Stimulated Raman scattering of the multi-Gaussian beam in relativistic plasma

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Abstract

In the present paper, we investigate the propagation of multi-Gaussian laser beams in underdense plasma. We studied the stimulated Raman backward scattering and investigated the effect of relativistic Lorentz factor and intensity profile on the amplitude of scattered laser beam. The self-focusing of the laser has also been studied.

Introduction

The technique Inertial Confinement Fusion (ICF) has been facing many challenges [1]. Controlling the laser-plasma interaction is one of them. In this direction, the parametric instabilities is found to play greater role as these limit the interaction length. In the indirect drive on the national ignition facility, the largest instability gain corresponds to stimulated Raman scattering on the inner core of beams inside the Hohlraum [2]. In the present article, we studied the non-linear characteristics of the propagation of a multi-Gaussian laser beam in an underdense collisionless plasma, which will have application to the said fields.

Mathematical Model

Consider the propagation of multi-Gaussian laser beam (polarise in the x-direction)

$$\overrightarrow{E_L} = A_0 exp\left(-i(\omega_0 t - k_0 z)\right)\hat{x} \qquad (1)$$

Here $$\left|\overrightarrow{E_L}\cdot\overrightarrow{E_L^*}\right| = A_0^2 = \begin{cases} (nA_{00})^2 exp\left(-\frac{\delta^2}{r_0^2}\right) exp\left(-\frac{r^2}{r_0^2}\right)\left(1+\frac{\delta^2 r^2}{4r_0^4}\right)^2; & z=0 \\ \left(\frac{nA_{00}}{f[z]}\right)^2 exp\left(-\frac{\delta^2}{r_0^2}\right) exp\left(-\frac{r^2}{f[z]^2 r_0^2}\right)\left(1+\frac{\delta^2 r^2}{4r_0^4 f[z]^2}\right)^2; & z>0 \end{cases} \qquad (2)$$

here n is the number of Gaussian beam, δ is initial eccentric displacement parameter, r_0 is the beam width and *f[z]* is dimensional beam width parameter. In the presence of density perturbation, the propagating laser induces the current density. The current density oscillates at frequencies $(\omega_0 \pm \omega_e)$ and produce two side bands, namely Stokes and Anti-Stokes. The electron density in the electron plasma density (EPW) is given by

$$\frac{\partial^2 N_e}{dt^2} + 2\Gamma_e \frac{\partial N_e}{\partial t} - v_{th}^2 \nabla^2 N_e + \frac{\omega_{P0}^2}{\gamma}\frac{n_e}{n_0} N_e = 0 \quad (3)$$

Here v_{th} is the thermal speed of electrons, γ is relativistic Lorentz factor and Γ_e is the Landau damping factor. The solution of Eq.(3) have an expression

$$N_e = N_{0e}\exp[-i(\omega_e t - k_e z)] \quad (4)$$

Here k_e and ω_e are the wave vector and frequency of EPW, that follow the relation

$$\omega_e^2 = \frac{\omega_{P0}^2}{\gamma}\frac{n_e}{n_0} + k_e^2 v_{th}^2 \quad (5)$$

The solution of differential equation can be realized by WKB approximation and paraxial approach.

The comprehensive electric field in the plasma is

$$\vec{E} = \overrightarrow{E_L} + \overrightarrow{E_S} \quad (6)$$

$$\vec{E} = \overrightarrow{E_L} exp(i\omega_0 t) + \overrightarrow{E_s} exp(i\omega_S t) \quad (7)$$

Here $\overrightarrow{E_S}$ is the electric field scattered laser. The $\vec{E}$ assure the wave equation

$$\nabla^2\vec{E} - \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial\vec{J}}{\partial t} \quad (8)$$

Substituting (7) into (8) and separating out the term at scattering frequency, we get

$$\nabla^2\overrightarrow{E_S} + \frac{\omega_S^2}{c^2}\left[1 - \frac{n_e}{n_{0e}}\frac{\omega_{P0}^2}{\gamma\omega_S^2}\right]\overrightarrow{E_S} = \frac{1}{2}\frac{\omega_{P0}^2}{c^2}\frac{\omega_S}{\omega_0}\frac{n_e}{n_{0e}}\overrightarrow{E_L} \quad (9)$$

