Collective particle dynamics under interaction with Localized Wavepackets

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Introduction

Wave-particle interactions are ubiquitous phenomena in fusion and astrophysical plasmas and they modify the particle velocity distributions, resulting in momentum and energy transport. In realistic applications, the waves have the form of spatially localized beams (wavepackets). We investigate the implications of these interactions in a large area of the parameter space consisted of the wavepacket amplitude and width, both numerically and analytically.

Interaction with electrostatic Gaussian beams

Particle dynamics under interaction with a electrostatic wavepacket with amplitude A, spatial width σ, wavenumber k, and phase velocity v_p is governed by the following Hamiltonian:

$$H=\frac{p^2}{2}+Ae^{-z^2/2\sigma^2}\sin\left[k\left(z-v_p t\right)\right].$$

The velocity distribution of an ensemble of particles is strongly modified due to the localized interaction with the wavepacket as shown in Fig. 1.

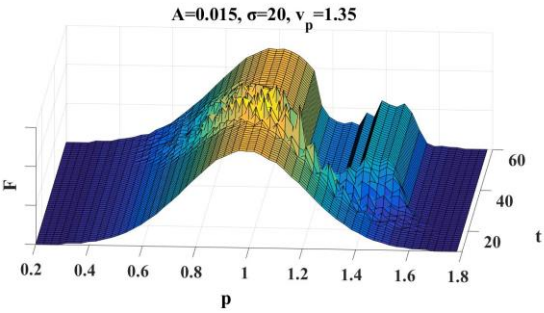

Figure 1: Evolution of the particles' velocity distribution function during the interaction with the wavepacket.

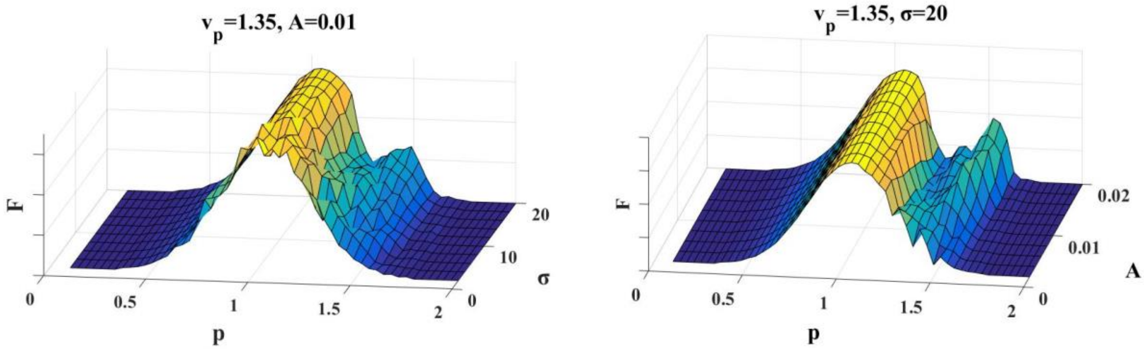

Figure 2: (Left) Dependence of the velocity distribution function's output profile on the beam width (σ). (Right) Dependence of the velocity distribution function's output profile on the beam amplitude (A).

The wave-particle interaction has a resonant character for particles with velocities close to the phase velocity of the wavepacket (similar to Landau damping). After exiting the wavepacket the distribution function has an invariant output form [Fig. 1]. The output form strongly depends on the wavepacket characteristics, namely its amplitude A and width σ, as shown in Fig. 2.

Small interaction strength (Canonical Perturbation Theory)

For a small interaction strength, the wavepacket is considered as a perturbation to the free particle motion. In this section we consider a uniform initial velocity distribution, which can be further utilized to obtain the effect of the interaction on any initial velocity distribution, by utilizing appropriate weighting factors. Canonical transformations lead to the calculation of approximate invariants of the motion, that provide analytical expressions for the extreme and mean values of momentum variation (Δp) [1], [2]. Comparisons have been made between the numerical and the aforementioned analytical expressions resulting to a significant agreement [Fig. 3].

