Relativistic collisionless shocks : microphysics and long-time dynamics

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Introduction

It is thought that relativistic collisionless shocks have a major role in producing the high-energy tail of the cosmic ray spectrum. However, the microphysical details of ion Fermi acceleration and the long-time behaviour of such relativistic collisionless shocks are still not yet fully understood [1]. Understanding how particles get accelerated and connecting their acceleration with the cosmic radiation measured on Earth is a topic of high interest and a long-lasting mystery in Astrophysics. Both in laboratory Astrophysics experiments or in Plasma kinetic Particle-In-Cell (PIC) simulations, the generation of these shocks is reproduced and studied by the interaction, mediated by collisionless instabilities, of two counter-propagating and collisionless plasma flows [2]. In the particular case of exactly symmetric and cold plasma slabs collision moving with a Lorentz factor $\gamma_0 > \sqrt{3/2}$, the dominating instability is the purely electromagnetic Weibel instability [3]. However, the most extensive PIC simulations using this approach to date can follow the plasma dynamics for time intervals not long enough to fully capture the proton acceleration efficiency, the magnetization level and the coherence length scale evolution in the resulting shock structure. It seems in addition that all latter physical processes increase with time in the simulations. We introduce here a novel PIC simulation setup that relaxes the PIC simulation constraints. It consists in simulating with a moving window the collision of the two symmetric electron-proton plasma slabs in the backward-propagating electron-proton plasma Slab Rest Frame (SRF). We will note throughout the paper, n_0, T_0, γ_0 and $\pm\mathbf{v}_0$ the initial densities, temperatures, Lorentz factors and velocities of the two symmetric plasma slabs as seen in the Center of Mass Frame (CMF), n_s (n_r), T_s (T_r), γ_s (γ_r) and $\mathbf{v}_s$ ($\mathbf{v}_r$) the initial density, temperature, Lorentz factor and velocity of the streaming plasma slab (respectively the plasma slab initially at rest) as seen in the SRF and ρ_{shock} ($\rho^{\pm}_{\mathrm{shock}}$), γ_{shock} ($\gamma^{\pm}_{\mathrm{shock}}$) and v_{shock} ($v^{\pm}_{\mathrm{shock}}$) the density, Lorentz factor and velocities of the two shocks as seen in the CMF (respectively in the SRF). Thanks to the moving window that follows the front shock structure, our simulation setup allows for observing the shock propagation on unprecedented time scales. As a proof-of-principle, we show in this conference 2D PIC simulation results that are performed

Space contraction and time dilation			
$\Delta_x = \Delta x_0$	$\Delta_y = \Delta y_0$	$\Delta_z = \gamma_0 \Delta z_0$	$\Delta_t = \Delta t_0/\gamma_0$
Streaming plasma slab properties			
$\gamma_s = 2\gamma_0{}^2 - 1$	$\mathbf{v}_s = \frac{2\gamma_0{}^2}{\gamma_s}\mathbf{v}_0$	$n_s = \frac{\gamma_s}{\gamma_0} n_0$	$T_s = \frac{\gamma_0}{\gamma_s} T_0$
Immobile plasma slab properties			
$\gamma_r = 1$	$\mathbf{v}_r = 0$	$n_r = \frac{n_0}{\gamma_0}$	$T_r = \gamma_0 T_0$
Relativistic collisionless shocks properties			
$\mathbf{v}^{\pm}_{\text{shock}} = \frac{\mathbf{v}_0 \pm \mathbf{v}_{\text{shock}}}{1 \pm \frac{v_{\text{shock}} v_0}{c^2}}$		$n^{\pm}_{\text{shock}} = \gamma_0 n_{\text{shock}} \left(1 \pm \frac{v_{\text{shock}} v_0}{c^2}\right)$	

Table 1: Space-time contraction/dilation, plasma flows and shock hydrodynamic moments.

with the highly parallelized code PIC OSIRIS [4]. Table (1) summarizes the space-time contraction/dilation, the plasma flow densities, velocities and temperatures as well as the shock velocities and densities as seen in the SRF.

