Nonlinear structures in a protoplanetary disk

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I. Introduction

Balbus and Hawley, in 1991, were able to explain that accretion disks undergo a powerful shearing instability transmitted by a weak magnetic field that is responsible for the origin of turbulent viscosity in accretion disks, which they termed as magnetorotational instability (MRI) [1]. In a protoplanetary disk system (PPD), various nonlinear magnetic activities that are taking place, are strongly subjected to non-ideal magnetohydrodynamics (MHD) effects due to the low ionisation of the disk plasma.

Inutsuka and Sano, in 2005, showed that in a weakly ionised PPD, the MRI-driven turbulence produces a strong electric field in the neutral co-moving frame which leads to plasma heating at some parts of the disk [2]. In 2015, Okuzumi and Inutsuka reported that this plasma heating reduces the electric conductivity J/E before the onset of impact ionization. Also, the influence of the plasma heating on ionisation degree of the gas results the Ohm's law to be nonlinear in $\boldsymbol{E}$ [3]. It has been observed that this plasma heating triggered by the electric field eventually leads to an asymmetric electron distribution in the protoplanetary disk, which can be represented by the Davydov distribution function [4].

In this paper, we investigate how this asymmetry in electron distribution plays a significant role in the behaviour of electrostatic solitary waves (ESW) that are produced in the PPD. In Section II, we give a brief description of MRI turbulence and the resultant plasma heating in the disk as well as electron heating in a PPD. In Section III, we put forward our basic formalism of the Sagdeev potential. Finally in Section IV, we summarise the work.

II. MRI turbulence and plasma heating

Consider a weakly ionised plasma with an abundance of neutrals. In absence of any accelerating field, the electrons tend to thermalize with the neutrals. However, when there is an external electric field, electrons are accelerated heavily due to their mobility and low energy transfer during binary collisions. So, there exists an equilibrium in such a plasma, where the external electric field $E \gg E_{crit}$ such that the random thermal energy of the electrons greatly exceeds that of the neutrals, namely $v_{the} \gg \sqrt{(T_n/m_n)}$, where the `n' subscript denotes the neutrals, and

$$E_{crit} = \left(\frac{T_n}{el_e}\right)\sqrt{\frac{6m_e}{m_n}} \quad (1)$$

where $l_e = (n_n\sigma_{en})^{-1}$ is the electron mean free path and σ_{en} is the electron-neutral momentum-transfer cross section [2]. For a hydrogen-rich environment $\sigma_{en}{\sim}10^{-15}\text{cm}^2$ at electron energy < 10 eV we have

$$E_{crit}{\sim}10^{-9}T_{100}n_{12}\ \text{esu cm}^{-2}. \quad (2)$$

In the above relation, T_{100} is the temperature measured in terms of 100 K and n_{12} is the electron density measured in terms of 10^{12}cm^{-3}. It can be shown that

$$\frac{E_{MRI}}{E_{crit}} \approx \frac{200}{\Lambda}\left(\frac{100}{\beta_z}\right)n_{12}^{-\frac{1}{2}}, \quad (3)$$

which is independent of the gas temperature T. In the above expression, $\Lambda = v_{Az}^2/\eta\Omega$ is the Elsasser number, β_z is the plasma β of the vertical magnetic field, and v_{Az} is the corresponding Alfvén velocity. For electron heating, we must have $E_{MRI} \gg E_{crit}$, which puts an upper limit on Λ. At the same time, for MRI turbulence to be sustained so that E_{MRI} is maintained sufficiently strong, one must have $\Lambda > \Lambda_{crit}$. So, for sustained MRI turbulence heating of the electrons, we should have,

$$\Lambda_{crit} \leq \Lambda \leq 200(100/\beta_z)\, n_{12}^{-\frac{1}{2}}. \quad (4)$$

We have $\Lambda{\sim}0.1 - 1$ and so the above condition can be satisfied in the parameters are in the range $n_{12}{\sim}10^2 - 10^6$ and $\beta_z{\sim}100 - 1000$ which are typical of PPDs.

Electron distribution with heating

We now consider the Davydov distribution function in steady state [2], which is given by,

$$f_e(\boldsymbol{E}.\boldsymbol{v}_e) = \left(1 - \frac{eEl}{T}\frac{\epsilon\widehat{\boldsymbol{E}}.\widehat{\boldsymbol{v}}}{\epsilon + \chi T}\right)f_{e0}(E, v) \quad (5)$$

where, $\widehat{\boldsymbol{E}} = \boldsymbol{E}/E$, $\widehat{\boldsymbol{v}} = \boldsymbol{v}/v$ and

$$f_{e0} = \left(\frac{m}{2\pi T}\right)^{3/2}\frac{(\epsilon/T + \chi)^{\chi}}{W(\chi)}e^{-\epsilon/T} \quad (6)$$

with

$$W(\chi) = \chi^{3/2+\chi}U\left(\frac{3}{2}, \frac{5}{2} + \chi, \chi\right)$$

where, $U(x, y, z)$ is the confluent hypergeometric function of the second kind. In the above equation f_{e0} is the symmetric part of f_e that depends on the magnitude of $\boldsymbol{E}$ and $\boldsymbol{v}$ but not on the angle between them $\cos^{-1}(\widehat{\boldsymbol{E}}.\widehat{\boldsymbol{v}})$. In the limit of weak electric field $(E \ll E_{crit})$ f_{e0} reduces to the familiar Maxwellian distribution and in the limit of strong electric field

