Ion acceleration in a non-equilibrium plasma flow expanding from a magnetic mirror

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Magnetic field expansion is successfully used for suppression of longitudinal electron heat flux in most of presently operating big linear machines for fusion-aimed investigations [1–3] and numerous applications based on compact mirror traps [4–6]. Besides reducing the density of energy flow such inhomogeneity of the magnetic field results in the variation of electrostatic potential along the field line that reflects most of passing electrons back to the trap body, thereby reducing collisional heat flux to the end walls, and accelerates ions [7]. The expansion also prevents secondary electrons generated at the end wall from reaching and cooling the plasma.

The basic physics of the potential formation in the expander region is well known and summarized by Ryutov in his seminal paper [8]. However, there are still some discrepancies between the theory and experiments. Particularly, the potential drop in the Debye sheath near the plasma collector is up to order of magnitude higher than measured at the GDT facility in Budker Institute [9]. The discrepancy is traditionally explained by plasma interaction with a residual neutral gas. However, this is not consistent with the recent experiments that demonstrate a weak dependence of plasma parameters on the background gas density [10,11]. In this paper, we propose an explanation of such discrepancy assuming the negligible influence of neutrals but considering ion acceleration by varying ambipolar electric field. Being taken into account consistently with the plasma potential formation, it allows to develop a relatively simple model that predicts the potential drop compatible with the experiment.

We consider a stationary flow of singly ionized plasma expanding in a divergent magnetic field. The cross-section S of the flow varies following the conservation of the magnetic flux, $\int B\,dS = \text{const}$. The cross-section S monotonously increases from $S_{\min}$ at the magnetic plug to $S_{\max}$ at the conductive wall collecting the plasma. Near the plug, we assume that the velocity distribution of the electrons is close to the Maxwellian distribution. Near the conductive wall, the plasma is so rarefied that collisions are weak and the expansion results in a strongly anisotropic distribution function of electrons. Somewhere between these two regions, there is an area that provides a smooth transition from isotropic to strongly anisotropic electron velocity distributions. We consider this transition area to be of zero width, thus it may be replaced with the boundary cross-section S_{b} at which the plasma flow and potential are continuous. The ions

are described with the quasi-one-dimensional stationary fluid equations:

$$\frac{d}{dS}\left(Snu_{\mathrm{i}}\right)=0, \tag{1}$$

$$\frac{d}{dS}\left(Sm_{\mathrm{i}}nu_{\mathrm{i}}^{2}\right)+S\frac{d}{dS}\left(nT_{\mathrm{i}}\right)=-Sne\frac{d\varphi}{dS}, \tag{2}$$

where u_{i}, T_{i} and m_{i} are, correspondingly, the directed velocity, temperature and mass of ions, e is the elementary charge, φ is the plasma (ambipolar) potential, n is the plasma density. Here we embed the quasi-neutrality condition: ion and electron densities are equal, $n=n_{\mathrm{i}}=n_{\mathrm{e}}$, everywhere except the Debye sheath near the conductive wall at $S_{\max}$. Thus, the electron dynamics enters these equations through a particular $n(S,\varphi)$ defined from the following expression

$$n(S,\varphi)=\int f_{\mathrm{e}}\,d^{3}\mathbf{v}, \tag{3}$$

where $f_{\mathrm{e}}(S,\mathbf{v})$ is a given electron distribution function in a velocity space at cross-section S.

In the collisional expansion region, $S_{\min}\leq S\leq S_{\mathrm{b}}$, the electron distribution function is assumed to be Maxwellian with temperature T_{e} equal to the electron temperature inside the trap and maintained constant in the region due to high electron thermal conductivity. It results in the Boltzmann's law, $e\varphi=T_{\mathrm{e}}\ln\left(n/n_{\mathrm{p}}\right)$, where n_{p} is the plasma density at the plug. The second condition needed to solve (1)–(2) is followed from known analogy with Laval's nozzle [12]: plasma velocity is equal to isothermal ion acoustic velocity at minimal cross-section, $u_{\mathrm{i}}(S_{\min})=c_{\mathrm{s}}\equiv\sqrt{(T_{\mathrm{e}}+T_{\mathrm{i}})/m_{\mathrm{i}}}$. Then solutions of equations (1)–(2) determine a self-consistent variation of the ion velocity, plasma density and potential in collisional expansion region, thus providing their values at the boundary cross-section S_{b}.