Further, we can break this in the following manner

$$\overrightarrow{E_S} = \overrightarrow{E_{0S}}\exp[ik_{0S}z] + \overrightarrow{E_{1S}}\exp[-ik_{1S}z] \quad (10)$$

Here E_{0S} and E_{1S} are slowly varying real functions of *r* and *z*. Also,

$$k_{0S}^2 = \frac{\omega_{0S}^2}{c^2}\varepsilon_{0S} \quad (11)$$

and $\omega_S = \omega_0 - \omega_e$ and $\overrightarrow{k_S} = \overrightarrow{k_0} - \overrightarrow{k_e}$.

Using Eqs. (10, 11) in Eq. (9), we get

$$-k_{0S}^2 E_{0S} + 2ik_{0S}\frac{\partial E_{0S}}{\partial z} + \nabla^2 E_{0S} + \frac{\omega_S^2}{c^2}\varepsilon_S E_{0S} = 0 \quad (12)$$

and $$-k_{1S}^2 E_{1S} - 2ik_{1S}\frac{\partial E_{1S}}{\partial z} + \nabla^2 E_{1S} + \frac{\omega_S^2}{c^2}\varepsilon_S E_{1S} = \frac{1}{2}\frac{\omega_{P0}^2}{c^2}\frac{\omega_S}{\omega_0}\frac{n_e}{n_{0e}}E_L \quad (13)$$

Under WKB approximation, from Eq. (13) one can get

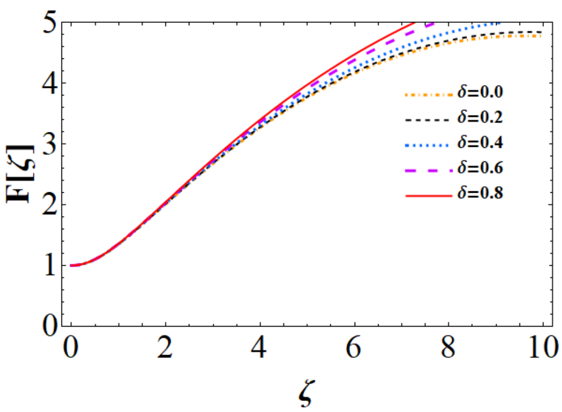
$$E_{1S} = -\frac{1}{2\,\varepsilon_{0S}}\frac{n_e}{n_{0e}}\frac{\omega_S}{\omega_0}\frac{E_L}{\left[\frac{k_{1S}^2}{k_{0S}^2} - \left(1 + \frac{1}{\varepsilon_{0S}}\left(1 - \frac{n_e}{\gamma n_{0e}}\right)\right)\right]} \quad (14)$$

Here $$\gamma = \left(1 + \frac{P_0^2}{f[z]^2}\left[\left(\left(\frac{r\delta}{2r_0^2 f[z]}\right)^2 + 1\right)^2\right]\exp\left(\frac{-\delta^2}{r_0^2}\right) exp\left(\frac{-r^2}{r_0^2 f[z]^2}\right)\right)^{\frac{1}{2}} \quad (15)$$

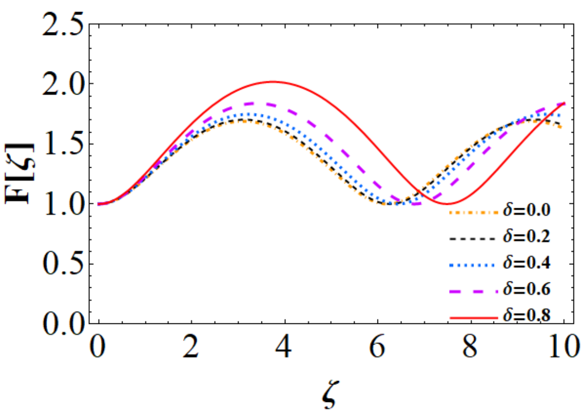
Following the approach introduced by Akhmanov et al. [3] and extended by Liu and Tripathi [4], we obtain the variation of beam width parameter with the distance of propagation. The complex behaviour of dimensionless beam width parameter is taken out through the numerical solution of the equation

