Some Issues in Realizing the RF Current Condensation Effect

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Introduction: The stabilization of magnetic islands produced by the neoclassical tearing mode (NTM) is critical to reaching economical fusion using the tokamak approach. These modes were found to be the single most common cause of disruptions in the JET tokamak [1]. However, theoretical calculations as early as 1983 showed that magnetic islands produced by tokamak tearing modes might be stabilized by rf waves driving current preferentially at the island center. [2]. Since then, a great number of experimental, theoretical, and computational efforts have been exerted. The most studied rf current drive methods for producing these currents for stabilizing the tearing mode, particularly the neoclassical tearing mode, are lower hybrid current drive (LHCD) [3] and electron cyclotron current drive (ECCD) [4]. Both can be localized to stabilize the neoclassical tearing mode, although current produced through ECCD might be more easily localized. Both exploit the fact that a high current drive efficiency is obtained when the rf waves are damped in plasma by superthermal electrons. It is thought that the neoclassical tearing modes must be stabilized before they grow too large, because the current required to stabilize large islands would be correspondingly greater and therefore more expensive. Thus, both for ECCD and LHCD, the requirement to stabilize islands while they are small makes more severe the requirement for precise localization of the power deposition.

However, recently a possibly very favorable current condensation effect was identified [5]. Power dissipated within the islands tends to lead to a temperature peaking at the island center, which induces more dissipation at the center. This positive feedback leads to a current condensation effect, where the current tends to condense on the island center, exactly where it is most effective at stabilizing the neoclassical tearing mode. This effect makes it possible both to stabilize larger islands and to do so with less precise localization. Since this condensation effect relies on the sensitivity of the power deposition to the electron temperature, the condensation will tend to be most pronounced when the current is carried by the fastest electrons, which is also where the current drive efficiency is highest. Here we explore some of the issues in reaching this regime of both high rf current drive efficiency and effective rf current condensation, both for lower hybrid current drive and for electron cyclotron current drive.

Power Deposition: First, let us explain the sensitivity to temperature. The power deposition by both lower hybrid waves [6] and electron cyclotron waves [7] is very sensitive to the temperature, because both of these waves deposit their energy on the electron tail. Let v_0 be the the electron speed at the location in velocity space of greatest power deposition. The deposition in that region is proportional to the number of electrons there, $P_{rf} \propto \exp(-w^2)$, where $w \equiv v_0/v_T$, $mv_T^2/2 = T$, and T is the electron temperature. For a small temperature perturbation, $\tilde{T}$, the change in the local power deposition produced by the perturbation is given by

$$P_{rf} \propto \mathrm{e}^{-w_0^2} = \mathrm{e}^{-w_0^2}\,\mathrm{e}^{w_0^2\tilde{T}/T_0} = \mathrm{e}^{-w_0^2}\,\mathrm{e}^{u} \tag{1}$$

where T_0 is the unperturbed temperature; w_0 is the value of w in the absence of the temperature perturbation; and we defined $u = w_0^2\tilde{T}/T_0$. Notice that the power deposition is exponential in $u = w_0^2\tilde{T}/T_0$, so that for w_0^2 large, as is the case for efficient current drive, the power deposition is sensitive even to a small $\tilde{T}/T_0$. For later use, we define P_0' by $P_{rf} \equiv P_0'\mathrm{e}^u$. Typically $w_0^2 \approx 10$ for ECCD and $w_0^2 \approx 20$ for LHCD. The rf driven current also grows exponentially with $u = w_0^2\tilde{T}/T_0$, while the rf current driven per power dissipated grows with w_0^2.

Island Temperature: To calculate the temperature in the island interior relative to that at the separatrix, we assume that the island is sufficiently large that the temperature is constant on the flux surfaces in the island. The electron and ion temperatures equilibrate quickly, so that $T_e = T_i = T$. Thus, while the rf power is deposited into electrons, the ions quickly share that energy. The heat transport out of the island will then be dominated by the ion heat diffusion. For simplicity, the densities and heat conductivities are all taken constant over the island.

The key physical effects can now be captured by a simple slab model, with $x' = 0$ representing the O-line and $x' = \pm W_i/2$ the separatrix. The 1D heat diffusion equation then becomes

$$3n\frac{\partial}{\partial t}kT = \kappa_\perp \frac{\partial^2 kT}{\partial x'^2} + P_{rf} + Q' \tag{2}$$

where $\kappa_\perp$ is the perpendicular ion heat diffusion coefficient; x' is the length across the island; k is the Boltzman's constant; and Q' represents any additional heating or cooling. It is convenient to normalize: $x' = x(W_i/2)$; $t = \tau(4\kappa_\perp/3nW_i^2)$; $P_0' = P_0(4\kappa_\perp kT_s/w_s^2W_i^2)$; and $Q' = Q(4\kappa_\perp kT_s/w_s^2W_i^2)$, so, together with expressions for u and P_{rf}, we now can write

$$\frac{\partial}{\partial \tau}u(x,\tau) = \frac{\partial^2 u}{\partial x^2} + P_0\,\mathrm{e}^u + Q \tag{3}$$

We supplement Eq. (3) with the homogeneous boundary conditions $u = 0$ at $x = 1$ and $\partial u/\partial x = 0$ at $x = 0$. For constant P_0, this equation was noted to have bifurcation solutions [5]. The bifurcation occurs as follow: Suppose that the rf heating term is the only input of heat into the island so that $Q = 0$. Constant P_0 corresponds to uniform rf power spread over the island width, except that where the electrons are hotter, more of the power will be absorbed. As discussed above, the extra power absorbed is exponential in u. Thus, if very little power is absorbed, namely in the limit $P_0 \ll 1$ so that $u \ll 1$, then the exponential term can be ignored. In that case, it can readily be seen the $u = P_0(1-x^2)/2$ satisfies the steady state equation.