In the kinetic expansion region, $S_{\mathrm{b}}\leq S\leq S_{\max}$, a solution of collisionless Boltzmann equation is found as an arbitrary function of two integrals of motion, with

$$f_{\mathrm{e}}=F(\mathscr{E},\mu),\qquad \mathscr{E}=\frac{m_{\mathrm{e}}\upsilon_{z}^{2}}{2}+\frac{m_{\mathrm{e}}\upsilon_{\perp}^{2}}{2}-e\varphi,\quad \mu=\upsilon_{\perp}^{2}S. \tag{4}$$

Here $\mathscr{E}$ is the electron energy, υ_{z} and $\upsilon_{\perp}$ are the longitudinal and transverse electron velocities with respect to the magnetic field lines (paraxial approximation used), μ is the magnetic moment related to the electron gyromotion. We assume μ to be an adiabatic invariant, because the divergence of $S(z)$ is slow compared to the electron Larmor radius.

To reconstruct f_{e}, let us consider $S=S_{\mathrm{b}}$ and assume that the electron distribution function here is still Maxwellian. Obviously, we can say so for the electrons with $\upsilon_{z}>0$. Its distribution would be a half-Maxwellian corresponding to $\upsilon_{z}>0$ with the same temperature as in the collisional region. Now let us introduce the potential φ_{w} of the plasma absorbing wall placed at $S_{\max}$. Electrons with low enough kinetic energy can not penetrate through the potential well

$e(\varphi_\mathrm{b}-\varphi_\mathrm{w})$ and do not reach the wall. Thus, f_e at S_b is expressed as $f_\mathrm{e}=A\exp(-\mathscr{E}/T_\mathrm{e})$ which is defined in the velocity region

$$\mathscr{V}_\mathrm{b}=\begin{cases} \upsilon_\perp<\left[\frac{2e(\varphi_\mathrm{b}-\varphi_\mathrm{w})/m_\mathrm{e}-\upsilon_z^2}{1-S_\mathrm{b}/S_\mathrm{max}}\right]^{1/2} & \text{if } \upsilon_z<0 \\ \text{any } \upsilon_\perp & \text{if } \upsilon_z>0 \end{cases} \tag{5}$$

and zero outside of this region. After that there are only two unknown parameters, the norm A and the wall potential φ_w, that may be recovered from the conditions of quasi-neutrality and zero net current of ions and electrons at the cross-section S_b:

$$\int_{\mathscr{V}_\mathrm{b}} 2\pi\upsilon_\perp f_\mathrm{e}\,d\upsilon_\perp d\upsilon_z=n_\mathrm{b},\quad \int_{\mathscr{V}_\mathrm{b}} 2\pi\upsilon_\perp\upsilon_z f_\mathrm{e}\,d\upsilon_\perp d\upsilon_z=n_\mathrm{b}u_\mathrm{ib}. \tag{6}$$

All these allow to fully determine the electron distribution function for the area of the velocity space that is causally related to the cross-section S_b. However, for $S>S_\mathrm{b}$ there is an area in velocity space that is not related to the collisional region because corresponding particles are both reflected by the magnetic mirror and by the plasma potential in their traveling to the wall. However, the trapped electrons may influence the potential profile. Following Ryutov, we assume that the velocity-space density of trapped electrons is not much different from that of the passing electrons [8]. Thus, finally, we consider $f_\mathrm{e}=A\exp(-\mathscr{E}/T_\mathrm{e})$ with A from (6) and defined in an extended region

$$\mathscr{V}(S)=\begin{cases} \upsilon_\perp<\upsilon_1 & \text{if } \upsilon_z<0 \quad \upsilon_1=\left[\frac{2e(\varphi-\varphi_\mathrm{w})/m_\mathrm{e}-\upsilon_z^2}{1-S/S_\mathrm{max}}\right]^{1/2} \\ \upsilon_\perp<\max(\upsilon_1,\upsilon_2) & \text{if } \upsilon_z>0 \quad \upsilon_2=\left[\frac{2e(\varphi-\varphi_\mathrm{b})/m_\mathrm{e}-\upsilon_z^2}{1-S/S_\mathrm{b}}\right]^{1/2} \end{cases}$$

and zero outside of this region. Then (3) may be expressed in explicit form in terms of the error function and is used for numerical solution of ion equations (1)–(2).

