Edge plasma turbulence influence on X-mode beam propagation in the framework of CTS in tokamaks

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Introduction It was shown that a microwave beam can be significantly scattered by plasma edge turbulence in a fusion machine [1-2]. This effect can deteriorate the neoclassical tearing modes stabilization efficiency [3], it may also result in widening of spatial resolution and complexity of data interpretation for the microwave plasma diagnostics. In this work we focus on one of the microwave diagnostics - the collective Thomson scattering (CTS) description with respect to the presence of the plasma turbulence. A simplified analytical model of the CTS is developed and the numerical simulation is done in order to justify prediction of the model.

X-mode beam broadening in turbulent plasma. Analytical description Here we describe an X-mode microwave beam propagation in turbulent magnetized plasma. We assume a slab geometry which is normally good enough for characterizing microwave beams propagation. The Cartesian coordinates are chosen as follows: x is the direction of plasma inhomogeneity, a probing beam is launched along this axis; z axis corresponds to lines of external magnetic field $\mathbf{B}$; y axis is perpendicular to the x and z and coincides with polarization of X-mode in vacuum. The wave equation in this geometry can be written as

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\left(x\right)+\delta k^2\left(x,y\right)\right]E_y=0 \tag{1}$$

where the X-mode wave number $k^2=\frac{\omega^2}{c^2}\frac{\varepsilon^2-g^2}{\varepsilon}$, components of the permittivity tensor in the "cold plasma" approximation $\varepsilon=1-\frac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2}$ and $g=\frac{\omega_{ce}}{\omega}\frac{\omega_{pe}^2}{\omega^2-\omega_{ce}^2}$, ω - probing frequency, ω_{pe} - plasma frequency, ω_{ce} - electron cyclotron frequency, perturbation of the wave number due to the turbulence $\delta k^2=-\frac{\omega_{pe}^2}{c^2}\frac{\left(\omega^2-\omega_{ce}^2\right)\left(\omega^2-2\omega_{pe}^2\right)+\omega_{pe}^4}{\left(\omega^2-\omega_{ce}^2-\omega_{pe}^2\right)^2}\frac{\delta n}{n}$, n - background plasma density and δn - the density turbulence. The launched probing beam is Gaussian $E_y(x=0,y)=E_0e^{-\frac{y^2}{\delta^2}}$. The equation (1) can be analysed applying the geometrical optics approach and the perturbation theory. The solution of (1) is

$$E_y\left(x,y\right)=E_0\sqrt{\pi}\delta\sqrt{\frac{\omega}{ck\left(x\right)}}\int_{-\infty}^{+\infty}\frac{dk_y}{2\pi}e^{-\frac{k_y^2\delta^2}{4}}e^{ik_yy}e^{i\int_0^x dx'k(x')-i\frac{k_y^2d^2(x)}{2}+i\delta\phi} \tag{2}$$

where the random phase $\delta\phi = -\int_0^x dx' \frac{1}{k(x')}\frac{\omega_{pe}^2}{2c^2}\frac{\left(\omega^2-\omega_{ce}^2\right)\left(\omega^2-2\omega_{pe}^2\right)+\omega_{pe}^4}{\left(\omega^2-\omega_{ce}^2-\omega_{pe}^2\right)^2}\frac{\delta n(x',y'(x,y,x',k_y))}{n(x')}$ and $d^2(x) = \int_0^x dx' \frac{1}{k(x')}$. The phase perturbation adds up along the wave trajectory which is described by the geometrical optics $y'(x,y,x',k_y) = y - k_y l^2(x',x)$, here $l^2(x',x) = \int_{x'}^x ds \frac{1}{k(s)}$. Assuming strong phase modulation regime $\left\langle \delta\phi^2(x,y,k_y)\right\rangle \gg 1$ one can derive the average intensity by means of averaging E_y^2 in (2)

$$\left\langle E_y^2(x,y)\right\rangle = E^2\frac{\omega}{ck(x)}\frac{\delta}{\sqrt{2}W(x)}e^{-\frac{y^2}{W^2(x)}} \tag{3}$$

the average beam width is specified as $W^2(x) = \frac{\delta^2}{2} + \frac{2d^4(x)}{\delta^2} + 4\int^x dx' D(x') l^4(x,x')$ where

$$D(x) = \frac{1}{8}\frac{1}{k^2(x)}\frac{\omega_{pe}^4}{c^4}\frac{\left[\left(\omega^2-\omega_{ce}^2\right)\left(\omega^2-2\omega_{pe}^2\right)+\omega_{pe}^4\right]^2}{\left(\omega^2-\omega_{ce}^2-\omega_{pe}^2\right)^4}\frac{\left\langle \delta n^2(X)\right\rangle}{n^2(X)}\int\frac{dq_y}{2\pi}\left|n_{0,q_y}\right|^2 q_y^2 \tag{4}$$

the definition for the introduced local relative turbulence amplitude $\frac{\sqrt{\left\langle \delta n^2(x)\right\rangle}}{n(x)}$ and the turbulence spectrum $\left|n_{q_x,q_y}\right|^2$ is $\left\langle \frac{\delta n(X,Y)}{n(X)}\frac{\delta n(X+\Delta x, Y+\Delta y)}{n(X+\Delta x)}\right\rangle = \frac{\left\langle \delta n^2(X)\right\rangle}{n^2(X)}\int\frac{dq_x dq_y}{4\pi^2}\left|n_{q_x,q_y}\right|^2 e^{i\Delta x q_x + i\Delta y q_y}$. We refer the readers to [4] for the detailed derivation.