Hysteresis Effect: In Fig. 1, we show the steady state solution of Eq. (3), with $Q = 0$ and P_0 constant. For large enough P_0, no steady state solution exists. For small P_0, the lower solution is stable and the upper solution is unstable. This means that if P_0 exceeds a critical value, then the island heats up without reaching a steady state. At some point, the physical assumptions would of course change. For example, if only finite rf power were available, then it would be important to recognize that for example by taking $P_0 = P_0(u)$ in Eq. (3). Alternatively, radiation losses might be accommodated by $Q = Q(u)$. In such cases, where u cannot increase without bound, there then appears a third root giving the upper branch solution as shown in Fig. 2. The lower and upper branches are stable, whereas the middle branch is not. This case can exhibit hysteresis; as P_0 increases, the central temperature jumps from the lower branch to the upper branch. However, once on the upper branch, as P_0 decreases, the upper stable circle in the figure can be reached. This stable steady state solution would correspond to a much more peaked, and higher central temperature, with larger power deposition balanced by larger transport.

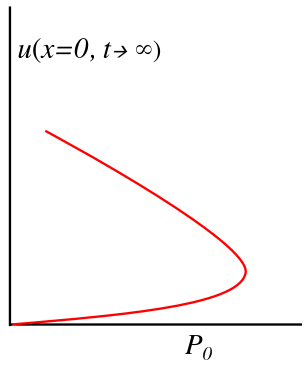

Figure 1: Normalized steady state central island temperature $u(0,t \to \infty)$ vs. normalized rf power P_0, showing bifurcation.

One way of reaching equilibrium is by limiting the rf power available. This can be accomplished in a very physically relevant way by introducing a finite rf power at the island periphery, and then accounting for the depletion of this power as the wave damps within the island [9]. Indeed, as shown in [9], this model exhibits both the predicted current condensation and the hysteresis effect. What was also identified was a limiting effect in certain regimes, in which the island central temperature grew enough that the peripheral temperature also grew substantially. Power arriving from the boundary might then be disadvantageously shaded from reaching the center.

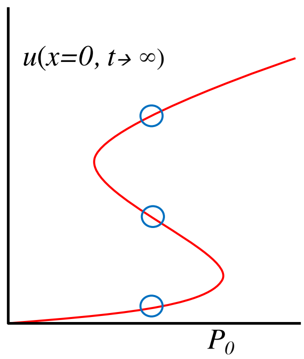

Figure 2: Normalized steady state central island temperature $u(0,t \to \infty)$ vs. normalized rf power P_0, showing hysteresis.

Discussion: This observation highlights the importance of comparing the issues and opportunities that might arise in achieving the current condensation effect through ECCD on one hand and through LHCD on the other hand, particularly in achieving it robustly or together with the favorable hysteresis effects. In both ECCD and LHCD, power is brought from the periphery, so in both cases the available power is largest at the periphery. With ECCD, there is the possibility of localizing deposition through the cyclotron resonance, thereby avoiding encountering resonant electrons at the periphery. With LHCD, there is the possibility of up-shifting the parallel wavenumber, but the control is less straightforward. On the other hand, an opportunity available with LHCD in a tokamak reactor is to amplify the wave within the plasma through the α-channeling effect [10]. In particular, lower hybrid wave trajectories launched from the high-field side of the tokamak can advantageously absorb α particle power [11, 12]. Apart from the interest for efficient current drive, such a mechanism could avoid shielding at the island periphery, since the wave power could be largest within the island rather than at the boundary. This particular opportunity would not be available to ECCD, since the interactions with ions are minimal. However, in a reactor sustained by α particle heating, the channeling of any power to the island center via this effect has the added feature of limiting the availability of that power to the island periphery, thus increasing the temperature differential. Another distinction between ECCD and LHCD lies in the tendency of lower hybrid waves to damp at higher parallel phase velocities, which not only gives higher current drive efficiencies. but also allow reaching at lower power dissipated the high-u regimes that feature the largest condensation effects as well as the possibilities for capitalizing on the hysteresis effect.

While ECCD and LHCD are the most promising candidates for stabilizing the NTM, and also most likely to exploit the rf current condensation effect, other waves should not be discounted, particularly those that might interact advantageously with both electrons and α particles.

Acknowledgments: Work supported by DOE DE-SC0016072. and DE-AC02-09CH11466.

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