Nonlinear multi-species collision operator for gyrokinetic codes

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Introduction

The collision frequency is often small compared to other typical frequencies in the core of tokamaks. It is nevertheless important to correctly describe the effect of collisions as they impact the level of turbulence either directly (TEM) or via zonal flow damping (ITG). Collisions are furthermore instrumental for neoclassical physics which is important for large scale flows and impurity transport. Collisions also damp small scales fluctuations in the velocity space, allowing for long time simulations.

In the edge, the collision frequency and the turbulence intensity increase compared with the one of the core. Hence the linearization of the collision operator is questionable for edge simulations.

In this context, an approximated version of the nonlinear Coulomb operator in the drift kinetic limit has been derived and implemented in the gyrokinetic code ORB5 [1]. This operator, which is based on a moment approach to compute the Rosenbluth potentials, is valid for arbitrary species (mass, charge and concentration).

Description of the collision operator

The collision operator of a species a colliding on a species b can be written as a Fokker-planck operator

$$C_{ab}\left(F_a,F_b\right)=\frac{\gamma_{ab}}{m_a}\partial_v\cdot\left[\partial_v\cdot\left(\frac{1}{2}\partial_v\partial_v G_b F_a\right)-\left(1+\frac{m_a}{m_b}\right)\partial_v H_b F_a\right]$$

where $\gamma_{ab}=\frac{e_a^2e_b^2}{4\pi\varepsilon_0^2 m_a}\ln\Lambda$ with $\ln\Lambda$ the Coulomb logarithm, e_s, m_s and F_s are respectively the charge, mass and distribution function associated with the species s. G_b and H_b are the Rosenbluth potentials associated with the species b and are defined as

$$G_b\left(v'\right)=\int dv' F_b\left(v'\right)\left|v'-v\right| \qquad\qquad H_b\left(v'\right)=\int dv'\frac{F_b\left(v'\right)}{\left|v'-v\right|}$$

The computation of the Rosenbluth potentials requires to know the distribution function of the species b which is numerically challenging. We make the assumption that the distribution function can be approximated by:

$$F_b^t=F_{Mb}\left\{1-\left(\frac{V_{\|b}}{v_{Tb}}\right)^2\left(1-\frac{2}{3}s_b^2\right)+2\frac{v_\|}{v_{Tb}}\left[\frac{V_{\|b}}{v_{Tb}}-\frac{q_{\|b}}{N_bT_bv_{Tb}}\left(1-\frac{2}{5}s_b^2\right)\right]\right\}$$

where F_{Mb} is an unshifted Maxwellian, $s_b = \frac{v}{v_{Tb}}$ and $v_{Tb} = \sqrt{\frac{2T_b}{m_b}}$ is thermal speed of the species b, $V_{\parallel b}$ is its mean parallel velocity and $q_{\parallel b}$ its mean parallel heat flux. This expansion is consitent with the neoclassical theory. The Rosenbluth potentials can be computed analytically with this approximated distribution function [3].

For simplicity, the drift kinetic limit is assumed, ensuring that the collision operator acts only in the velocity space instead of the 5D phase space. This approximated collision operator has been implemented in ORB5 [2] by using the equivalence between a Fokker-Planck operator and a Langevin equation.

Basic properties of the collision operator

A correction term has been implemented in the ORB5 code in order to ensure that the collision operator conserves the density, the total momentum and total energy to machine precision inside spatial bins. This correction term respects the velocity dependence of the collision operator to avoid an unphysical modification of the disctribution function. More details about this correction term can be found in [1].

The collisional exchange of momentum (respectively energy) between species is shown on Fig.1 (respectively Fig.2) and compared with theoretical predictions derived in [1]. An excellent agreement is found in both cases. This is an important property of inter-species collisions.

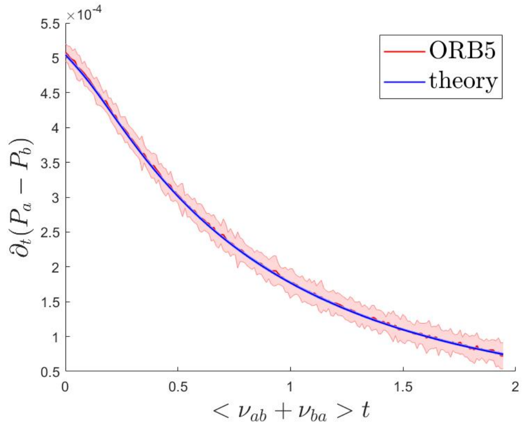

Figure 1: Exchange rate of momentum between species

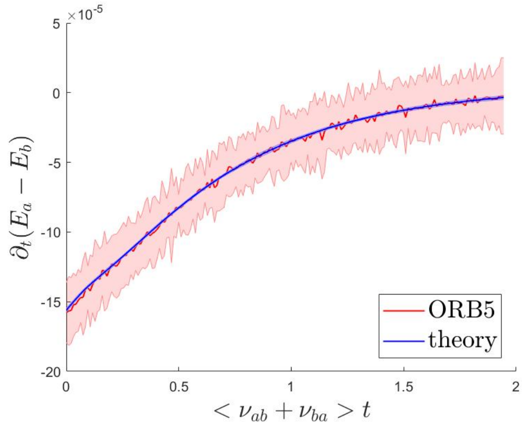

Figure 2: Exchange rate of energy between species

Neoclassical benchmark

In this section, ORB5 simulations are performed by keeping only axisymmetric components of the electric potential, hence removing turbulence. An adiabatic electron response is assumed. An adhoc MHD equilibria with circular concentric flux surfaces is used. The nonlinear collision operator and its linearized counterpart are compared with theoretical predictions of the neoclassical theory. As expected from neoclassical theory, the difference between the linear and nonlinear collision operators is small for these tests.