CTS description in turbulent plasma We apply the reciprocity theorem for calculation the scattered off the core fluctuations signal

$$E^s = \frac{E_{unit}^s}{4}\int d\mathbf{r}\frac{E^+ + E^{+*}}{2}j^{nl} \tag{5}$$

where E^s is the scattered off the core fluctuations field, E_{unit}^s is the received filed normalized to unit energy flux, E^+ is the receiver antenna radiation normalized to unit power, $j^{nl} = \sigma\frac{E_{fl}+E_{fl}^*}{2}\frac{E_p+E_p^*}{2}$ is the nonlinear current resulted from interaction of the probing E_p and core fluctuations E_{fl} radiation, σ is the quadratic conductivity. The space distribution of the radiation E_p and E^+ is described by (2), but one should keep in mind that the equation (2) described E_p and E^+ in different frames of reference. So far as the three radiation amplitudes in (5) $\left(E_p, E_{fl}, E^+\right)$ come to the intersection domain from different areas of a fusion machine, they are statistically independent. Then the average power of the received signal

$$\left\langle P^s\right\rangle = \frac{|\sigma|^2}{64}\int\int d\mathbf{r}d\mathbf{r}'\left\langle E_p(\mathbf{r})E_p(\mathbf{r}')\right\rangle\left\langle E_{fl}(\mathbf{r})E_{fl}(\mathbf{r}')\right\rangle\left\langle E^+(\mathbf{r})E^+(\mathbf{r}')\right\rangle \tag{6}$$

We consider the core fluctuations model as the sum $E_{fl} = \sqrt{\left\langle E_{fl}^2\right\rangle}\sum_{jl} S_{fl} e^{i\Delta k jx + i\Delta k ly} e^{i\varphi_{j,l}}$ with a spectrum S_{fl} and random phases $\varphi_{j,l}$, Δk is the wave number step. The correlator in (6) gives

$\left\langle E_{fl}(\mathbf{r})E_{fl}(\mathbf{r}')\right\rangle = \left\langle E_{fl}^2\right\rangle \sum_{jl}\left|S_{fl}\right|^2 e^{i\Delta kj(x-x')+i\Delta kl(y-y')}$. Since $E_p^+ \propto e^{i\int d\mathbf{r}\mathbf{k}_p^+}$ the main contribution to the registered power comes from the resonance harmonic $\mathbf{k}_{fl} = -\mathbf{k}_p - \mathbf{k}^+$. Therefore the expression (6) can be written as

$$\langle P^s\rangle = \frac{|\sigma|^2}{64}\left\langle E_{fl}^2\right\rangle \left|S_{fl}\left(\mathbf{k}_{fl}\right)\right|^2 \int\int d\mathbf{r}d\mathbf{r}' e^{i\mathbf{k}_{fl}(\mathbf{r}-\mathbf{r}')}\left\langle E_p(\mathbf{r})E_p(\mathbf{r}')\right\rangle \left\langle E^+(\mathbf{r})E^+(\mathbf{r}')\right\rangle \qquad (7)$$

We will consider a simplified scattering model for the further analysis. The first assumption is that the plasma density and magnetic field profiles are considered as constant in the area of scattering, the second one is that we neglect the probing E_p and receiver E^+ radiation diffraction on the scale of the scattering area. The scattering angle - α. In the framework of the simplified model we substitute E_p^+ (which are described by (2) with respect to different frames of reference) into (7) and calculate 8 integrals. Finally the obtained expression

$$\langle P^s\rangle = \frac{|\sigma|^2\pi^2}{32}\left|S_{fl}\left(\mathbf{k}_{fl}\right)\right|^2 \left\langle E_{fl}^2\right\rangle E_{0p}^2 E_0^{+2}\frac{\omega^2}{c^2k^2(\mathbf{x}^*)}\frac{\delta_p\delta^+}{\sin^2\alpha\sqrt{\sigma_p^2\sigma^{+2}}} \qquad (8)$$

$\mathbf{x}^*$ - point of the two beams $\left(E_p^+\right)$ axes intersection. The functions $\sigma_p^{+2} = \frac{2}{\delta_p^{+2}} + 4\int_{l_p^+} dx'D(x')$ have the meaning of an angular beam broadening due to the turbulence, l_p^+ is trajectory of the beams axes. It is seen from the equation (8) that presence of the edge turbulence reduces the registered signal amplitude. The relative expression

$$R = \frac{\langle P^s\rangle\Big|_{turb\neq 0}}{\langle P^s\rangle\Big|_{turb=0}} = \frac{1}{\sqrt{\left(1+2\delta_p^2\int_{l_p} dx'D(x')\right)\left(1+2\delta^{+2}\int_{l^+} dx'D(x')\right)}} \qquad (9)$$

allows to estimate the signal reduction rate.

