Acceleration of a Two Species Plasma from Moving Walls

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Introduction

The Fermi acceleration model was first used as a means to describe how cosmic ray particles are accelerated to great speeds by interacting with moving magnetic fields [1]. Since then, many variations of the model have been studied which simplify Fermi acceleration down to the interaction of bouncing balls and moving walls [2-4]. One well known example is the Fermi-Ulam model which describes the acceleration of an ensemble of noninteracting particles bouncing between a moving wall and a stationary wall. In our study, we present another variation of the model where a two species plasma interacts with a moving walls. Assuming that one species is much more massive than the other, this species can be taken to be stationary, and the lighter species will pitch angle scatter on these massive particles while conserving energy. Introducing this stochastic effect into the system will influence the frequency at which particles interact with the moving wall, and therefore also affect the total evolution of the plasma distribution function. In particular, due to the relationship between the mean free path for Coulomb collisions and the speed of a particle, the rate at which a particle is accelerated by the moving wall may be heavily dependent on its initial speed and the density of the background species. This would imply that such a system could be tuned with these parameters to accelerate distributions of particles in a desired way to achieve a peaked energy distribution. This non-thermal phenomena could be of interest to p-^{11}B fusion and astrophysical research due to the system setup and assumptions of the relative masses of the two species. We have begun to analytically and numerically investigate this system from different perspectives, including the evolution of the distribution function and adiabatic invariants to gain a more accurate description of the acceleration profile. A basic 1D velocity space model has been explored for different collisional limits and a numerical simulation was used to show similar behaviors in the 3D case. For the highly collisional limit, we see an interesting inverse relationship between the change in velocity and energy from compression and the initial energy, giving rise to narrower distributions compressed in phase space.

Theoretical Model

A 1D case of the problem being considered consists of an ensemble of particles in a box interacting with a rigid wall moving at speed v_w, much smaller than any particle speed. A second, more massive ensemble is assumed to be nearly stationary in the background and the primary type of collision is pitch angle scattering between the two species. In the collisionless limit, the particles bounce back and forth between the moving wall and a stationary wall at the other end of the box, separated by distance L. In the highly collisional limit where collisions are considered to be the primary mechanism of changing a particle's pitch angle, the particles travel one mean free path ($\lambda_{mfp} = \alpha v^4$) into the box before being reflected. In this case there is no stationary wall on the other side of the domain. A graphic of both cases is shown in Fig. 1.

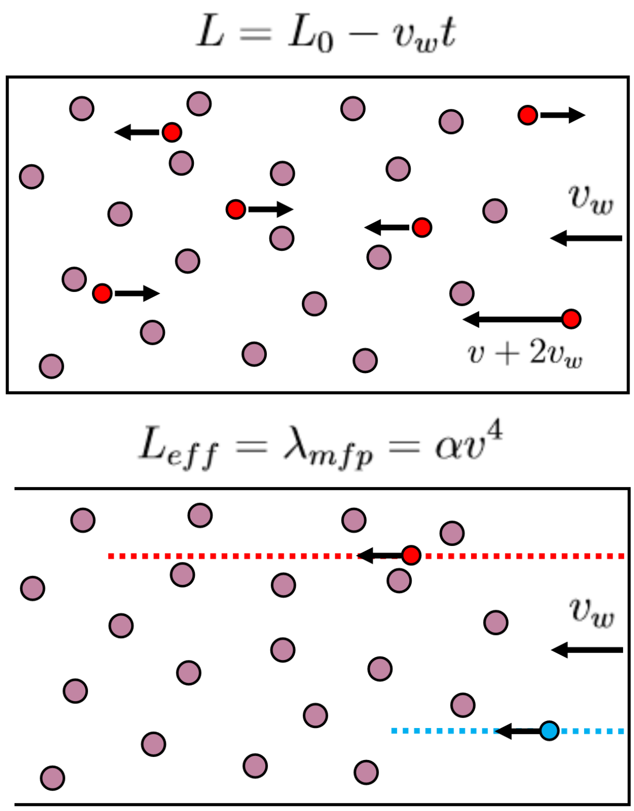

Figure 1: Particles interacting with a moving wall in a box of fixed length L (top) and being reflected after traveling one mean free path (bottom).

Since v_w is small compared to particle speed, a justified way to examine this problem is in the context of adiabatic invariants. The adiabatic invariant for the collisionless case is the well known $J_1 = vL$, and for the highly collisional limit it turns out that the invariant is $J_2 = \Delta L + \frac{1}{4}\alpha v^4$, where ΔL is the distance compressed. These two invariants are starkly different physically since particles which conserve J_1 will experience a greater acceleration rate if they begin with a greater initial velocity, while the opposite is true for J_2. This relationship occurs because in the highly collisional limit, fast particles spend a greater time away from the wall since the mean free path scales with v^4. The fact that slow particles achieve a greater increase in energy over some compression time could result in some unique non-thermal distributions, and a similar behavior is expected in systems of higher dimension.

