From Lawson to Burning Plasmas: a Multi-Fluid Analysis with Simplified Impurity Effects

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In experimental devices for controlled nuclear fusion, in particular for the magnetic confinement approach, a critical figure of merit is given in terms of the "Lawson product" $p_{tot}\tau_E$, where p_{tot} is the total (ions plus electrons) plasma pressure and τ_E the energy confinement time. The Lawson criterion [1] provides a temperature-dependent value for the Lawson product that needs to be reached for the plasma to ignite, i.e. to continue producing fusion reactions once all the external heating power is turned off. Recent work [2] extended the zero-dimensional, single-fluid Lawson criterion to a one-dimensional model, in which ions, electrons and alphas are treated as separate fluids. More important to current and future experiments is the analysis of plasmas with a finite production of fusion power, but in the presence of external heating. The ratio between the total fusion power and total heating power is called gain factor and indicated with the symbol Q. $Q = 1$ is the breakeven condition (where the fusion power is equal to the external heating power) and $Q = 5$ is the threshold value for the "burning plasma" condition, where the fusion power deposited into the plasma is equal to the external heating power. This is correct only for the deuterium-tritium (DT) fusion reaction, the most likely one to be used in the next generation of $Q \geq 1$ devices, due to the fact that in that reaction 80% of the fusion power is carried away by neutrons, which are not interacting with the plasma. Finite-Q plasmas were analyzed in Ref. [3].

In the work in both Refs. [2] and [3] several simplifying assumptions were made. Among others, it was assumed that the plasma is composed of only hydrogen ions (in equal parts deuterium and tritium) and electrons. On the other hand, experimental plasmas always contains impurities, i.e. other ions coming from imperfect cleaning of the experimental chamber or from sputtering from the first wall. It is well known that the presence of impurities degrades the performance of plasmas in two fundamental ways. First, additional radiation losses are produced. Second, if plasma quasi-neutrality is assumed, the concentration of hydrogen ions is reduced with respect to the one of a pure plasma. These effects will be briefly considered in this work. In a more subtle way, the temperature profiles for electron and hydrogen ions are altered through transport equations by the presence of additional heat losses. For the purpose of this work, temperature profiles will be assumed to not depend on the presence of impurities.

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In this work, we will consider ions, electrons and alpha particles. Their behavior is governed by the equations

$$\frac{3}{2}n_i\frac{\partial T_i}{\partial t} = S_{hi} - \frac{3}{2}\frac{p_i}{\tau_{Ei}} + \frac{3}{2}\frac{n_i(T_e - T_i)}{\tau_{eq}} \tag{1}$$

$$\frac{3}{2}n\frac{\partial T_e}{\partial t} = S_{he} - \frac{3}{2}\frac{p_e}{\tau_{Ee}} + \frac{n_\alpha}{\tau_\alpha}E_\alpha - C_B\frac{p_e^2}{T_e^{3/2}} + \frac{3}{2}\frac{n(T_i - T_e)}{\tau_{eq}} \tag{2}$$

$$\frac{\partial n_\alpha}{\partial t} = \frac{n_i^2}{4}<\sigma v> - \frac{n_\alpha}{\tau_\alpha} - \frac{n_\alpha}{\tau_{E\alpha}}, \tag{3}$$

where T_e, T_i, p_e, p_i are electron and hydrogen ion temperatures and pressures, n is the electron number density, n_i the hydrogen ion number density, S_{hi} and S_{he} the external heating power deposited into ions and electrons, τ_{Ei} and τ_{Ee} the energy confinement times for ions and electrons, τ_{eq} the ion-electron equilibration time, n_α the energetic alpha particle number density, τ_α the alpha slowing down time, E_α the energy of an alpha particle created by a DT fusion reaction, C_B the coefficient of Bremsstrahlung, $<\sigma v>$ the distribution function averaged cross section for fusion reactions and $\tau_{E\alpha}$ the energy confinement time for alpha particles. In all our calculations we assign the electron and impurity density, from which the ion density is calculated as $n_i = n - Zn_{imp}$, where Z is the impurity charge and n_{imp} the impurity number density. It should be noticed that we take into account impurities in the calculation of Bremsstrahlung radiation, but for simplicity do not include recombination in the energy balance.

