Interpreting Radial Correlation Doppler Reflectometry using Gyrokinetic Simulations

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Radial correlation Doppler reflectometry (RCDR) [1] is a well established diagnostic technique to measure the radial correlation function in the turbulence, implemented via cross-correlation of neighboring Doppler backscattering signals. In Doppler backscattering, a microwave beam incident into the plasma at a finite incidence angle makes the backscattered signal sensitive to a specific turbulence wavenumber $\mathbf{k}_\perp$, which is related to the incident microwave beam wavenumber $\mathbf{k}_i$ by the Bragg condition for backscattering at the cutoff $\mathbf{k}_\perp = -2\mathbf{k}_i$ [2, 3]. If the Doppler backscattering signal is sensitive to only one turbulence wavenumber $\mathbf{k}_\perp$, what does this mean for the measured radial correlation length? In this work, we study the dependence of the radial correlation length on the measured $k_\perp = |\mathbf{k}_\perp|$, and the potential diagnostic resolution effects on radial correlation length measurements via RCDR in the linear response regime [4, 5]. We use a synthetic model DBS applied to nonlinear gyrokinetic simulation corresponding to a highly unstable ETG regime in a moderate-β NSTX H-mode (analyzed in [6, 7, 9, 10]), and to an ITG-driven core turbulence regime in a JET L-mode (analyzed in [11]).

The model DBS implemented for gyrokinetic simulation assumes the scattered amplitude is a linear response with respect to the density fluctuations $A_s(\mathbf{r_0}, \mathbf{k}_0, t) = e^{-i\mathbf{k}_{\perp\mathbf{0}}\cdot\mathbf{r_0}} \sum_{\mathbf{k}_\perp} \delta\hat{n}(\mathbf{k}_\perp, \theta, t) W(\mathbf{k}_\perp - \mathbf{k}_{\perp 0})$, where W is a filter in $\mathbf{k}$-space applied to the density fluctuation field $\delta\hat{n}$ at the outboard midplane $\theta = 0$ (details in [12]). The perpendicular wave-vector is expanded along its normal k_n and binormal components k_b as $\mathbf{k}_\perp = k_n\mathbf{e_n} + k_b\mathbf{e_b}$, and $W(\mathbf{k}_\perp - \mathbf{k}_{\perp 0})$ is Gaussian in k_n and k_b, with characteristic radial resolution W_n and binormal resolution W_b. The model can be derived from a first principles, linear response, beam-tracing DBS model developed by Hall-Chen *et al*. [13].

We make the distinction between three different correlation functions. CCF^{real} is the *real* turbulence cross-correlation function, computed via radial correlation of the full density field δn from the gyrokinetic code for different radial separations Δx, and knows about the full turbulence spectrum (all $\mathbf{k}_\perp$). CCF^{k_b} is the scale-dependent correlation function corresponding to a specific turbulence wavenumber k_b, and informs about the characteristic radial correlation of a specific turbulent eddy with binormal wavenumber k_b. This quantity does not know about the full turbulence spectrum, but is intrinsic to k_b. Each binormal wavenumber k_b in the turbulence, ie. each binormal wavelength, has a specific radial structure associated to it, and it is given by CCF^{k_b}. The varying CCF^{k_b} for different k_b is an intrinsic characteristic of the turbulence, and it is *not* related to a diagnostic effect. CCF^{syn} is the synthetic cross-correlation function, computed via radial correlation of the scattered amplitude A_s. Notably this cross-correlation function is directly dependent on specific experimental parameters, which in this model are k_{b0}, W_n and W_b (here we use experimentally relevant $W_b \gg 2\pi/k_{b0}$). We will define the correlation length as the $1/e$ value of the corresponding radial correlation function.