$$\frac{\partial^2 f[\xi]}{\partial \xi^2} = \frac{1}{f[\xi]^2} - \frac{\rho_2^2\Omega^2 P_0^2}{f[\xi]^3}\left(\frac{\delta^2}{2r_0^2} - 1\right)\left(1 + \frac{P_0^2}{f[\xi]^2}\exp\left(-\frac{\delta^2}{2r_0^2}\right)\right)^{-3/2} \quad (16)$$

Here $\xi = \frac{z}{kr_0^2}$, $\rho = \left(\frac{\omega r_0}{c}\right)$, and $\Omega = \frac{n_e}{n_{Cr}}$, together with n_{Cr} as the critical density. The second order differential equation (16), governs the behaviour of beam width parameter under relativistic non-linearity. We solve Eq. (16) numerically for 4 different cases, as below in Figure [1].



(a)



(b)

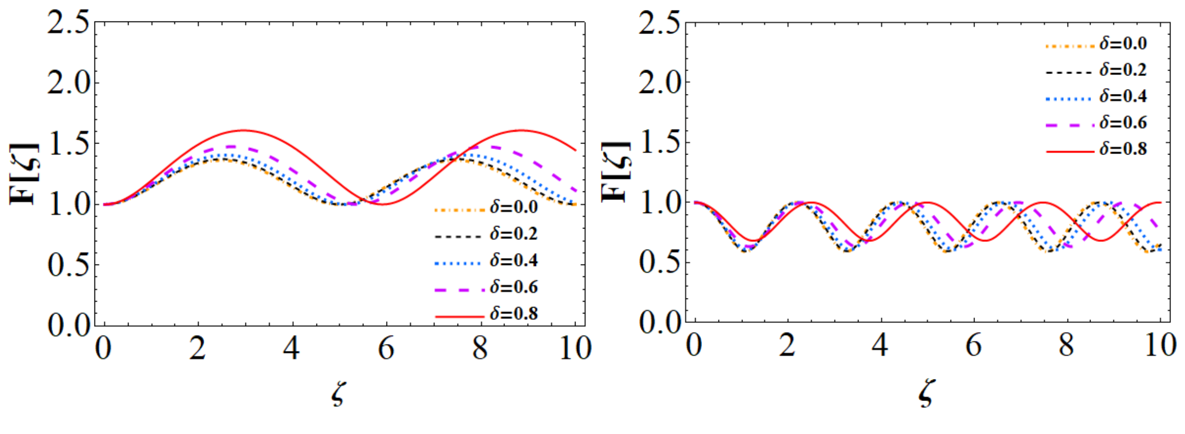


Fig. 1 The variation of dimensionless beam width parameter with normalized distance of propagation (normalised by Rayleigh length of laser) (a) $\Omega = 0.03, P_0 = 10, \rho_0 = 40$ (b) $\Omega = 0.06, P_0 = 12, \rho_0 = 40$ (c) $\Omega = 0.03, P_0 = 14, \rho_0 = 100$ and (d) $\Omega = 0.12, P_0 = 10, \rho_0 = 40$.

Results

- The amplitude of scattered laser beam is found to be proportional to ω_s/ω_0 in Raman backscattering [Eq. (14)].
- For higher the relativistic Lorentz factor, the amplitude of scattered laser beam is found to be enhanced. In other words, self-focusing enhances the amplitude of scattered laser beam, provided that other criteria of scattering are fulfilled [Eqs. (14,15)].
- For a given laser parameter, there exists a critical density ratio Ω, above which self-focusing occurs.
- The laser beam with larger eccentric displacement parameter δ get more defocused comparable to the laser having lower value of δ.
- A high intensity laser beam in less underdense plasma, only get focus if its initial beam width parameter ρ_0 is enough large [Fig. 1(c)].

References

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