Theory and PIC Simulation

The number of plasma (macro-)particles is necessarily finite in a (PIC-simulated) plasma. Consequently, there is necessarily statistical fluctuations between the exact 3D-3V phase-space densities of (macro-)particles and their distribution function. As a result, instabilities, that are seeded by natural statistical fluctuations, develop at the available spatial frequency $\mathbf{k}$ for which the instability growth rate is maximum. Assuming equilibrium distribution functions $\langle f_{s_a} \rangle = n_s \delta^3 \left[\mathbf{p}_{s_a} - \gamma_s m_a \mathbf{v}_s\right]$ and $\langle f_{r_a} \rangle = n_r \delta^3 \left[\mathbf{p}_{r_a}\right]$ for respectively the streaming electrons ($a = e$) and protons ($a = p$) and the ones initially at rest and linearizing the Maxwell equations self-consistently coupled to the Klimontovich equations for each species in the small perturbation parameter $1/\gamma_s$, one finds by neglecting collisions in the cold approximation the following dispersion relation for these linearly growing plasma fluctuations

$$\left[\underline{\omega}^2 - \underline{k}^2 - \left(1 + \mu + \frac{a}{\gamma_s}\right)\right]\left[1 - \frac{1+\mu}{\underline{\omega}^2} - \frac{a}{\gamma_s{}^3}\frac{1}{\left(\underline{\omega} - \beta_s \underline{k}_z\right)^2}\right] = -\frac{a}{\gamma_s}\frac{(1+\mu)\,\beta_s{}^2 \underline{k}_\perp{}^2}{\underline{\omega}^2\left(\underline{\omega} - \beta_s \underline{k}_z\right)^2}. \quad (1)$$

It is similar to the one found by [5] concerning the electromagnetic oblique instability of a relativistic electron beam propagating in a denser plasma when neglecting the magnetic field generated by the former. Here, $\underline{\omega} = \omega/\omega_{p_r}$ and $\underline{\mathbf{k}} = k\omega_{p_r}/c$ are normalized to the rest plasma frequency $\omega_{p_r} = \omega_{p_0}/\sqrt{\gamma_0} = \omega_{p_s}/\sqrt{\gamma_s}$ with $\omega_{p_0} = \sqrt{4\pi n_0 e^2/m_e}$, $\beta_s = v_s/c$ and $a = (n_s/n_r)(1+\mu) \approx \gamma_s$. In the limit $\mu = m_e/m_p \to 0$ and $\gamma_s \gg 1$, the oscillations building up with the largest growth rate are those whose frequency is close to the rest plasma frequency. Retaining consequently only the term depending on the wave vector in first bracket of Eq. (1), imposing $\underline{\omega}^2 = 1$ in the denominator of the right hand side term and assuming $\underline{k} \neq 0$, one finds the dispersion relation $1 - 1/\underline{\omega}^2 - \varepsilon/\left(\underline{\omega} - \underline{k}_z\beta_s\right)^2 = 0$ where $\varepsilon = (a/\gamma_s)[(1/\gamma_s{}^2)(k_z{}^2/k^2) + (k_\perp{}^2/k^2)]$. Similarly as for the two-stream instability, this oblique instability occurs whenever $k_z v_s < \omega_{p,r}[1+\varepsilon^{1/3}]^{3/2}$. For a fixed wave vector component perpendicular to the streaming propagation direction $k_\perp$, the growth rate progressively increases with increasing k_z from $\delta^{\text{OI}}(k_z = 0, k_\perp)$ until reaching its maximum value

$$\delta^{\text{MFI}} = \delta^{\text{OI}}_{\text{max}}(k_\perp) = \omega_{p,r}\frac{\sqrt{3}}{2}\left(\frac{\varepsilon}{2}\right)^{1/3} \tag{2}$$

when electrostatic waves propagates along the streaming propagation direction in phase with the streaming plasma $k_z = \omega_{p,r}/v_s$. Then, it abruptly decreases to zero. This oblique instability is mainly electrostatic : the electric field component transverse to the wave vector is small compared with the longitudinal one in a ratio $k_\perp k_z \delta^{\text{OI}}(\mathbf{k})/k^2\omega_{p,r}$.

