On the role of non-equilibrium, relativistic hot electron population in Target Normal Sheath Acceleration

A. Maffini[1], A. Formenti[1], M. Passoni[3]

[1] *Politecnico di Milano, Department of Energy, Milano, Italy*

Laser interaction with solid density targets is a topic of great interest in a number of research fields that have to deal with plasmas, such as laser-based radiation sources, astrophysics, gas discharges and plasma-wall interaction in fusion devices. If the laser pulse is ultra-intense (I$\geq 10^{18}$ W/cm^2) and ultra-short ($\tau \leq 100$ fs) and the target is thin enough, then the interaction results in the conversion of the transverse, oscillatory electric field carried by the laser pulse into a longitudinal, quasi-stationary electric field that can be exploited to accelerate an ion bunch up to energies of the order of 100 MeV[1].

Many different acceleration schemes have been identified in the literature, depending on the laser and target parameters. Among them, the one that has proven to be the most robust is the Target-Normal-Sheath Acceleration (TNSA) mechanism. According to TNSA, a relativistic “hot" electron cloud is generated through many absorption processes during the laser-matter interaction. The ion acceleration is then driven by this hot electron population that, separating from the bulk target, generates a strong accelerating field, of the order of some MV$/\mu$m. The hot electron population is usually characterized by an average kinetic energy of the order of few MeV, hence the name *hot*. Getting insights on the dynamics of hot electrons is key to understand and control the ion acceleration process under the TNSA scheme.

In general, laser-ion acceleration is massively investigated with both numerical simulations and analytical models. Numerical simulations are useful for theoretical understanding of the processes at play, but have limitations in their predictive capability and are subject to a trade off between accuracy and computational cost, often affected by specific features of interest. On the other hand, models represent a complementary approach that allow to retrieve general trends and dependencies with simplified pictures. Several models to describe the TNSA process exist that rely on different physical features, such as Isothermal Expansion Model [2], Adiabatic Expansion Model [3] and quasi-static models. Among the quasi-static models, the Passoni-Lontano allows to take into account kinetic and relativistic effects in a self-consistent way [4, 5, 6].

This approach assumes that, following the laser pulse absorption, the hot electron population reaches a stationary state which lasts for a time of the order of hundreds of fs. During this time, both target bulk ions and light contaminant ions (i.e., the ions on the rear target surface that are accelerated by the TNSA field) can be considered frozen because of their inertia. The stationary state reached by the hot electron population is described by a time-independent distribution function $f(\mathbf{x},\mathbf{p})$, which depends on the spatial variable $\mathbf{x}$ through the electrostatic potential $\varphi(\mathbf{x})$ generated by the hot electron charge displacement itself.

Under the assumption of invariance with respect to the spatial coordinates in the target plane (a reasonable assumption if the transverse length scale is $\ll$ of the laser spot size), the self-consistent potential (and thus the accelerating field) can be found by solving the 1-dimensional self-consistent Poisson equation inside ($x < 0$) and outside ($x > 0$) the target:

$$\begin{cases} \Delta\varphi = 4\pi e(n_{trap}(x) - n_{trap}(x^*)) & \text{if } x < 0 \\ \Delta\varphi = 4\pi e n_{trap}(x) & \text{if } x > 0 \\ \varphi(x) = \varphi^* & \text{if } x = x^* \\ \dfrac{\mathrm{d}\varphi}{\mathrm{d}x}(x) = 0 & \text{if } x = x^* \\ \dfrac{\mathrm{d}\varphi}{\mathrm{d}x}(0^+) = \dfrac{\mathrm{d}\varphi}{\mathrm{d}x}(0^-), \end{cases} \tag{1}$$

where it is assumed that only the trapped hot electrons,i.e. the bounded electrons whose total energy (kinetic and electrostatic potential energy) is negative, contribute to the build up of the potential:

$$n_{trap}(x) = \int_{-p_{cutoff}}^{+p_{cutoff}} f(x,p)\mathrm{d}p \tag{2}$$

$$p_{cutoff} = m_c c\sqrt{\left(\frac{e\varphi(x)}{m_e c^2}+1\right)^2 - 1} \tag{3}$$

The last equation comes from the condition $-e\varphi(x) + m_e c^2(\gamma(p) - 1) \leq 0$ and it ensures that $\varphi \to 0$ as $x \to \infty$.

