Mapping of power deposition zone of electron Bernstein waves externally excited via mode conversion in tokamak plasmas

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1. Introduction

In toroidal plasmas electron Bernstein (EB) waves are excited at the upper hybrid resonance (UHR) layer via mode-conversion from the X waves and propagate toward the higher field side and are cyclotron-damped away at the Doppler shifted frequency with driving a local toroidal current before arriving at the electron cyclotron (EC) resonance layer. The power deposition profiles were analysed using ray tracing technique for EB wave propagation and absorption in inhomogeneous magnetized plasmas [1-3]. The ray tracing equation is composed of ordinary differential equations that give the wave trajectory and the refractive index vector at every location on the trajectory by step-by-step numerical integration. It looks difficult to forecast the power deposition zone without doing ray tracing calculation since the ray tracing involves complex dispersion characteristics of EB waves.

It is shown in this paper that in axisymmetric tokamak plasmas the ray tracing equation of EB waves can be cast into the form of equation of particle motion in a potential field. The particle trajectory corresponds to the wave trajectory and its momentum does to the refractive index. The potential field reflects the dispersion characteristics of EB waves, that is, the perpendicular refractive index strongly increases as the waves approach to the ECR layer from the lower field side, making a deep trench of potential along the ECR layer. Therefore the EB waves are perpendicularly pulled toward the ECR layer. This picture of EB wave propagation enables us to map the power deposition zone without ray tracing as described in this paper.

2. OXB wave trajectory in a Solov'ev tokamak plasma

Figures 1 (a) and (b) show the wave trajectories projected on the poloidal cross section and those projected on the mid-plane, respectively, in the case of OXB mode conversion scheme in a Solov'ev tokamak plasma [4] . The plasma has a major radius R_0=1 m, a minor radius a=0.4 m, vertical elongation κ=1.7, triangularity δ=0.32, safety factor q_{edge}=4.0 and the vacuum toroidal field at the major radius B_{Tvac}=1 T. Various profiles on the mid plane are shown in figures 1 (c) and (d). Wave frequency is $\omega/2\pi$ = 31 GHz. The electron density and temperature have peaks at the magnetic axis, with n_{em}=2.03x10^{19}m^{-3} and T_{em}=0.85 keV, respectively. Subscript 'm' denotes the value at the magnetic axis. The plasma is moderately over dense with ω_{pe}^2/ω^2=1.71 at the magnetic axis.