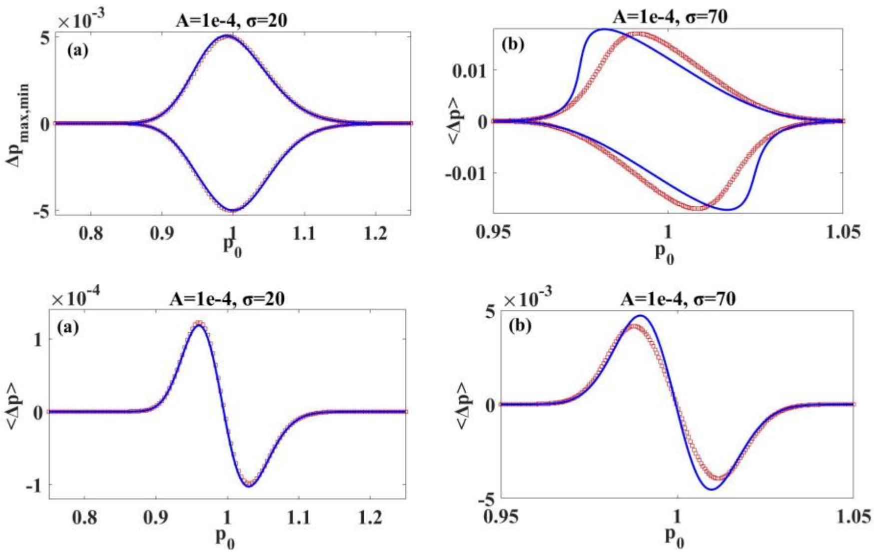

Figure 3: Analytical results (blue line) compared with numerical ones (red squares) for the extreme and mean values of the momentum variation as functions of the initial momentum p_0. For sufficiently small perturbation strength (left), namely sufficiently small or spatially short beams, there is an excellent agreement. For higher values of the effective perturbation strength (right), the agreement between the mean values is better than the agreement between the extreme values.

Infinitely long wavepackets (O'Neil's plane-wave approximation [3])

In the case of an infinitely long wavepacket (plane wave) the Hamiltonian is transformed to the well-known Hamiltonian of a pendulum:

$$H = \frac{p^2}{2} + A sin\left[k\left(z - v_p t\right)\right] \;\Rightarrow\; H' = \frac{P^2}{2} - A cos Q$$

The separatrix width can be used for an estimation of the corresponding momentum variation. Based on the comparison with analytical expressions of the momentum variation for small interaction strengths, the two limiting cases are comparable for values (A, σ) according to the formula $= \sqrt{2/\pi A}$, as shown in Fig. 4. In such cases, analytical solutions of the pendulum Hamiltonian, in terms of Jacobi elliptic functions, can be used.

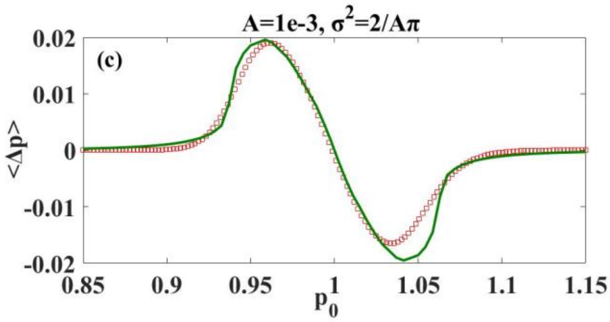

Figure 4: Numerical results (red squares) compared with the solutions of the pendulum Hamiltonian (green line) for parameter values (A,σ) satisfying the condition $\sigma = \sqrt{2/\pi A}$.

General case (fully nonlinear interaction)

In the general case of arbitrary values of the beam parameters (A,σ) the interaction is fully nonlinear and no analytical results are available.