Figure 1.**a)** shows the scale-dependent radial correlation length $l_r(k_b)$ computed from CCF^{k_b} for each individual k_b in the NSTX electron-scale gyrokinetic simulation (blue curve, where the $E\times B$ shearing rate $\gamma_E = 0$) along with the real correlation length of the turbulence (green dashed line, computed from CCF^{real}). The radial correlation length is largest for a finite

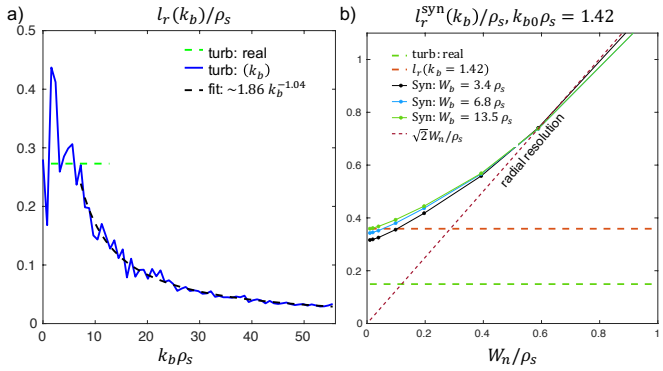

Figure 1: a) Scale-dependent $l_r(k_b)$ for the NSTX electron scale turbulence case. **b)** Synthetic correlation length as a function of W_n.

$k_b\rho_s \approx 1.5$, and it becomes a decreasing function of k_b for larger wavenumbers. A power law $\sim Ck_b^{-\alpha}$ is fitted to the l_r curve for larger wavenumbers ($k_b\rho_s \gtrsim 7$), giving an exponent $\alpha \approx 1.04$. This fit characterizes the scale-by-scale dependence of the radial correlation length corresponding to each binormal wavenumber k_b (or binormal wavelength). This shows that it is of crucial importance to take into account the scale by scale dependence of the radial correlation length as only one wavenumber k_b is selected by a Doppler reflectometer system. The relationship $l_r \sim Ck_b^{-\alpha}$ with $\alpha \approx 1$ implies that the radial characteristic length l_r and binormal wavenumber k_b preserve the same proportionality relation scale by scale (for $k_b\rho_s \gtrsim 7$). The constant of proportionality C can be interpreted as a rough measure of the eddy aspect ratio in the perpendicular (x, y) plane to $\mathbf{B}$. In this case we find $l_r/l_b \approx 1.54$, where $l_b \approx 1/k_b$. This estimate is only accurate when there is negligible tilt on the turbulence ($\gamma_E \approx 0$), and suggests that ETG-driven eddies measured by DBS would exhibit an aspect ratio $l_r/l_b \approx 3/2$ in the (x, y) plane (at the outboard midplane of this specific NSTX condition).

Next we seek to understand the effect of the filter's radial dimension W_n on the NSTX electron-scale turbulence simulation. Figure 1.**b)** shows the synthetic correlation lengths as a function of W_n (computed from CCF$^{\text{syn}}$) for different experimentally relevant values of the binormal spot size $W_b = 3.4, 6.8, 13.5\ \rho_s$. The correlation length for small W_n converges to the correlation length for $k_b\rho_s = 1.42$ (red dashed line). However, for increasing W_n, the synthetic correlation length is shown to asymptote linearly to the purple dashed line, which is the radial resolution $\approx W_n$. This suggests that the diagnostic radial resolution W_n can itself strongly affect the value of the 'measured' correlation length in DBS experiments for large enough radial resolution W_n, when $W_n > l_r(k_{b0})$. The value of W_n in experiments remains an open question for Doppler backscattering. Previous work seems to suggest values on the order of the width of the Airy lobe W_{Ai} or the beam width, whichever is larger. It is well known that standard normal incidence reflectometry experiments are characterized by a radial spot size given by the width of the Airy lobe $W_{\text{Ai}} \approx 0.5L_\epsilon^{1/3}\lambda^{2/3}$ [14,15], where L_ϵ is the permittivity scale length, equal to the density gradient scale length L_n in linear profile, slab geometry and O-mode polarization, and λ is the vacuum wavelength of the incident microwave beam. The issue of the width of the beam near the reflection point is discussed analytically in [16, 17], which seems to be supported by ray tracing, beam tracing and full-wave simulations carried out for realistic toroidal geometries of AUG [18, 19], additionally suggesting that the Airy width W_{Ai} should be considered as a minimum value of W_n in experiments. In the present conditions $W_{\text{Ai}} \approx$ 2-4 ρ_s, which is notably larger than the largest correlation length in this strongly-driven ETG simulation condition ($l_r \approx 0.4\ \rho_s$, figure 1.**a)**). This suggests that radial correlation length measurements carried out for electron-scale $k_\perp$ DBS measurements are likely to be dominated by the radial resolution W_n, making impossible the measurement of the radial correlation length $l_r(k_b)$.