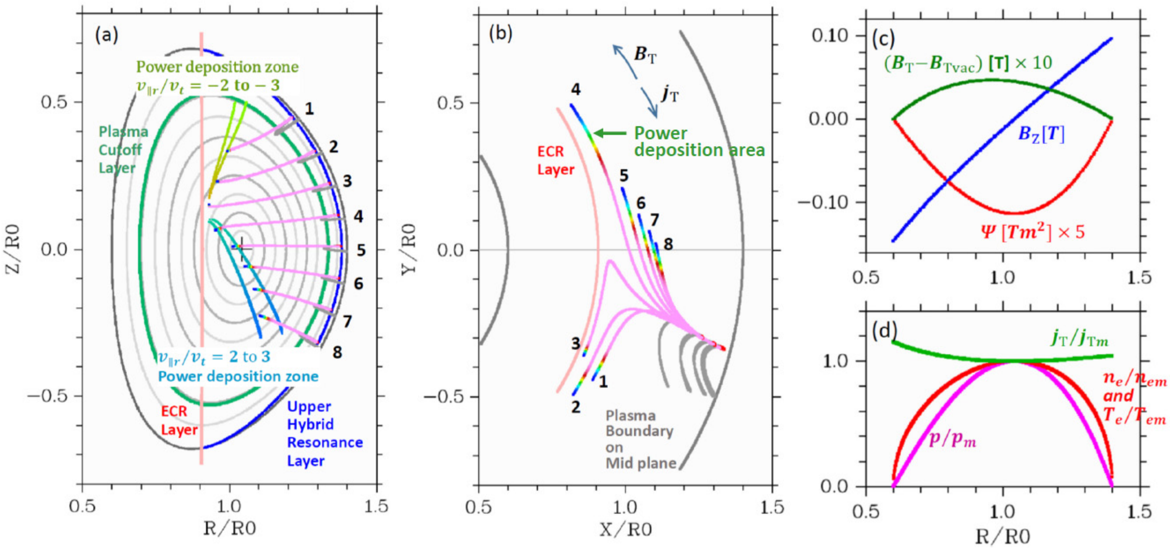

Figure 1 (a) and (b): Appearance of wave trajectories and power depositions for various vertical locations of OXB mode conversions by oblique O wave injection at optimal angle (N_ϕ~0.81) from the lower field side. Pink lines denote EB wave trajectories and the rainbow colour denotes power deposition area. Red, yellow, green, light and dark blue denote, respectively, that injected power has decreased from 1 to 0.8, 0.6, 0.4, 0.2 and 0.01. Grey circles in (a) denote normalized radius ($\rho=[(\psi-\psi_m)/(\psi_{edge}-\psi_m)]^{1/2}$) increasing by 0.1 step from zero at the magnetic axis. Power deposition zones in (a) are estimated and mapped using equations (5) and (10) directly from the plasma profiles without ray tracing.
(c) and (d): various radial profiles on mid-plane.

EB wave trajectories in figure 1 were calculated using ray tracing equations with the local dispersion function G (G=0 gives the local dispersion relation):

$$\frac{d\boldsymbol{r}}{ds} = \frac{\partial G}{d\boldsymbol{k}}, \quad \frac{d\boldsymbol{k}}{ds} = -\frac{\partial G}{dr} \text{ and } \frac{dt}{ds} = -\frac{\partial G}{d\omega} \qquad (1)$$

Here G has the following function form,

$$G = G\left(\omega/\Omega_{ce}, k_\parallel \rho_e, k_\perp \rho_e, \omega_{pe}^2/\Omega_{ce}^2\right) \qquad (2)$$

with $\rho_e=v_t/\Omega_{ce}$ where $v_t=(2T_e/m_e)^{1/2}$ is the electron thermal velocity and

$$k_\parallel = \frac{[k_R B_R + (k_\phi/R)B_T + k_Z B_Z]}{B} \qquad k_\perp = \sqrt{k_R^2 + (k_\phi/R)^2 + k_Z^2 - k_\parallel^2} \qquad (3)$$

Here the cylindrical coordinate system (R, ϕ, Z) is used with corresponding components of wave number, k_R, k_ϕ, k_Z, respectively. Since the plasma is axisymmetric, k_ϕ is conserved during propagation. Refractive indexes $N_R=ck_R/\omega$, $N_\phi= ck_\phi/\omega$, $N_Z=ck_Z/\omega$ are also used.
The EC resonance condition is given by

$$\omega - k_\parallel v_\parallel - \Omega_{ce}/\gamma \qquad (4)$$

While the formula of EB wave dispersion relation used for ray tracing is non relativistic, we employ the relativistic EC resonance formula (4) for estimation of wave damping and current drive efficiency using the quasi linear theory. The EC resonance condition makes a resonance curve in the electron velocity space. The foot point of the curve on the parallel velocity axis, $v_{//r}$, referred to as the parallel resonance velocity, decreases as the wave propagates toward the ECR layer. Power deposition appears when $v_{//r}/v_t$ decreases to ~3 and is completed in a short distance before $v_{//r}/v_t$ decreases to 2. The formula for $v_{//r}$ is given by

$$\frac{\gamma v_{\parallel r}}{c} = \frac{(N_\parallel/|N_\parallel|)\sqrt{(\Omega_{ce}/\omega)^2 + N_\parallel^2 - 1} - (\Omega_{ce}/\omega)N_\parallel}{N_\parallel^2 - 1} \quad with\ \gamma = \sqrt{1 + (\gamma v_\parallel/c)^2} \qquad (5)$$

