagation and absorption of electromagnetic waves in a homogeneous and stationary plasma are usually characterized by the angular wave frequency ω and the wave number vector $\vec{k}$. In a weakly inhomogeneous plasma where the inhomogeneity scale length L satisfies $kL \ll 1$, the approach of geometrical optics, ray and beam tracing method, is widely used for wave analysis. When the wave length is comparable to or longer than L or the wave is evanescent, however, full wave analysis solving boundary-Value problm of Maxwell's equation is required.

In the full wave analysis, the response of plasma is represented by a dielectric tensor $\overleftrightarrow{\varepsilon}$. In a cold plasma, $\overleftrightarrow{\varepsilon}$ is local and independent of the wave structure. In a hot plasma, however, particle's thermal motion leads to the dependence on the wave structure, usually represented by the wave number $\vec{k}$ formally defined in a uniform plasma. Since the wave structure is not known a priori, there are several approaches to include the hot plasma effects in $\overleftrightarrow{\varepsilon}$:

1. Cold-plasma wave number approximation: wave number is estimated by the cold plasma approximation; only applicable for single propagating waves, not for a standing wave.
2. Differential operator approach: wave number $\vec{k}$ in $\overleftrightarrow{\varepsilon}$ for uniform plasma is replaced by a spatial differential operator $-\mathrm{i}\vec{\nabla}$; limited to a long wave length and practically up to the second order of $\vec{\nabla}$.
3. Fourier transform approach: wave electric field and inhomogeneous dielectric medium are Fourier decomposed in space; large computational resource is required to include all Fourier modes.
4. Integral operator approach: Plasma response is represented by an integral $\int \varepsilon(x-x')\cdot E(x')\,\mathrm{d}x'$ and free from $\vec{k}$; interaction is localized in space and less computational resource required.

In this paper, we discuss a systematic approach to kinetic full wave analysis based on integral form of dielectric tensor. We assume stationary wave electric field in the form of $\vec{E}(\vec{r},t) = \vec{E}(\vec{r})\,\mathrm{e}^{-\mathrm{i}\omega t}$ where ω is a complex in general and constant in time. Maxwell's equation in an inhomogeneous and dispersive medium is written as

$$\nabla\times\nabla\times\vec{E}(\vec{r}) - \frac{\omega^2}{c^2}\int \mathrm{d}\vec{r}'\,\overleftrightarrow{\varepsilon}(\vec{r},\vec{r}';\omega)\cdot\vec{E}(\vec{r}') = \mathrm{i}\,\omega\mu_0\vec{J}_{\mathrm{ext}}(\vec{r}) \tag{1}$$

First we consider a simple one-dimensional case and derive an dielectric tensor in a uniform plasma. Free motion of a particle with velocity v_x, $x = x' + v_x(t - t')$, leads to a variable transformation from a velocity variable to a position variable

$$v_x = \frac{x - x'}{t - t'}. \tag{2}$$

Substituting this variable transformation into the perturbed distribution function

$$\tilde{f}(x, \vec{v}) = \frac{n}{(2\pi T/m)^{3/2}} \frac{q}{T} \int_0^\infty \mathrm{d}\tau \, \vec{v} \cdot \vec{E}(x') \exp\left[-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2T} + \mathrm{i}\,\omega\tau\right] \tag{3}$$

with $\tau = t - t'$, we obtain an expression for perturbed current induced in a plasma

$$\vec{J}(x) = q \int \mathrm{d}\vec{v}\, \vec{v} \tilde{f}(x, \vec{v}) = \int \mathrm{d}x' \, \overleftrightarrow{\sigma}(x - x') \cdot \vec{E}(x') \tag{4}$$

and the xx component of conductivity tensor is given by

$$\sigma_{xx}(x - x') = \frac{nq^2}{m\omega} \int_{-\infty}^{\infty} \mathrm{d}\hat{x}' \, \xi^2 U_{-2}(\xi), \quad \cdots \tag{5}$$

where $v_\mathrm{T} = \sqrt{T/m}$, $\hat{x} = \omega x / v_\mathrm{T}$, $\hat{\tau} = \omega\tau$, $\xi = \omega(x - x')/v_\mathrm{T}$. The function U_n is defined by

$$U_n(\xi) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \mathrm{d}\hat{\tau}\, \hat{\tau}^{n-1} \exp\left[-\frac{1}{2}\frac{\xi^2}{\hat{\tau}^2} + \mathrm{i}\,\hat{\tau}\right] \tag{6}$$

and localized within an excursion length of particles, v_T/ω. It should be noted that this kernel function is related to the inverse Fourier transform of the plasma dispersion function.