Similar features to the electron-scale turbulence can be observed in the JET ITG-driven ion-scale turbulence case, but this time for larger correlation lengths. The $1/e$ correlation length for each k_b is plotted in figure 2.**a)**, in blue for $\gamma_E = 0$ and in red for $\gamma_E = \gamma_E^{\text{exp}}$. Both show a peak correlation length for $k_b\rho_s \approx$ 0.1-0.2, and decreasing for larger k_b. The $\gamma_E = 0$ case exhibits larger l_r (γ_E can induce radial decorrelation), and yields slightly different values for the fit parameters. Inferring an eddy aspect ratio from the γ_E^{exp} case would yield an inaccurate estimate for l_r/l_b: the proportionality constant C in $l_r \sim Ck_b^{-\alpha}$ is only an accurate estimate the eddy aspect ratio when $\gamma_E \approx 0$ (due to the tilting of eddies induced by γ_E, details in [12]). The effect of finite diagnostic radial resolution W_n for ion-scale turbulence is similar to electron-scale turbulence, however, the condition $W_n < l_r(k_{b0})$ is now more easily satisfied. Figure 2.**b)** shows the variation of the $1/e$ synthetic correlation length with W_n. The correlation length converges to the correlation length of $k_{b0}\rho_s = 0.15$ for all values of W_b when $W_n \ll l_r(k_{b0})$,

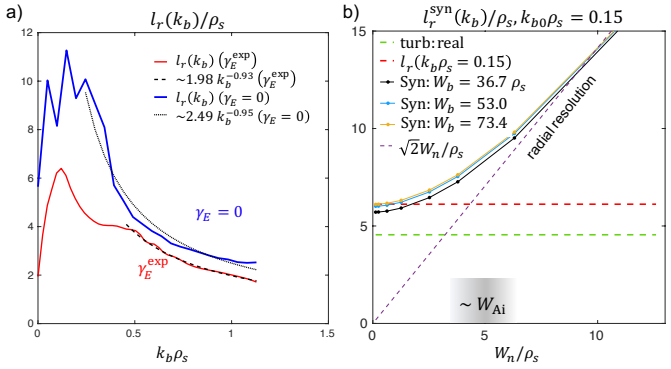

Figure 2: a) Scale-dependent $l_r(k_b)$ for the JET ion-scale turbulence case. **b)** Synthetic correlation length as a function of W_n.

and it asymptotes to the radial resolution W_n for $W_n \gg l_r(k_{b0})$. The condition $W_n \approx l_r(k_{b0})$ happens for a larger correlation length $l_r(k_{b0}\rho_s = 0.15) \approx 5\ \rho_s$. Evaluating the reflectometry estimate for the radial spot size for the current conditions gives $W_n \approx W_{\rm Ai} \approx$ 4-6 ρ_s. This suggests that experimentally relevant W_n are likely to affect a measured RCDR correlation length, but in a less dramatic way than for electron-scale fluctuations. The effect is not be completely negligible, as demonstrated by figure 2.**b)**.

The dependence of the radial correlation length on the binormal scale $l_r(k_b) \sim C\ k_b^{-\alpha}$ is encoded in the ensemble averaged turbulent wavenumber spectrum of the density fluctuations $\langle|\delta\hat{n}(k_n, k_b)|^2\rangle_T$. This dependence can only be captured for non-separable density fluctuation spectra in k_n and k_b, ie. $\langle|\delta\hat{n}(k_n, k_b)|^2\rangle_T \neq h_n(k_n)h_b(k_b)$ ([12]). Using a separable wavenumber spectrum, as in previous works [1, 4, 5] results in $l_r(k_b) = l_r^{\rm real}$ for all k_b, which does not accurately represent magnetized plasma turbulence in the tokamak core. A proposed analytical expression for non-separable, density fluctuation wavenumber power spectra inspired from the power-law spectra characteristic of gyrokinetic simulations, and which contains the scale-by-scale dependence of the radial correlation length, is as follows

$$\langle|\delta\hat{n}/n(k_n, k_b)|^2\rangle_T \sim \frac{A}{1 + \left(\frac{|k_n|}{w_{k_n}}\right)^{\gamma} + \left(\frac{|k_b - k_{b*}|}{w_{k_b}}\right)^{\beta}} \quad \Longrightarrow \quad l_r(k_b) \sim \frac{\text{const.}}{w_{k_n}} \frac{1}{\left[1 + \left(\frac{|k_b - k_{b*}|}{w_{k_b}}\right)^{\beta}\right]^{1/\gamma}}. \tag{1}$$