3. Purely perpendicular propagation in no poloidal field plasmas

In low beta tokamak plasmas poloidal fields B_R and B_Z are one order of magnitude smaller than the toroidal field and it may be a useful approximation to treat $B_R=B_Z=0$ in ray tracing analyses since equation (3) is greatly simplified as follows

$$k_\parallel = k_\phi/R \qquad k_\perp = \sqrt{k_R^2 + k_Z^2} \qquad (6)$$

Further we consider the case of purely perpendicular propagation. Then, the dispersion relation of EB waves reads

$$G = G\left(\omega/\Omega_{ce},\ k_\parallel = 0,\ k_\perp\rho_e,\ \omega_{pe}^2/\Omega_{ce}^2\right) = 0 \qquad (7)$$

The solution is plotted in figure 2. Here, all of Ω_{ce}, ρ_e and $\omega_{pe}^2/\Omega_{ce}^2$ are functions of $\boldsymbol{X}=(R, Z)$ in tokamak plasmas and ω is an externally given wave parameter. Therefore, above dispersion relation indicates that $N_\perp$ is given as a function of $\boldsymbol{X}$.

We introduce a function $U(\boldsymbol{X}) = -N_\perp^2/2$ as a function of $\boldsymbol{X}$. Then the following function is a dispersion function of EB waves propagating in no poloidal field plasmas.

$$G_\perp(\boldsymbol{N}_\perp,\boldsymbol{X}) = (\boldsymbol{N}_\perp \cdot \boldsymbol{N}_\perp)/2 + U(\boldsymbol{X}) \quad with \quad \boldsymbol{N}_\perp = (N_R, N_Z) \qquad (8)$$

Then, ray tracing equations read

$$d\boldsymbol{X}/ds = \boldsymbol{N}_\perp \quad and \quad d\boldsymbol{N}_\perp/ds = -\nabla U(\boldsymbol{X}) \qquad (9)$$

Equations (8) and (9) are equivalent to the Hamiltonian and the Hamilton equation, respectively, for a unit mass particle in the potential U. $\boldsymbol{X}$ represents the particle trajectory and $\boldsymbol{N}_\perp$ does the momentum.

Various particle trajectories on the potential U for the case shown in figure 1(a) are plotted in figure 3(a). The EB wave trajectories in figure 1(a) are replotted in the figure, indicating that the trajectory in the simplified model well describes the trajectory in the full model. The potential U is zero on the UHR layer and strongly sinks toward the ECR layer, making a deep trench of potential along the ECR layer. Therefore the waves perpendicularly leave the UHR layer and then are perpendicularly pulled toward the ECR layer. Such a deep trench of potential originates from the strong increase of $k_\perp\rho_e$ with ω/Ω_{ce} decreasing toward 1 (figure 2).

4. Mapping of power deposition zone and summary

Thus the parallel refractive index is approximated by the formula

$$N_R = N_\perp \quad and \quad N_\parallel = N_\phi/R + N_R B_R/B \qquad (10)$$

near the ECR layer and is calculated without ray tracing (see figure 3(b)). Then the parallel resonance velocity (equation (5)) is also directly estimated from the plasma profile and possible power deposition zone where

$$2 \le \left|v_{\parallel r}/v_t\right| \le 3 \qquad (11)$$

can be mapped from related plasma profiles without ray tracing as shown in figure 1(a).

In summary propagation of EB waves in tokamak plasmas looks particle motion in the potential field $U(X) = -N_{\perp}^2/2$, which provides us with above picture of EB wave propagation and enables us to map the power deposition zone without ray tracing. The mapping is useful to guide the ray racing analyses and cross-check the ray tracing results.