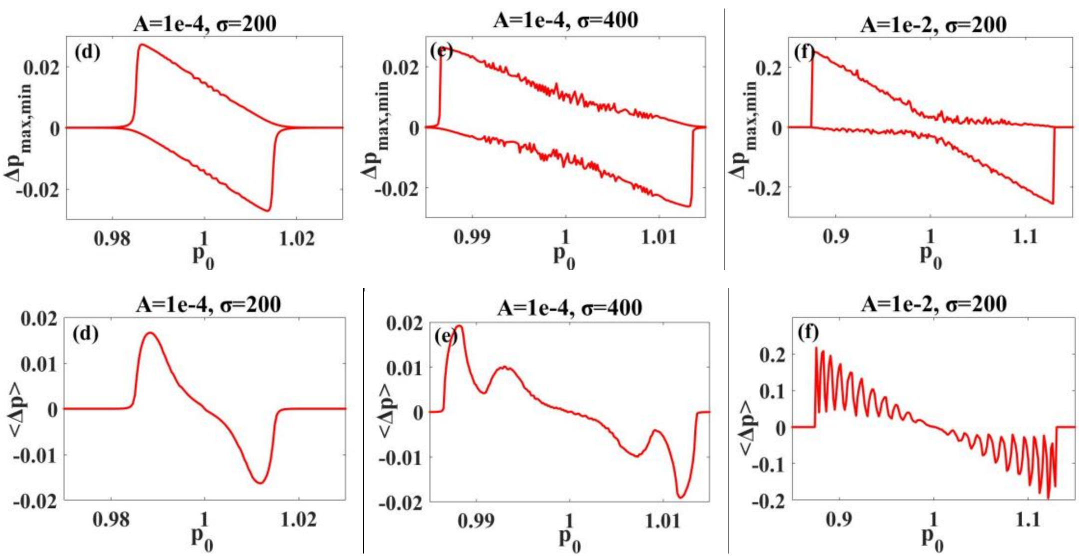

Figure 5: Numerical results of the extreme and mean values of momentum variation for an increasing spatial width given a certain wavepacket amplitude (first and second columns) and for an increasing wavepacket amplitude, given a certain spatial width (first and third columns).

Larger values of the wavepacket amplitude A and/or the spatial width σ, result in sharp transitions for the extreme values of the momentum variation and multiple local maxima for the mean momentum variation, with respect to the initial particle momentum p_0 [Fig. 5].

Parametric investigation and domains of validity of the analytical results

The aforementioned characteristic cases, are pinpointed in the parameter space (A,σ), as shown in Fig. 6. Analytical results are available in the limit of small interaction strength and infinitely long wavepackets. The rest of the parameter space can be investigated through numerical integration of the equations of motion.

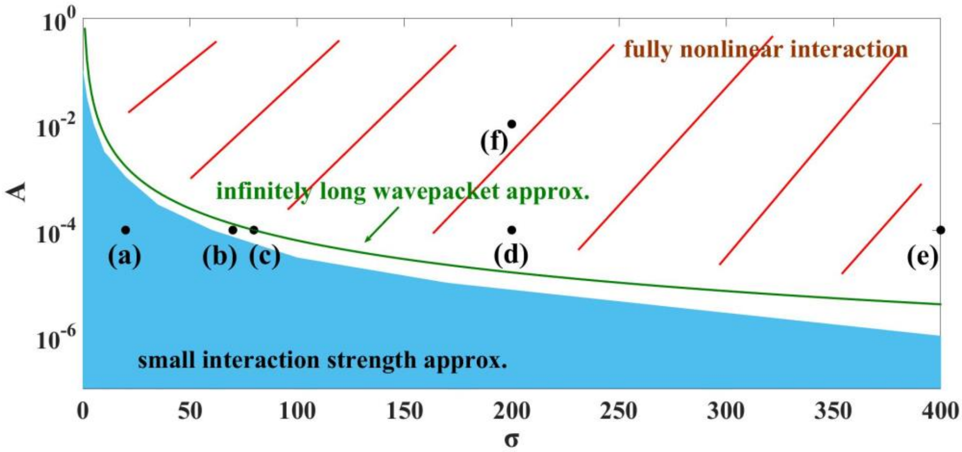

Figure 6: Systematic investigation of the parameter space (A, σ) and domains of validity of the analytical results.

Summary and Future Work

The interaction of particles with localized electrostatic wavepackets is systematically studied for the full range of wavepacket amplitude and spatial width values. Analytical results have been obtained for the limiting cases of small interaction strength and infinitely long wavepackets. The domains of validity of the analytical results have been identified by detailed comparisons with direct numerical integration of the equations of motion. We focus on uniform initial velocity distribution functions, since they serve as a basis to obtain any other distribution with the utilization of appropriate weight factors. The above findings provide intuition for the validity of the various approximations in particle interactions with localized wave modes in more general and realistic configurations, including the consideration of a uniform or nonuniform magnetic field, electromagnetic waves and relativistic particle dynamics.

[1] Y. Kominis, A.K. Ram, and K. Hizanidis, *Interaction of charged particles with localized electrostatic waves in a magnetized plasma*, Phys. Rev. E 85, 016404 (2012).

[2] Y. Kominis, K. Hizanidis, and A.K. Ram, *Transient dynamics of charged particles interacting with localized waves of continuous spectra*, Phys. Rev. Lett. 96, 025002 (2006).

[3] T. O'Neil, *Collisionless Damping of Nonlinear Plasma Oscillations*, Phys. Fluids 8, 2255 (1965).

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