In a magnetized plasma, cyclotron motion perpendicular to the magnetic field is expressed as

$$x = x_0 - \frac{v_\perp}{\omega_\mathrm{c}} \sin\theta_g, \qquad x' = x_0 - \frac{v_\perp}{\omega_\mathrm{c}} \sin(\theta_g + \omega_\mathrm{c}\tau) \tag{7}$$

and the variable transformation from the velocity variables $(v_\perp, \theta_g)$ to the past particle position x' and the guiding center position x_0 is required. Introducing a kernel function

$$F_n^{(i)}(X, Y) = \frac{1}{2\pi^2} \int_0^\pi \mathrm{d}\theta \left[-\frac{X^2}{1 + \cos\theta} - \frac{Y^2}{1 - \cos\theta}\right] f_n^{(i)}(\theta) \tag{8}$$

where $f_n^{(1)} = \cos n\theta / \sin\theta$, $f_n^{(2)} = \sin n\theta$, $f_n^{(3)} = \sin n\theta / \sin^2\theta$, $f_n^{(4)} = \cos\theta \sin n\theta / \sin^2\theta$, we obtain a dielectric tensor for cyclotron motion localized within the Larmor radius $v_\mathrm{T}/\omega_\mathrm{c}$.

As an application of the integral form of dielectric tensor, we consider the O-X-B mode conversion of electron cyclotron (EC) wave in a tokamak plasma. EC waves launched from a low density region are often reflected at the cutoff density of the O-mode or the X-mode unless the wave frequency is high enough. In some experiments, however, electron heating in over-dense plasma has been observed. One explanation of this phenomena is the O-X-B mode conversion[1]. If the parallel wave number is the optimum, the electron densities at the O-mode cutoff and the X-mode cutoff become same, and the mode conversion from the O-mode to the fast X-mode occurs. The fast X-mode is reflected at higher density and mode-converted to the slow-X mode. Finally the slow-X mode is reflected near the lower-density upper-hybrid-resonance and mode-converted to the electron Bernstein wave (EBW). The EBW penetrates into the high density region and absorbed at the cyclotron fundamental or harmonic resonance. Since there exists an evanescent layer between the O-mode cutoff and the fast X-mode cutoff when the parallel wave number is not optimum, ray tracing analysis has difficulty.

We have developed a one-dimensional kinetic full wave code TASK/W1 [2] and applied it to the analysis of O-X-B mode conversion in a small-size spherical tokamak. In the previous analyses, the wave is excited by antenna located in the low field side, and the O-mode excitation has a possibility of direct excitation of EBW. In the present analysis, we have implemented waveguide excitation on th wall, and pure O-mode excitation is confirmed. In addition, we have extended the analysis to the two-dimensional and obtained preliminary results.

We employed the plasma and wave parameters of the spherical torus LATE: $R_0 = 0.22\,\mathrm{m}$, $a = 0.16\,\mathrm{m}$, $B_0 = 0.08\,\mathrm{T}$ $n_e(0) = 1\times10^{17}\,\mathrm{m}^{-3}$, $f = 2.45\,\mathrm{GHz}$. Figure 1 shows the dispersion relation, perpendicular wave number versus major radius, and Fig. 2 illustrates the wave structure in the major radius direction for three values of parallel wave numbers. Waves are excited by the O-mode waveguide with parallel wave electric field (E_z). By Fourier composing in the uniform magnetic direction (z), we obtain preliminary results of two-dimensional wave structure as illustrated in Fig.3 for cold plasma and Fig.4 for hot plasma.

References

[1] Hansen et al., Plasma Phys. Control. Fusion **27**, 1077 (1985)

[2] J.A. Khan et al., Plasma and Fusion Res. **11**, 2403070 (2016)

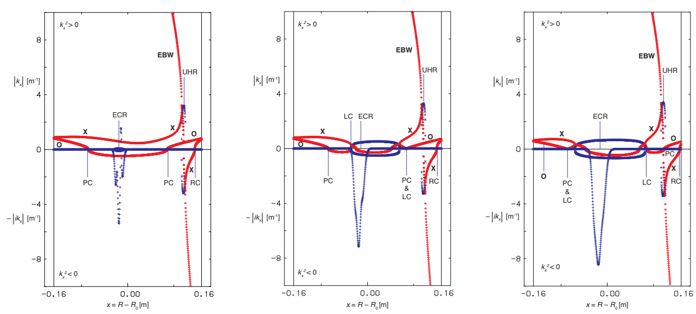

Figure 1: Dispersion relation (perpendicular wave number versus major radius) for $k_{\parallel} = 24\,\mathrm{m}^{-1}$ (no X cutoff), $k_{\parallel} = 32\,\mathrm{m}^{-1}$ (Optimum), $k_{\parallel} = 40\,\mathrm{m}^{-1}$ (O cutoff)