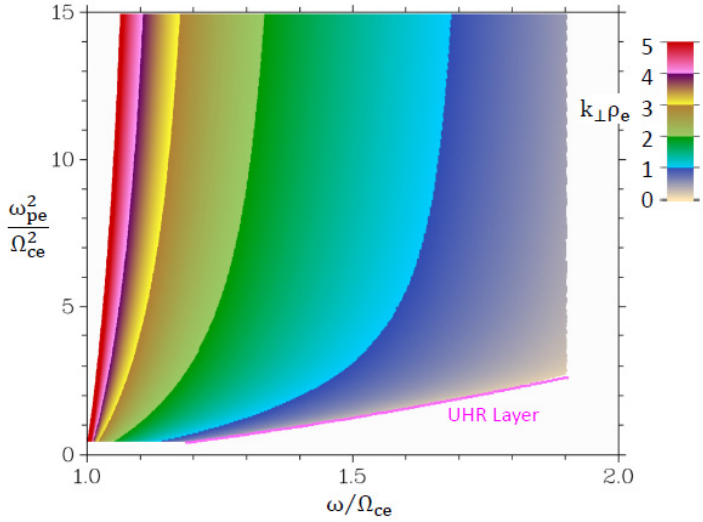

Figure 2
Dispersion relation of purely perpendicular EB waves plotted as a contour plot of $k_{\perp}\rho_e$ on (ω/Ω_{ce}, $\omega_{pe}^2/\Omega_{ce}^2$) plane.

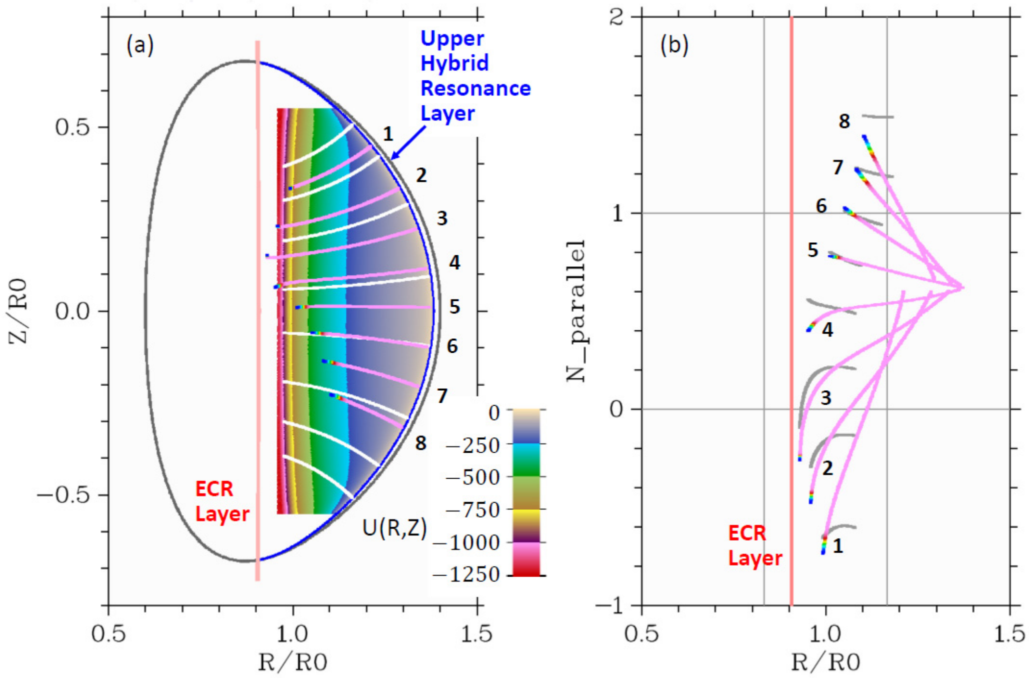

Figure 3 (a) EB wave trajectories (pink lines) in figure (a) are replotted and compared with particle trajectories (white lines) on a contour plot of potential $U = -[N_{\perp}(R,Z)]^2/2$ for the plasma in figure 1. (b) Variations of $N_{//}$ along EB wave trajectories 1 to 8 in pink lines from full ray tracing calculation are compared with those evaluated by the mapping method (equation(10)) in grey lines.

[1] Maekawa T., Tanaka S., Terumichi Y. and Hamada Y. 1980 J. Phys. Soc. Jpn 48 247-257.

[2] Forest C.B., Chattopadhyay P.K., Harvey R.W. and Smirnov A.P. 2000 Phys. Plasmas 71352-1355.

[3] Nagasaki K. and Yanagi K. 2002 Plasma Phys. Control. Fusion 44 409-422.

[4] Cerfon A.J. and Freidberg J.P. 2010 Phys. Plasmas 17 032502.