Distribution Function

To gain some insight on the effect of true pitch angle scattering in a system of higher dimensions, we can consider some distribution $p(v,t')$ of the time a particle spends between bounces. The evolution of the particle distribution function $f(v,t)$ during some small compression is then described by the advection-diffusion equation

$$\bar{t}\frac{\partial f(v,t)}{\partial t} + 2v_w\frac{\partial f(v,t)}{\partial v} = \frac{\delta t^2}{2}\frac{\partial^2 f(v,t)}{\partial t^2}, \tag{1}$$

where $\bar{t}$ and δt are the mean and deviation of the bounce time distribution $p(v,t')$. For both collisional limits, the solution mean follows the respective adiabatic invariant with variance $\sigma^2 = \delta t^2 v_w(v - v_0)/2$. Fig. 2 shows the evolution of a uniform energy distribution during compression as it conserves the adiabatic invariant J_2 with no time variance in the bounce distribution ($\delta t = 0$). Since the particles with greater energy experience a lesser degree of acceleration, the distribution is compressed in phase space, resulting in a narrower peaked distribution.

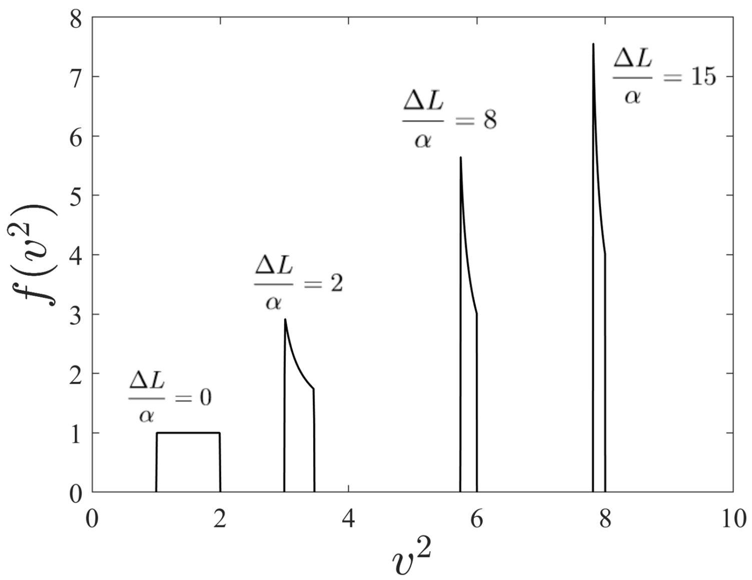

Figure 2: Evolution of a step energy distribution from compression while conserving the adiabatic invariant $J_2 = \Delta L + \frac{1}{4}\alpha v^4$.

Numerical Results

A particle simulation was written to investigate the effect of Coulomb pitch angle scattering in the 3D case. A total of 10^5 particles were initialized at the surface of the moving wall with velocities directed away from the wall. The particles were divided into ten different initial velocities in order to see how the velocity increase scaled with the initial state. There is no stationary wall on the other side of the simulation domain, so the pitch angle scattering is the only mechanism responsible for turning particles back towards the moving wall. Collisions were simulated by randomly changing a particle's pitch angle every time it traveled one mean free path, $\lambda_{mfp} = \alpha v^4$. Fig. 3 shows the average change in velocity for each subgroup as a function of their initial velocity after a compression consisting of around 2000 collision times.

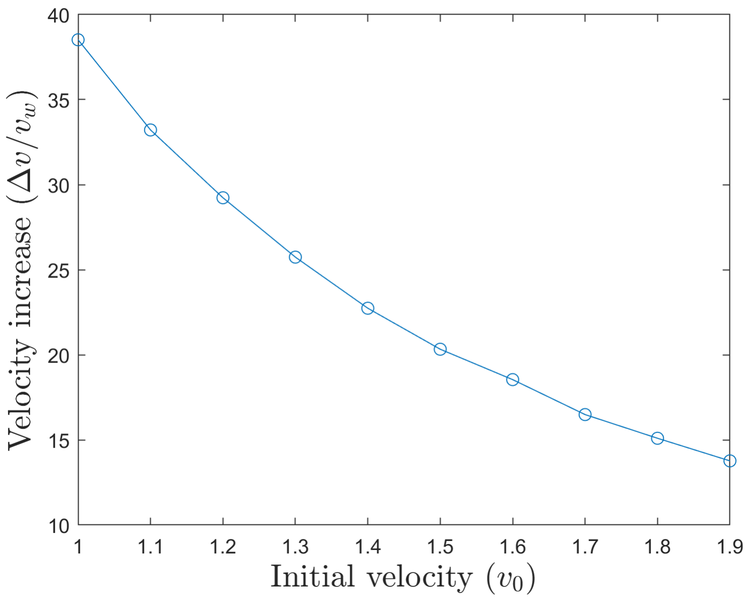

Figure 3: Velocity increase following compression as a function of initial velocity. In this simulation, $\alpha = 0.1$, $v_w = 10^{-9} v_0$, and $T = 2000\tau_{v_0}$.

Clearly the result shows an inverse relationship between the two variables as shown earlier in the 1D case and exhibits a scaling near $\Delta v \sim v_0^{-3/2}$. This inverse scaling is the key to obtaining the non-thermal peaked distributions shown in Fig. 2 from phase space compression. The particular scaling with $v_0^{-3/2}$ rather than with the collision frequency (v_0^{-3}) is likely due to the scaling of the fraction of particles which do not interact with the wall over the compression timescale. Future work will focus on better understanding the scaling behavior of the velocity and energy increase due to compression and identifying any adiabatic invariants of the 3D system.

Acknowledgements

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