In the original work of [2], both zero- and one-dimensional results were presented. One-dimensional profiles are also considered in this work. Density and temperature profiles are parametrized as:

$$n(r,t) \equiv n_0(t)\left[\hat{n}_{edge} + (1 - \hat{n}_{edge})\left(1 - r^\theta\right)^\eta\right] \tag{4}$$

$$T(r,t) \equiv T_0(t)\left[\hat{T}_{edge} + (1 - \hat{T}_{edge})\left(1 - r^\nu\right)^\mu\right], \tag{5}$$

with $0.1 \leq (\mu;\eta) \leq 2$ and $1.1 \leq (\nu;\theta) \leq 4$. For impurities, we consider two profiles: a very peaked profile with high impurity concentration at the edge, $n_{imp}(r) \propto r^8$ and a perfectly flat one with no radial dependence. As detailed later, we use a fraction of the central electron density as peak value for impurities. Since the electron and ion density go to small values at the edge, when needed impurity density is adjusted at the edge to ensure that quasi-neutrality is preserved.

Several configurations were considered, with different values of Q, up to the limit of $Q \to \infty$ corresponding to ignition. Particular attention was devoted to the threshold of the burning plasma state, $Q = 5$. We will only focus on $Q = 5$ analysis in the rest of this work. Due to the limited space available in this work, only the temperature profile used in Refs. [2-3] to represent H-mode-like tokamak discharges ($\mu = 0.5$ and $\nu = 1.5$) will be discussed. As done in Refs. [2-3], we vary the density profile from a very peaked one ($\theta = 1.1$, $\eta = 2$) to a very flat one ($\theta = 4$, $\eta = 1$). For each density profile, the minimum Lawson product needed to reach $Q = 5$ in the presence of impurities is calculated numerically. Sample results are presented in Fig. 1 for the two impurity profiles introduced above, the profile peaked at the edge (left) and the flat profile (right). Different concentrations of impurities are used in the two cases to allow us to evaluate both the effect of concentration and profile. For the peaked impurity profile the maximum value of the impurity number density is set to 0.1% of the central electron density. For the
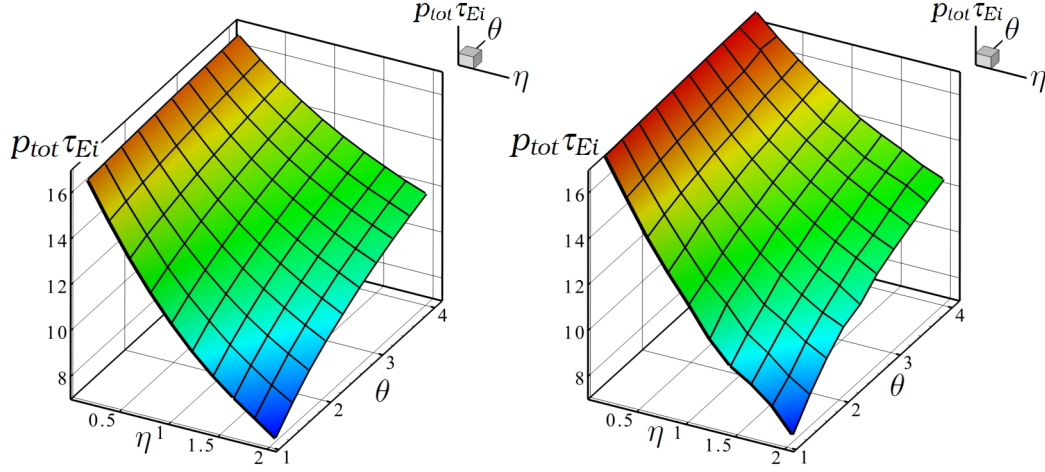
Figure 1: Triple products for $Q = 5$ and different impurity profiles

flat profile the maximum value of the number density is se to 1% of the central electron density. We only consider the case of pure carbon as impurity.

Perhaps surprisingly, the results for the two profiles are strikingly similar. As seen in Fig. 1 the minimum Lawson product needed to reach the burning-plasma condition in the presence of impurities is larger for flat density profiles and lower for peaked profiles. The same behavior was observed in the case without impurities in Ref. [3]. A numerical comparison of the Lawson product with impurities and the one without shows that the ratio between the two is with good approximation constant with respect to the (η, θ) parameters, with different constants for the two cases. In the low-concentration, peaked-profile case, the ratio is very close to unity for all the considered electron and ion density profiles. In the higher-concentration, flat-profile case the ratio is ~ 1.07. Similar ratios are found for the energy confinement times.