Numerical simulation of the CTS in turbulent plasma To estimate the signal reduction in a CTS experiment due to the plasma turbulence and to verify the analytical model we demonstrate the results of the CTS simulations under the conditions of a simplified model described in the previous section. Plasma parameters are chosen similar for the parameters expected on ITER, the probing frequency $f = 60$ GHz also corresponds to the chosen frequency in the CTS experiment for ITER. Magnetic field $B = 4.25$ T is a constant in the domain of simulation, the 1D density profile $n(x)$ and the 1D turbulence envelope $\sqrt{\langle\delta n^2(x)\rangle}$ are shown on the figure 1. The 1D turbulence is generated as $\delta n(x) = \sqrt{\langle\delta n^2(x)\rangle}\sqrt{\frac{\Delta q l_c}{2\pi}}\sum_j e^{-\frac{l_c^2\Delta q^2 j^2}{4}} e^{i\Delta q j x} e^{i\varphi_j}$, where φ_j - random phase, Δq - step of the wave number discretisation, $l_c = 1.5$ cm - the correlation length. The model core collective fluctuations $\delta\tilde{n}$ are represented by just one mode which provide scattering at angle $\alpha = \pi/2$. The relative amplitude $\frac{\delta\tilde{n}}{n_{max}} = 0.0003$. The 2D profiles are constructed as 2D mesh $n(x) \times n(y)$, $\delta n(x) \times \delta n(y)$. The simulation scheme is illustrated on the figure 2: the turbulence map $\delta n + 100\delta\tilde{n}$ (the factor 100 is used in order to see the core fluctuations) and evolution of the average probing beam width. The receiver antenna position is on the right hand side of the figure 2 at poloidal coordinate $y^r = 45$ cm, according to the analytical prediction the received signal is independent on the receiver radial coordinate. The registered field E^{reg} is determined as a product of the scatted field E^s radial distribution with the receiver radiation diagram $E^{reg}(x) = E^s(x, y^r)\frac{1}{\sqrt{\pi}\delta_+}e^{-\frac{x^2}{\delta+2}}$.

To estimate the registered power reduction we calculate (9) for the specified plasma parameters, that amounts to $R \approx 0.11$. The numerical evaluation of the reduction rate $R^{num} = \int dx E^{reg2}\big|_{turb\neq 0} / \int dx E^{reg2}\big|_{turb=0}$ gives the value $R^{num} \approx 0.15$ for different radial positions of the receiver as it was expected from the analytical analysis. The signal decrease in turbulent plasma amounts to nearly 90%. The numerical simulation is in good agreement with the analytical prediction.

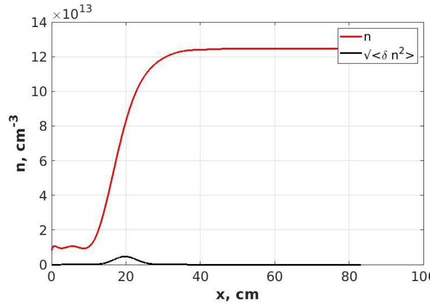

Figure 1: The density and turbulence envelope profiles

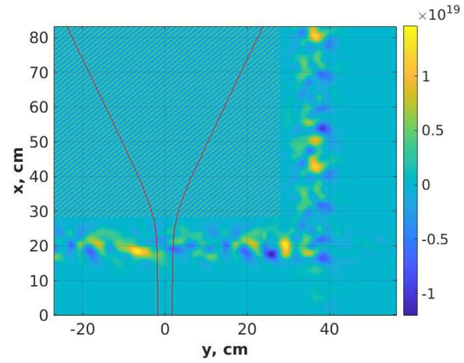

Figure 2: Scheme of the simulation: 2D domain region with an overlaid turbulence map $\delta n + 100\delta\tilde{n}$. The red lines are evolution of the average probing beam width with respect to the plasma turbulence.

Conclusion In this work it was demonstrated that presence of the edge turbulence can significantly disturb an X-mode microwave beam. The provided analytical and numerical analysis of the reduction the registered signal in a CTS experiment revealed that this value is sensitive to the turbulence properties (9). An estimate of this value was obtained for plasma conditions and probing frequency relevant for a planned CTS experiment on ITER. Mitigation of the registered power is up to 90%, it may be crucial to take this effect into account since the scattered radiation level is usually extremely low, of the order of noise rate.

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References

[1] E V Sysoeva, F da Silva, E Z Gusakov, S Heuraux and A.Yu. Popov, 2015 *Nucl. Fusion* **55(3)** 033016

[2] A Snicker et al, 2018 *Plasma Phys. Control. Fusion* **60** 014020

[3] E Poli et al, 2015 *Nucl. Fusion* **55** 013023

[4] P Tretinnikov, E Gusakov, S Heuraux 2021 *Plasma Phys. Control. Fusion* **63** 085003