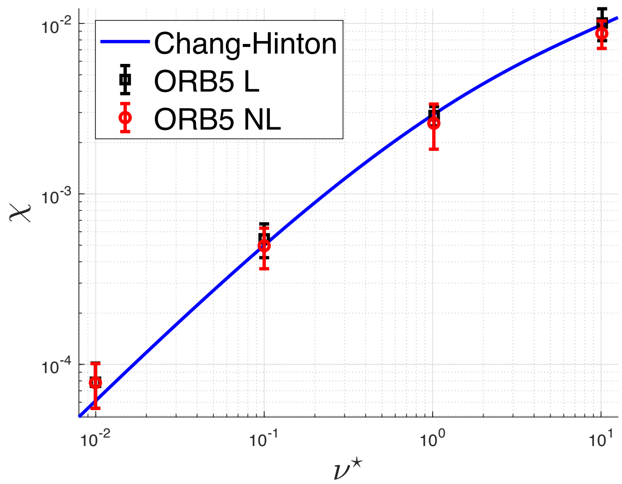

Figure 3: Comparison of the numerical heat diffusivity obtained with the linear (black square) and the non-linear (red circle) collision operator with the theoretical prediction of Chang–Hinton (blue curve).

The neoclassical heat diffusivity χ is represented on Fig.3 as a function of the collisionality. A good agreement is found with the theoretical prediction given by Chang-Hinton [4] in all collisonality regimes. Most of the time the neoclassical heat flux is small compared with the one induced by turbulence. It can however become non negligible in certain cases, for instance in transport barriers where turbulence is reduced. In this case, it is then important to have the right level of neoclassical heat transport.

On Fig.4, the coefficient $k_{neo} = \frac{V_\theta e<B^2>_\psi}{B_\varphi \frac{\partial T}{\partial r}}$ is represented as a function of the collisionality and compared with three theoretical predictions [5, 6, 7]. A good agreement is found in the three collisionality regimes with Shaing's prediction. This is an important result as the change of sign in the poloidal rotation between the banana and the Pfirsch-Schlüter regimes can lead to an important shear of the neoclassical poloidal rotation. This shear has been proposed to be a key element in the L-H transition [8].

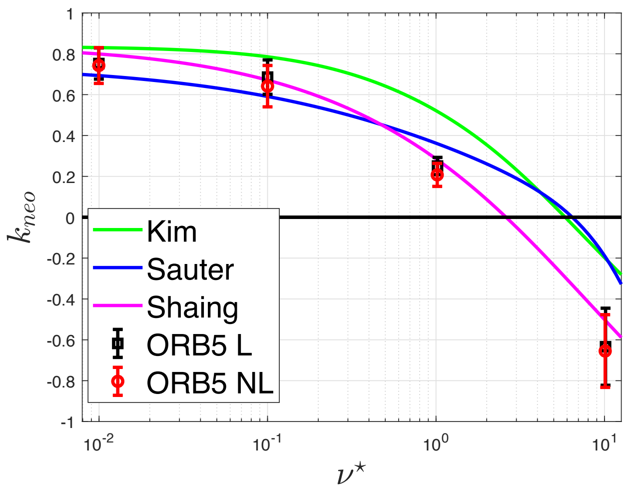

Figure 4: Comparison of the numerical k_{neo} obtained with the linear (black square) and the non-linear (red circle) collision operator with the theoretical prediction of Kim (green curve), Sauter (magenta curve) and Shaing (blue curve).

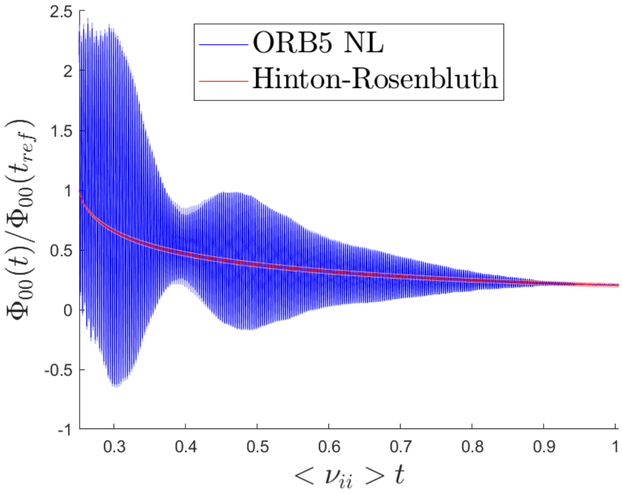

Figure 5: Collisional damping of ϕ_{00} in presence of collision and comparison with a theoretical prediction [9].

Zonal flows play a major in the saturation of turbulence. Hence it is critical to verify that the collisionnal damping of zonal flows is correct. Fig.5 shows the time evolution of the zonal component of the electric potential in presence of collision. A good agreement is found with the theoretical prediction of Hinton and Rosenbluth [9]. The oscillations of the potential in ORB5 on Fig.5 correspond to geodesic acoustic modes which have not been included in the theoretical prediction.

Conclusion

An approximated nonlinear multi-species collision operator has been derived in the drift kinetic limit [1] and implemented in the global gyrokinetic code ORB5 [2]. The conservation properties of the collision operator as well as the proper exchange rates of momentum and energy between species are retrieved with this collision operator. The collision operator has been benchmarked successfully against neoclassical theory.

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