In this expression, the spectral exponents γ and β in k_b are allowed to differ, w_{k_n} and w_{k_b} represent the spectral widths of the turbulence wavenumber spectrum, and the spectrum in k_b is allowed to peak at a finite $k_b = k_{b*}$, consistent with the linear drive for typical micro-instabilities in the tokamak core peaking at finite values of k_b, and representing the injection driving scale of the turbulence (eg. $k_{b*}\rho_s \approx$ 0.1-0.6 for ITG, $k_{b*}\rho_s \approx$ 2-30 for ETG, etc.). The analytic expression of the spectrum allows an analytic expression of $l_r(k_b)$, which has the functional dependence exhibited in equation 1.

In figure 3.**a)** are shown the scale-dependent correlation functions CCF^{k_b} of the gyrokinetic (thick lines) and analytic spectra fit (colored dots), corresponding to two different wavenumbers $k_b\rho_s = 3.22, 13.67$ in the NSTX electron-scale turbulence condition. The scale-dependent correlation functions are satisfactorily reproduced by the analytic expression for the spectrum. The scale-dependent correlation length $l_r(k_b)$ computed for all k_b is also shown to be well reproduced by the analytic spectrum in equation (1), as is shown in figure 3.**b)**. This discussion motivates a preferential use of realistic spectra, gyrokinetic or analytic expressions as in equation (1), above the use of separable spectra (Gaussian, Lorentzian, top-hat, etc.) in future modelling works of RCDR. Similar analysis for the JET ion-scale turbulence condition also shows satisfactory agreement of the analytic spectrum with the gyrokinetic spectrum.

The proposed relationship $l_r(k_b) \sim k_b^{-\alpha}$ appears to be a universal, robust characteristic of the turbulence, as provided by nonlinear gyrokinetic simulation of ETG and ITG-driven turbulence. This may explain, or contribute to the explanation of, the observed decrease of the measured radial correlation length with k_b in experiments [20] (denoted $k_\perp$ in the literature). The fact that analytical work [5] and full-wave simulations [21] show that separable spectra (Gaussia, top-hat, Lorentzian, etc.) yield a measured l_r dependent on k_b suggests that a combination of two effects, the scale-by-scale variation of l_r with $k_\perp$ and a diagnostic effect (forward scattering [5]) may explain the experimental observations [20]. For this reason, an analytic,

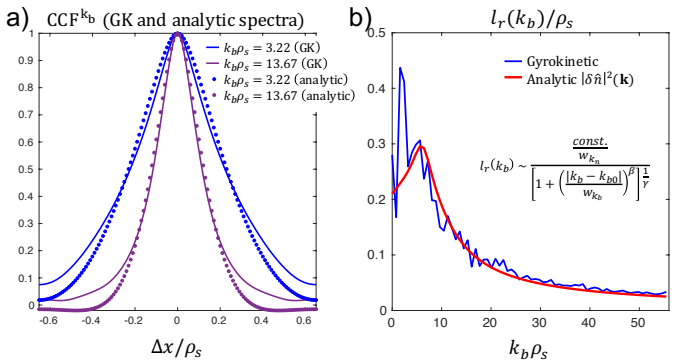

Figure 3: a) Scale-dependent correlation function CCF^{k_b} computed from gyrokinetic simulation (thick lines), compared to CCF^{k_b} using analytic fit to the wavenumber spectrum (colored dots). **b)** Scale-dependent correlation length for all wavenumbers k_b in the simulation.

realistic power-law spectrum (equation (1)) is proposed, which could be used in future works to model the DBS response using full-wave or analytical calculations.

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