The hot electron distribution function must solve the relativistic Vlasov equation for the stationary states, namely:

$$\frac{p}{\gamma(p)m_e}\frac{\partial f}{\partial x} - e\frac{\mathrm{d}\varphi}{\mathrm{d}x}\frac{\partial f}{\partial p} = 0. \tag{4}$$

The space profile of φ can be obtained by solving numerically equations 1 and 2. If one considers a light ion initially at rest on the rear target surface as a test particle that does not perturb φ, then $eZ\varphi(x=0) = eZ\varphi_0$ is the maximum energy it would acquire because of the self-consistent potential. Therefore, φ_0 can be considered as an estimate of the ion cut-off energy.

There are no specific requirements on the functional form of f, provided that the condition in eq. 4 is satisfied. A choice that has been widely employed in the literature by virtue of its simplicity is to assume an equilibrium distribution function such as the Maxwell-Jüttner distribution or its ultra-relativistic limit, by means of which equation 4 is fulfilled.

However, it is very likely that the hot electron population is brought to a non-equilibrium state upon the laser-target interaction, since this process is highly nonlinear. Moreover, the degree of collisionality for a diluted relativistic electron gas can quite low, so that some kind of non-equilibrium feature may survive long enough to have an impact on the light ion dynamics and acceleration. Therefore, an extension of self-consistent quasi-static TNSA models to include non-equilibrium electron distribution function is highly desirable.

Among the possible choices for $f(x,p)$, the Cairns distribution function is an interesting option to be considered. Firstly introduced by R. A. Cairns to explain density depletion in earth ionosphere [7], it is a non-relativistic distribution function that depends on a real parameter $\alpha \in (0,\infty)$ which accounts for some non equilibrium features. The Cairns distribution function has been proposed to describe the non-thermal electrons in TNSA modeling [8]; however its non-relativistic character is an intrinsic limitation as far as hot, relativistic electrons are concerned. Here we propose to explore the role of non-thermal effects on the TNSA process by extending the self-consistent quasi-static TNSA model making use of a relativistically correct Cairns-like distribution function, namely:

$$f(x,p) = \frac{n_0}{\mathscr{N}(T,\alpha)} \left\{ 1 + \alpha \left[\frac{m_e c^2(\gamma(p)-1) - e\varphi(x)}{T} \right]^2 \right\} \exp\left[-\frac{m_e c^2 \gamma(p) + e\varphi(x)}{T} \right], \quad (5)$$

where the normalization constant $\mathscr{N}(T,\alpha)$ is given by:

$$\mathscr{N} = 2K_1\left(\frac{m_e c^2}{T}\right) + \frac{\alpha {m_e}^2 c^4}{2T^2}\left(-4K_0\left(\frac{m_e c^2}{T}\right) + 7K_1\left(\frac{m_e c^2}{T}\right) - 4K_2\left(\frac{m_e c^2}{T}\right) + K_3\left(\frac{m_e c^2}{T}\right)\right). \quad (6)$$

This distribution function tends to the Maxwell-Jüttner equilibrium distribution for $\alpha \to 0$; for $\alpha \to \infty$ it describes two counter-streaming electron beams. In order to assess the effect of the non equilibrium features on the TNSA acceleration process one can numerically solve the model for different values of α and then compare the ion cut-off energy by observing how $\varphi_0(\alpha)$ varies with α. However, we point out that also the total number N_{tot} of electrons and their kinetic energy E_{tot} vary with α:

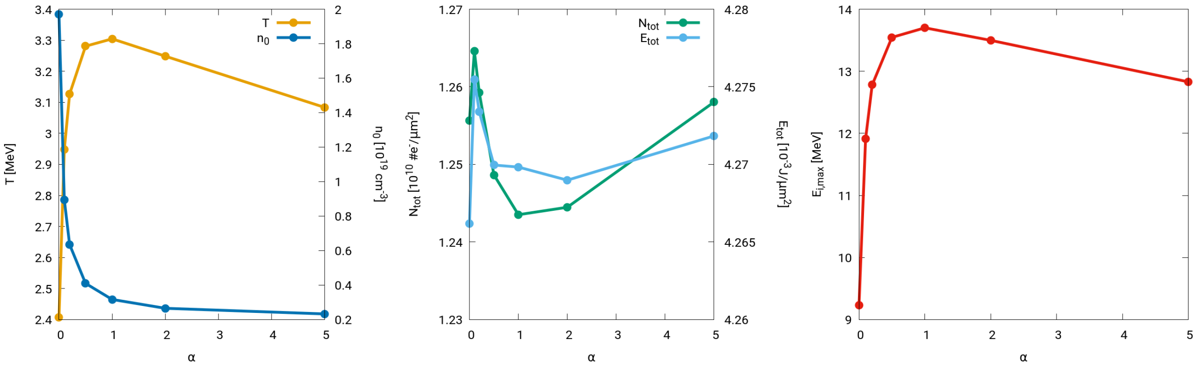

Figure 1: *Left: $n_0(\alpha)$, and $T(\alpha)$ for which the conditions $\frac{\partial N_{tot}}{\partial \alpha} = 0$ and $\frac{\partial E_{tot}}{\partial \alpha} = 0$ are verified. We assume $\varphi^*(\alpha) = 4.8T(\alpha)$. Middle: the corresponding values of N_{tot} and E_{tot} as a function of α. Right: the value of the potential at the target-vacuum interface φ_0*

$$\frac{\partial N_{tot}}{\partial \alpha} = \frac{\partial}{\partial \alpha} \int_0^\infty \int_{-d}^\infty f(x,p;\alpha)\mathrm{d}x\mathrm{d}p \neq 0 \tag{7}$$

$$\frac{\partial E_{tot}}{\partial \alpha} = \frac{\partial}{\partial \alpha} \int_0^\infty \int_{-d}^\infty m_e c^2(\gamma(p)-1)f(x,p;\alpha)\mathrm{d}x\mathrm{d}p \neq 0. \tag{8}$$

Therefore, in order to make a fair comparison, we adjusted the model parameters n_0, T and φ^* for each value of α in order to keep N_{tot} and E_{tot} constant. In figure 1(left) we show a particular set of $n_0(\alpha)$, $T(\alpha)$ and $\varphi^*(\alpha)$ for which the conditions $\frac{\partial N_{tot}}{\partial \alpha} = 0$ and $\frac{\partial E_{tot}}{\partial \alpha} = 0$ are verified within 1% accuracy (figure 1(middle)). The corresponding value of $\varphi_0(\alpha)$ is shown in figure 1(right). A significant difference (up to 40%) can be appreciated, signaling the fact that non-thermal features of the hot electron population can play a major role in the TNSA ion acceleration.

References

[1] A. Macchi, M. Borghesi and M. Passoni, Rev. Mod. Phys. **85**, 2 (2013): 751.

[2] P. Mora et al., Phys. Rev. Lett. **90** 18 (2003): 185002.

[3] S. Betti, et al., Plasma Phys. Controlled Fusion **47** (3) (2005) 521.

[4] M. Passoni and M. Lontano, Phys. Rev. Lett. **101** 11 (2008): 115001.

[5] M. Passoni et al., New J. Phys. **12** 4 (2010): 045012.

[6] M. Passoni et al. Phys. Plasmas **20** 6 (2013): 060701.

[7] R. A. Cairns et al. , Geophys. Res. Lett. **22** 20 (1995): 2709.

[8] A. Bahache et al., Phys. Plasmas **24** 8 (2017): 083102.