Turbulence-induced transport dynamo mechanism

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Abstract. Turbulence can induce random motions of particles. The random motions of particles can break the magnetic field lines. The flow associated with these particle motion can in turn generate the magnetic field. The transport magnetic field generation mechanism is different form the conventional turbulent dynamo mechanism in that it is mediated by particle transport, and thus can occur on a much slower time scale.

1. Introduction

The generation and amplification of magnetic fields are important in laboratory and astrophysical plasmas. A wide variety of interesting phenomena arise by the complex interplays between the magnetic fields and the plasmas. Many theories for the magnetic field generation have been proposed such as turbulent dynamo, Biermann battery, Weibel instability, thermoelectric current dynamo, etc. Underneath all these theories, there lies Cowling's anti-dynamo theorem. According to the anti-dynamo theorem by Cowling, generating the magnetic field by plasma flows in two dimensional geometry is not possible. Thus, the dynamo theories have evolved into three dimensional geometry, which has a long development history [2].

Cowling noted in his monograph [1] that the flow velocities required for the sustainment of the magnetic field in the Sun and the Earth are very small, but they have to be the relative ones to the field lines. There can exist many flow velocities much greater than these, but overcoming the line-tying effects is not easy in a highly conducting media [3]. Thus, one can say that the essence of the dynamo problem lies in finding a flow nearly frozen in field lines which increases the magnetic field.

Random motions of particles can break the magnetic field lines. Because of this particular nature, transport flows of particles deserve a special attention from a point of view of dynamo. Cross field plasma transport has been well investigated in the thermonculear fusion plasma research. The particle transport can be described by

$$\mathbf{\Gamma} = -D\nabla n + \mathbf{v}n, \tag{1}$$

where n is the density, D the diffusion coefficient, and $\mathbf{v}$ is the flow velocity. The plasma transport described by the coefficient D is determined by collision and turbulence. In the turbulent case, the transport is driven by the electric and magnetic fluctuations in microturbulence. In a fusion plasm device, the anomalous transport driven by turbulence can cause large plasma flows toward the wall cross the confining magnetic fields. Because the anomalous transport degrades the plasma confinement seriously, understanding the anomalous transport phenomena has been an important area in the thermonuclear fusion research. Although it has been well known that the anomalous transport crossing the field line is important, the fact that the same flows can actually generate the magnetic field by breaking the line-tying effects has not been much realized.

Turbulence can generate the magnetic field via the direct coupling between the magnetic and velocity components, but it can also generate the magnetic field by introducing random ion motions to break the magnetic field lines. In the present paper, the latter possibility of the magnetic field generation induced by turbulence is studied.

2. Turbulent dynamo mechanism

The basic ideas of the mean-field MHD dynamo can be briefly summarized as follows.

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\eta c^2}{4\pi} \nabla^2 \mathbf{B}. \tag{2}$$

The field and flow velocity variables can be decomposed into the mean and the fluctuating parts. By denoting the overbar as the average value, and the prime the fluctuation, the above equation can be written as

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{v}} \times \overline{\mathbf{B}}) + \nabla \times \mathcal{E} + \frac{\eta c^2}{4\pi} \nabla^2 \overline{\mathbf{B}}. \tag{3}$$

The average value is taken on a large scale. The small perturbations normally come from waves or turbulence. The coupling of small perturbations of velocity and magnetic fields can give rise to a non-zero mean EMF(electomotive force)

$$\mathcal{E} = \overline{\mathbf{v}' \times \mathbf{B}'} \tag{4}$$

for $\overline{\mathbf{v}'} = \overline{\mathbf{B}'} = 0$.

This formulation of the mean EMF is at the core of the mean-field MHD dynamo theory by turbulence [2]. The mean-field dynamo theory has been successful in reproducing the butterfly diagram of the sun spot in simulations. But, recently, questions have been raised about the existing solar dynamo model, which invoked the rethinking of the dynamo theory [4, 5]. The necessity of an additional α- effect was even suggested [6]. On the other hand, to find the experimental evidence, Tzeferacos et al. have studied the turbulent dynamo in colliding jet plasmas produced by using a high power laser [7]. However, Ryu pointed out that the Biermann battery effects could play an important role on a small scale turbulent dynamo [8].

One of the main weak points of the turbulent dynamo theory is that it cannot properly amplify the seed toroidal fields to the levels required [9]. It is generally believed that the active regions on the solar surface originates from a strong toroidal magnetic field deep seated in the solar convection zone. Thin flux tube models of emerging flux loops through the solar convective envelope requires a strong toroidal magnetic field generation at the base of the solar convection zone about $10-100$ times the kinetic energy [9]. However, the turbulent dynamo mechanism cannot increase the magnetic energy beyond the equipartition energy.