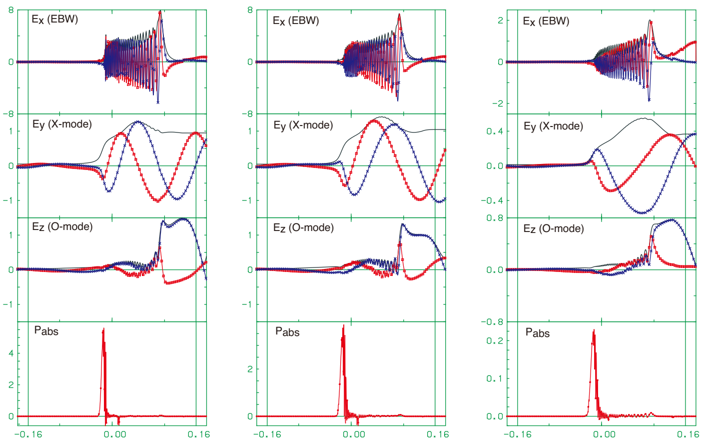

Figure 2: 1D wave structure for hot plasma: $k_{\parallel} = 24\,\mathrm{m}^{-1}$ (no X cutoff), $k_{\parallel} = 32\,\mathrm{m}^{-1}$ (Optimum), $k_{\parallel} = 40\,\mathrm{m}^{-1}$ (O cutoff)

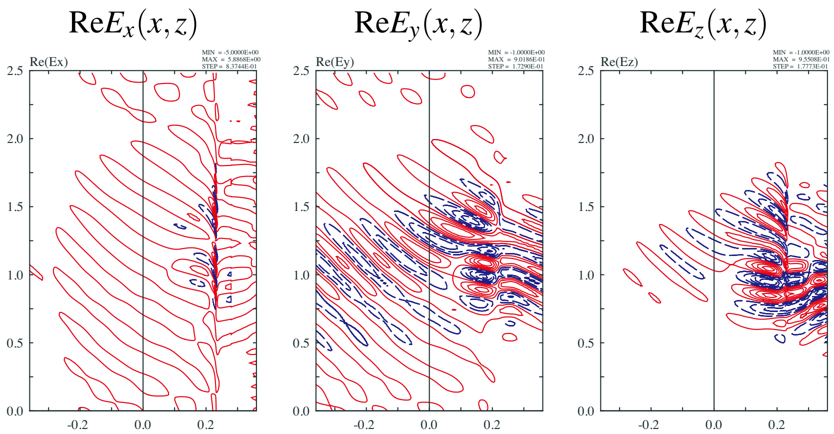

Figure 3: 2D wave structure for cold plasma

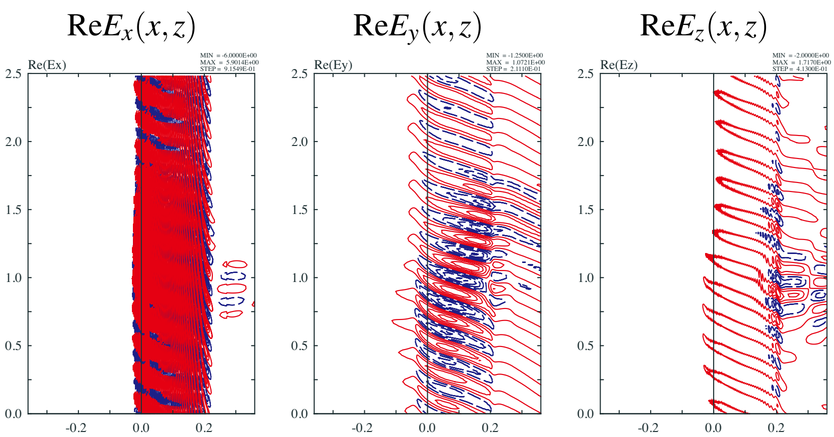

Figure 4: 2D wave structure for hot plasma