To illustrate our results, we present a PIC simulation considering $\gamma_0 = 2.12132$ and $k_B T_0 = 10^{-4} m_e c^2$ up to $L_t = 11500/\omega_{p,s}$ corresponding to $L_{t_0} = \gamma_0 L_t = 12562/\omega_{p,0}$ in the CMF. The 2D-3V phase-spaces (z, x, p_z, p_x, p_y) is sampled by $N_{\text{mpc}_s} = \gamma_s N_{\text{mpc}_r} = 32$ macro-particles per cell in such a way that all macro-particles have the same statistical weight. The time unit is fixed to $\omega_{\text{ref}} = \omega_{p,s}$ such that the normalized densities are $\underline{n}_r = n_r/n_s = 1/\gamma_s$ and $\underline{n}_s = 1$ in simulation units. The cells are squared with a size $\underline{\Delta}_z \times \underline{\Delta}_x = (\Delta\omega_{\text{ref}}/c)^2 = 0.05^2$ for a simulation box $\underline{L}_z \times \underline{L}_x = (L_z\omega_{\text{ref}}/c) \times (L_x\omega_{\text{ref}}/c) = 2000 \times 250$ leading to a total of $N_p = 72 N_z N_x = 1.44\,10^{10}$ macro-particles and $N_z \times N_x = 40000 \times 5000$ grid points. The simulation time step $\underline{\Delta}_t = \omega_{\text{ref}}\Delta_t = 0.98\underline{\Delta}/\sqrt{2} = 3.464\,10^{-2}$ respects the CFL stability/numerical heating condition $\Delta_t \leq \Delta/c < \Delta/v_{T_{r,e}} < \Delta/v_{T_{s,e}} < 1/\omega_{p_s} < 1/\omega_{p_r}$ where $v_{T_{r,e}}$ and $v_{T_{s,e}}$ are the thermal velocities of electrons initially at rest and streaming electrons, respectively. Leading to a total of $N_t = L_t/\Delta_t = 331987$ PIC loop iterations, the empirical computational cost of such a simulation is about $C \approx 500{,}000$ CPU$\times$hours and ≈ 400 GB of RAM memory with full OpenMP, MPI and AVX vectorization parallelization levels. We found good agreements between the theory and the PIC simulation concerning the growth rate of this almost purely Magnetostatic Filamentation Instability (MFI) by imposing $k_z \approx \omega_{p_r}/v_s$ and $k_\perp \approx \omega_{p_0}/c\sqrt{\gamma_0}$ in Eq. (1) leading to $\delta^{\text{MFI}} \approx 0.06/\omega_{ps}$. The shock formation time is about the inverse of streaming protons cyclotron frequency $\tau_f = \gamma_s m_p c/e\delta B_{\text{sat}} \approx 1469/\omega_{p_s}$ where $\delta B_{\text{sat}} \approx 10 m_e \omega_{p_s} c/e$ is the magnetic field at the MFI saturation time τ_s. Similarly as done by [6] for electron-positron colliding slabs in the CMF, one can estimate the latter according to $\tau_s \approx \ln\left(\delta B_{\text{sat}}{}^2/\delta B_{\text{flu}}{}^2\right)/2\delta^{MFI} \approx 106/\omega_{p_s}$ where $\delta B_{\text{flu}} \approx 0.9 m_e \omega_{p_s} c/e$ is the initial magnetic fluctuations amplitude level at $t \approx 10/\omega_{p_s}$. Finally, in agreement with the Lorentz transforms from the CMF to the SRF gathered in Table 1, with the density jump condition $n_{\text{shock}}/n_0 = \left(\Gamma_d + \gamma_0^{-1}\right)/(\Gamma_d - 1) \approx 3.94$ and with the shock velocity $v_{\text{shock}} = (\Gamma_d - 1)\sqrt{(\gamma_0 - 1)/(\gamma_0 + 1)} \approx 0.30c$ as seen in the CMF with the 2D adiabatic index $\Gamma_d = 3/2$, we obtain $n^+_{\text{shock}} \approx 22.4\,n_r \approx 2.80\,n_s$ and $v^+_{shock} \approx 0.93c$ as seen in the SRF.

Conclusion

Assuming a computer architecture allowing for η =1.2 double-floating-point instructions per ns as well as a time complexity of $73\,N_p$, $113\,N_p$, $445\,N_p$, $35\,N_z N_x$ and $135\,N_p$ double-floating-point instructions per PIC loop iteration for respectively the quadratic macro-particle Boris pusher, charge deposit, Esirkepov charge-conserving, Fei Maxwell solver [7] and fields interpolation numerical schemes that we haved used, we deduce a computational cost $C \approx \eta N_z N_x N_t \left[35 + 1532\left(N_{\text{mpc}_s} + N_{\text{mpc}_r}\right)\right] \approx 1{,}200{,}000$ CPU×hours. Performing the same analytical estimate for the cost C_0 of the equivalent simulation performed in the CMF with spatial cells size $\Delta_{z_0} \times \Delta_{x_0} = \Delta/\gamma_0 \times \Delta$, time step $\Delta_{t_0} \lesssim \Delta_{z_0}$, time duration $L_{t_0} = \gamma_0 L_t$, box $L_{z_0} \times L_{x_0} \approx cL_{t_0} \times L_x$ and using the same number of macroparticles per cell for simplicity, our simulation setup leads to a computational cost reduction of $C/C_0 \approx L_z/\gamma_0{}^4 cL_t$. However, the simulation box size along the propagation axis should be chosen sufficiently large in the SRF such as $L_z > c\tau_f$ for capturing the shock front at the shock formation time τ_f and the moving window velocity should be the closest as possible to the shock front velocity v^+_{shock}. As a conclusion, by noting τ_{f0} the shock formation time as seen in the CMF, our simulation setup allows for a computational cost reduction of $C/C_0 \approx \tau_{f0}/\gamma_0{}^4 L_{t_0}$ thanks to the use of the moving window technique coupled with the time dilation effects in the SRF.

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