3. turbulence-induced transport dynamo

When the transport is included, a new class of dynamo mechanism becomes possible, which cannot be conceived in the realm of convective flows [10, 11, 12, 13]. The temporal evolution of the flux $\Phi = \int \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{A}$ becomes

$$\frac{\partial}{\partial t} \int \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{A} = \oint \mathbf{v} \times \boldsymbol{B} \cdot \mathbf{d}\boldsymbol{l} - \frac{\eta c^2}{4\pi} \oint \nabla \times \boldsymbol{B} \cdot \mathbf{d}\boldsymbol{l}, \tag{5}$$

$$= -2\pi a v_D B_z + \frac{1}{2}\eta c^2 a \frac{\partial B_z}{\partial r}, \tag{6}$$

where $\mathrm{d}\boldsymbol{A}$ is the surface element and $\boldsymbol{l}$ denotes a line element at the boundary, and $v_D = -D\frac{1}{n}\frac{\partial n}{\partial r}$.

The generation of the magnetic field by a transport flow can occur only in a non-uniform plasma. To amplify the magnetic field, the density profile has to be hollow. This transport dynamo mechanism is different from the thermoelectric current dynamo, where thermal transport of electron occurs due to the temperature difference. If there are intense turbulent activities in plasmas, such as Alfvénic or drift turbulence, anomalous ion transport would occur. For a highly turbulent plasma, D can be estimated from the mixing length and the correlation time.

D_{class} is the classical diffusion coefficient due to the binary collision and D_{Bohm} is the anomalous diffusion discovered by Bohm experimentally. Thus, we can take $D = \frac{kTc}{eB}$ as the turbulent diffusion coefficient [14]. T is the temperature and c is the speed of light. The growth rate of the magnetic field in the induction equation can be estimated from the field induction as

$$\gamma \mathbf{B} \sim \frac{1}{L} v_D \mathbf{B} \sim \frac{1}{L^2}\frac{kTc}{eB}\mathbf{B}, \tag{7}$$

from which the magnetic field amplification rate can be obtained as

$$\gamma \sim \frac{1}{L^2}\frac{kTc}{eB}. \tag{8}$$

If the above growth rate is applied to a sunspot flux tube in the convection zone for the plasma parameters B=5 gauss, $T = 2.0 \times 10^6 K$, and the spatial scale $L = 1000 km$, then, we can get the magnetic field amplification rate $\gamma \sim 3.5 \times 10^{-7}$/sec ~ 0.03/day. The growth rate decreases as the magnetic field and the flux tube size increase.

Sustainment of the flux tube requires a source and a sink, outside and inside, respectively. Ionization and recombination of atoms could play such roles of the source and the sink. Then the density profile can be sustained for a long time via the recycling process. In a controlled thermonulcear fusion device, recycling of the working gas at the boundary of the plasma is necessary to keep the plasma longer than the particle confinement time [15]. Because of a high recycling rate, plasmas can be sustained for many particle confinement times without the injection of gas from the external sources in the vacuum system. A similar recycling process can take place for a sun spot flux tube, where The pressure balance can be written as $p_{\text{out}} = p_{\text{in}} + \frac{B^2}{8\pi}$. And thus, the pressure outside would be higher than the inside. Since $p = nT$, the density and temperature in the strong magnetic field region will be lower than those in the weaker region. Therefore, the temperature and density inside the flux tube would be lower than outside, which would make the density profile hollow with inward transport flow.

4. Conclusion

Turbulence-induced particle transport can be an interesting dynamo mechanism. The particle transport flow can break the flux-freezing condition, inducing magnetic fields. Transport induced by turbulence is a widespread phenomenon. This suggests that the diffusion with the magnetic field could have an interesting application in plasma physics. The magnetic field can be significantly increased or decreased by turbulence particle transport.

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6. References

[1] Cowling T. 1976 Magnetohydrodynamics, *Chap5, p86, Adam Hilgar, Bristol.*
[2] Brandenburg A. 2018 *J. Plasma Phys.* **84(4)** 735840404.
[3] Bondi H. and Gold T. 1950 *Month.,Not. R. Soc.* **110** 607.
[4] Charbonneau P. 2016 *Nature* **535** 500.
[5] Wright N. and Drake J. 2016 *Nature* **535** 526.
[6] Guerrero G. and Käpylä P. 2011 *A&A* **533**, A40.
[7] Tzeferacos, P. *et al.* 2018 *Nat. Comm.* **9 (1)**, 591.
[8] Ryu C.M. *et al.* 2019 *High Energ. Dens. Phys.* **3**, 74.
[9] Fan Y. *Living Rev. Sol. Phys.*, 2009 **6**: 4.
[10] Ryu C.M. 1984 *Los Alamos preprint*, LA-UR–84-1677; Ryu C.-M. 1987 *J. Astron. Space Sci.* **4(2)** 95.
[11] Ryu C.M. and Yu M. 1998 *Phys. Scr.* **57(5)** 601.
[12] Lee T.Y and Ryu C.M. 2004 *Phys. Plasmas* **11** 5462.
[13] Lee T.Y. and Ryu C.M. 2007 *J. Phys. D: App. Phys.* **40(19)** 5912.
[14] Wikipedia "Bohm Diffuison" *https://en.wikipedia.org/wiki/Bohm_diffusion.*
[15] Marmar E. 1978 *J. Nucl. Mater.* **59** 450.