$(E \gg E_{crit})$, f_{e0} reduces to the Druyvesteyn distribution function [2]. In the presence of an electrostatic potential ϕ, the distribution function becomes,

$$f_e(\boldsymbol{E}.\boldsymbol{v}_e) = \left(1 - \frac{eEl}{T}\frac{(\epsilon - \epsilon_p)\widehat{\boldsymbol{E}}.\widehat{\boldsymbol{v}}}{\epsilon - \epsilon_p + \chi T}\right) f_{e0}(E, v). \tag{7}$$

where, $\epsilon_p = e\phi$. The symmetric part of the distribution can be written as,

$$f_{e0} = \left(\frac{m}{2\pi T}\right)^{3/2} \frac{\left[(\epsilon - \epsilon_p)/T + \chi\right]^{\chi}}{W(\chi)} e^{-(\epsilon - \epsilon_p)/T}. \tag{8}$$

Integrating the above distribution in the spherical co-ordinate, we can derive the corresponding electron density, which can be written as

$$n_e = 4\pi \int_{-\infty}^{\infty} \boldsymbol{v}^2 f_e(\boldsymbol{E}, \boldsymbol{v})\, d\boldsymbol{v} \tag{9}$$

where the volume element of each spherical shell is $4\pi\boldsymbol{v}^2 d\boldsymbol{v}$. Integrating the symmetric and asymmetric part of the distribution individually we can derive the symmetric and asymmetric electron densities respectively. After combining both the densities, we can finally write the total electron density, in the form

$$n_e = n_{sym} + n_{asym}. \tag{10}$$

III. Nonlinear electrostatic waves

Our basic plasma model comprises of the ion fluid equations, Boltzmannian-like electrons obeying Davydov's asymmetric velocity distribution, and Poisson's equation, which is,

$$\epsilon_{\mathbf{0}} \frac{\partial^2 \phi}{\partial x^2} = e(n_e - n_i) = F(\phi) \tag{11}$$

where following energy conservation for the ions, the ion density n_i can be written as,

$$n_i = n_0 \left(1 - \frac{2e\phi}{m_i u_0^2}\right)^{-1/2} \tag{12}$$

where, ions are assumed to be cold. The electron density is given by eq. (10). The physical quantities are normalized accordingly. The Poisson equation can be re-casted in the form,

$$\frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 = -\int_0^{\phi} F(\phi)\, d\phi \equiv V(\phi), \tag{13}$$

where $V(\phi)$ represents Sagdeev potential. In case of Davydov electrons, Sagdeev potential *must* be calculated numerically, shown in the left panel of Fig.1 .

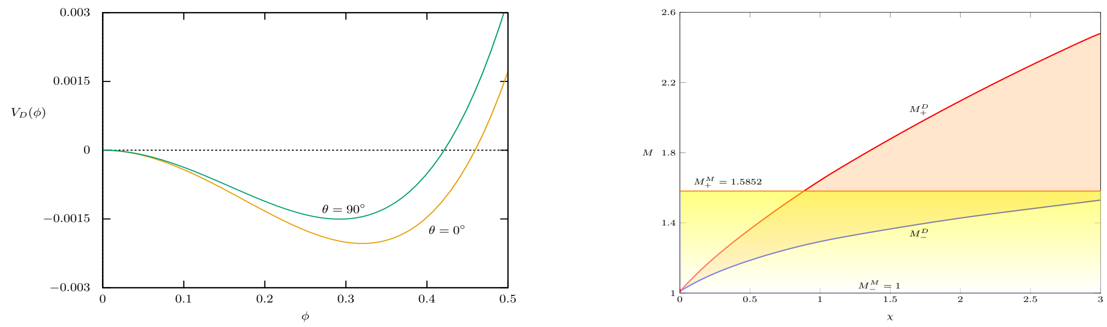

Fig 1: (Left panel) Pseudo potentials for Davydov electrons for $M = 1.5$ and $\chi = 1.51$ and (Right panel) limits on M and χ for Davydov electrons. The upper and lower limits are given by the curves labeled $M_{+,-}^D$

Limits on Davydov electrons

For $|\boldsymbol{\phi}| \ll \mathbf{1}$: We note that the condition for the Davydov electrons (V_D) can be found *only* for $\chi \ll 1$. Noting that $dV_D/d\phi = -F(\phi)$, we can expand $V_D'' = -dF(\phi)/d\phi$ around $(\phi = 0, \chi = 0)$, $V_D''(0) \cong 2\chi + (1/M^2) - 1$, from which, we can see that for a solitary structure to form, we *must* have $0 < \chi < 1/2$ and $M > (1-2\chi)^{-\frac{1}{2}}$, which puts an upper limit on χ for small ϕ.

For arbitrary $\boldsymbol{\phi}$***:*** For arbitrary ϕ, a determination of analytical limits is not possible and we need to establish the limits numerically. In the right panel of Fig.1, the upper and lower limits of M for Davydov electrons are represented by the curves labeled. So solitons can exist *only* in the shaded region between these two curves. The corresponding Maxwellian limits are represented by $M_{+,-}^M$ which is found to be $1 < M < 1.5852$.

IV. Summary and Conclusion

In this paper, we investigate the formation of solitons in a protoplanetary disk, using a psuedo potential analysis. The asymmetric electron distribution resulting from the plasma heating due to MRI-driven turbulence is governed by the Davydov distribution function. The domain of soliton existence becomes larger with increasing asymmetry χ. This signifies that we get fast moving and large number of solitons when the electron asymmetry becomes large in a protoplanetary disk.

References

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