However, the wall potential φ_w defined by (6) is inconsistent with the numerical solution $\varphi(S_\mathrm{max})$ of (1)–(2): in all cases $\varphi_\mathrm{w}<\varphi(S_\mathrm{max})<0$. This may be interpreted as follows. The quasi-neutrality condition is not met in the thin Debye sheath near the wall, however, the wall potential φ_w is determined accurately basing on principles independent of the sheath formation. The Debye sheath is typically narrow in comparison to the scale of the expansion region, so $\varphi(S_\mathrm{max})$ characterizes the potential just before the sheath. Therefore, we can introduce the potential drop inside the sheath as $\Delta\varphi_\mathrm{w}\approx\varphi_\mathrm{w}-\varphi(S_\mathrm{max})$, that makes our description consistent. Note that even slight shift of $\varphi(S_\mathrm{max})$ in comparison to φ_w due to ion acceleration may affect $\Delta\varphi_\mathrm{w}$ substantially. This fact is illustrated in figure 1(a), where two potential profiles are plotted: one is obtained within the described model while another corresponds to the case when equation (2) is replaced with $u_\mathrm{i}=\mathrm{const}$. We place the wall at $S_\mathrm{max}/S_\mathrm{min}=100$ and consider $S_\mathrm{b}/S_\mathrm{min}\to 1$.

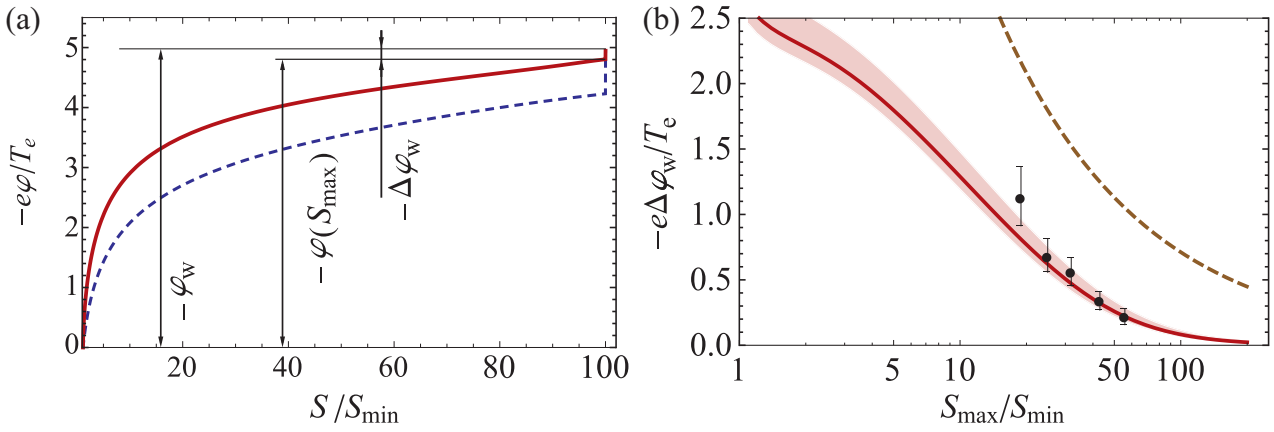

Figure 1: (a) The potential profile in expander with (solid) and without (dashed) ion acceleration considered. (b) The dependence of the wall potential drop on the total expansion ratio. Electron temperatures $0 < T_i < T_e$ are indicated with a cloud with the central curve for $T_i = 0.6T_e$. Dashed curve correspond to Ryutov's asymptotic. Black dots indicate the experimental results from [9].

Comparison of $|\Delta\varphi_w|$ resultant from our modelling to both theoretical estimations by Ryutov [8] and experimental data measured at the GDT [9] are shown in figure 1(b). One may find out that both $|\Delta\varphi_w|$ resultant from modelling and Ryutov's estimate predict the same tendency to decrease with the expansion ratio. However, our approach results in much smaller wall potential drop which is in good agreement with the experimental values.

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