It is stressed that the number of impurity particles in the system depends on the density profile. A careful check of the profiles shows that for both calculations in Fig. 1 the number of impurity particles is maximum for the peaked plasma density profiles (bottom right) and minimum for the flat plasma density profiles (top left), with a variation of a factor of 2.5-3, as shown later in Fig. 2. The high-impurity case has an impurity content about 50 times larger than the low-impurity case, with a maximum of about 3.5% of the number of electrons. Conversely, the low-impurity case has a maximum impurity content of less than 0.1% of the number of electrons.

Several elements come into play to determine the actual effect of impurities when analyzed within the limit of our simplified model. The total number of impurities determines the additional losses that need to be balanced by fusion power in order to reach the desired value of Q. On the other hand, a large number of impurities at the edge has a smaller influence than a large number of impurities in the center, because of two reasons. First, the electron temperature (responsible for the radiation losses in our simple model) is lower at the edge, resulting in lower losses. Second, a reduction of the ion density at the edge (due the quasi-neutrality constraint) effectively increases the peakedness of the ion density profile, which lowers the Lawson product needed for any value of Q. Thus the combination of a larger number of impurities and their weaker effect for peaked plasma density profiles results in a total modification of the Lawson product similar to the one obtained for flatter profiles, with fewer impurities but with a stronger effect.

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It should be emphasized that the results shown in Fig. 1 refer to the Lawson product calculated only using the ion and electron fluids. The calculation can be repeated including the impurity energy content. Since we do not solve an energy equation for impurities, this is done assuming that the impurity temperature is equal to the electron temperature. Under those assumptions, the Lawson product calculated using the total plasma energy increases on average by about 1% for the high-impurity case and is marginally modified in the low-impurity case.

In order to improve our analysis, one more case is considered in some detail. In this last case we assign an impurity density profile peaked at the edge as in the low-impurity case, but with a maximum value of 1% of the central electron density. With these assumptions it is found that the maximum number of impurities occurs at intermediate values of η, as shown in the lower section of Fig. 2. In the figure, the total number of impurity particles normalized to the plasma volume is plotted as a function of the electron density profile. The flat, high-impurity case is on the top, the edge-peaked, high-impurity case on the bottom. The peaked, low-impurity case (not shown) looks qualitatively similar to the flat, high-impurity case. The different profile obtained for the peaked, high-impurity case has a noticeable effect on the resulting Lawson product. In particular, it is found that the ratio between the Lawson products with and without impurities is larger for flat profiles and becomes approximately constant for $\eta \gtrsim 1.1$. It should be noted that in this case the number of impurity particles for peaked electron density profiles is only a factor of ~ 1.5 larger than the flat profile value. This explains the reduced effect of impurities for those profiles.
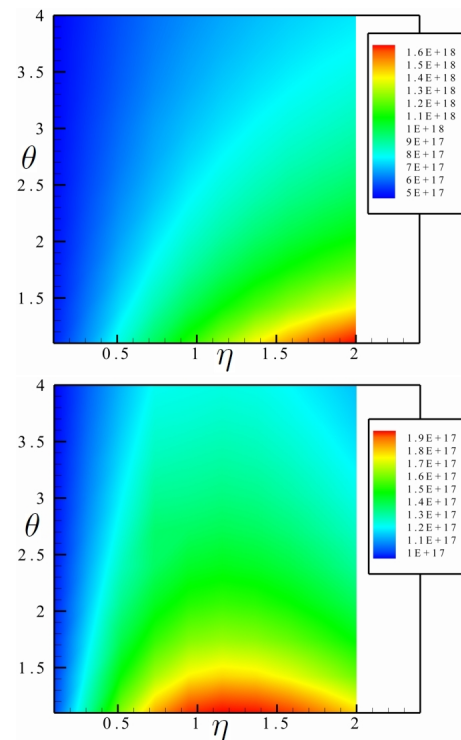

Figure 2: Impurity concentration

A more detailed analysis will be carried out in the future to better quantify the effect of impurities on the Lawson product required to reach the burning plasma state and for ignition. In the present work we have shown that a relatively small concentration of impurities causes a measurable reduction of the plasma performance by increasing the Lawson product needed to achieve the burning plasma condition. This is true even in the simplified model adopted in this work, where impurity charge exchange radiation is neglected. Remarkably, the net effect of the presence of impurities depends on the plasma density profile.

References

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