Confinement time of ion kinetic energy in a controlled nuclear fusion system

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Abstract

The single most important scientific question/challenge in fusion energy generation research may well be ion kinetic energy confinement time in a fusion plasma. A theoretical model is presented for ion kinetic energy confinement time in a material where fusion reactions occur. After review of ion stopping in neutral materials, a formula is derived for the confinement (or decaying) time of ion kinetic energy in neutral materials. Under the supportable assumption that ion stopping cross section in a neutral material is comparable to that in a plasma, an estimate is obtained for the confinement time of ion kinetic energy in a D-T plasma – and found to be orders of magnitude lower than required in the Lawson criterion. The estimate is compared with indirect indications from experiments at TFTR and Wendelstein 7-X. An experimental program is proposed for studying and greatly improving ion kinetic energy confinement time in plasmas – as will be required to achieve successful commercial fusion-based energy generation.

1. Introduction

The single most important question/challenge in fusion research may be ion kinetic energy confinement time in a fusion plasma [1]. In plasma physics, the energy confinement time of a plasma is defined as the thermal energy content of the plasma divided by the power loss,

$$\tau_{E,plasma} \equiv \frac{W}{P_{loss}}, \tag{1}$$

where W is the thermal energy of the plasma, and P_{loss} is the power loss. Until this paper, there has not been a theory for $\tau_{E,\,plasma}$ calculation. The fusion research community relies on derived empirical scaling laws for energy confinement times in fusion reactor designs [2].

In this paper, we discuss the confinement time of ion kinetic energy in a material where fusion reactions occur. After reviewing ion stopping in neutral materials, we derive a formula for the confinement (or decaying) time of ion kinetic energy in neutral materials. Making the supportable assumption that ion stopping cross section in a neutral material is comparable to that in a plasma, we estimate the confinement time of ion kinetic energy in a D-T plasma and find it to be orders of magnitude lower than required in the Lawson criterion [3]. We discuss the implications of this estimate in D-T fusion reactors, and review some of the most

Table 1. Values of fitting parameter A_1

Element	A_1	Element	A_1
Hydrogen (H)	1.262	Beryllium (Be)	2.248
Helium (He)	1.229	Boron (B)	2.474
Lithium (Li)	1.411	Carbon (C)	2.631

advanced thermonuclear fusion experiments, especially TFTR [1] and Wendelstein 7-X [4], and outline the direction for future research.

2. Ion stopping

As an energetic ion traverses a material, it loses its kinetic energy on average via ion stopping, a process dominated by transferring ion kinetic energy to the electrons in the material. The ion stopping cross section is defined by [5]

$$\epsilon = \frac{1}{n}\left|\frac{dE}{dl}\right| = -\frac{1}{n}\frac{dE}{dl}, \tag{2}$$

where n is the atom number density of the material, and $\left|\frac{dE}{dl}\right|$ is the magnitude of average ion kinetic energy loss per unit length the ion traverses the material.

For a hydrogen ion with a low energy (1-10 keV/ N_A), the ion stopping cross section is given by [6]

$$\frac{\epsilon}{10^{-15}\ \mathrm{eV}\cdot\mathrm{cm}^2} = A_1\sqrt{\frac{E}{\mathrm{keV}\cdot N_A}}, \tag{3}$$

where N_A is the atomic mass number of the atom in the material, and A_1 is a fitting parameter. The values of the fitting parameter are listed in Table 1 for various elements ranging from hydrogen to carbon.

3. Confinement time

Neutral materials: Making use of the definition $E = \frac{1}{2}mv^2$, the approximation $dl = vdt,$ and the definition $dE = -\epsilon n dl$, it is readily shown that

$$\frac{dE}{dt} = -\frac{E}{\tau_E} \text{ and } \frac{1}{\tau_E} \equiv \left(10^{-15}\ \mathrm{eV}\cdot\mathrm{cm}^2\right)A_1\left(\frac{2}{\mathrm{keV}\cdot N_A\, m}\right)^{1/2} n. \tag{4}$$

Here, τ_E is the *confinement time of ion kinetic energy.* For a hydrogen ion, the product of the atom number density and the confinement time is a constant, i.e.,

$$\frac{1}{n\tau_E} = \left(10^{-15}\ \mathrm{eV}\cdot\mathrm{cm}^2\right)A_1\left(\frac{2}{\mathrm{keV}\cdot N_A\, m}\right)^{1/2}. \tag{5}$$

A_1 is approximately the same for proton, deuteron and triton in hydrogen isotopes. Table 2 lists $n\tau_E$ of proton, deuteron and triton in several isotopes of hydrogen.

Table 2. $n\tau_E$ of proton, deuteron, and triton in isotopes of hydrogen

Isotope	$n\tau_E$ (proton)	$n\tau_E$ (deuteron)	$n\tau_E$ (triton)
Hydrogen (H)	1.8×10^{16} s/ m^3	2.6×10^{16} s/ m^3	3.1×10^{16} s/ m^3
Deuterium (D)	1.8×10^{16} s/ m^3	2.6×10^{16} s/ m^3	3.1×10^{16} s/ m^3
Tritium (T)	1.8×10^{16} s/ m^3	2.6×10^{16} s/ m^3	3.1×10^{16} s/ m^3

D-T plasmas: There is an indication that ion stopping cross section in a neutral material is comparable to that in a plasma [7,8]. Under the assumption that ion stopping cross section in a neutral material is comparable to that in a plasma, the confinement time of ion kinetic energy in a D-T plasma may be estimated by averaging $n\tau_E$(ideuteron on tritium) and $n\tau_E$(tritium ion on deuterium), i.e.,

$$n\tau_E(\mathrm{D-T\ plasma}) = 3 \times 10^{16}\ \mathrm{s/\ m^3}, \tag{6}$$

which is a constant, independent of the target ion density or the ion kinetic energy in the range of 2 keV to 30 keV.

4. Discussion

Implication in D-T thermonuclear fusion: The estimated confinement time of ion kinetic energy in Eq. (6) is found to be orders of magnitude below the Lawson criterion. For D-T fusion, the minimum of the Lawson curve occurs at 25 keV [3],

$$\left(n\tau_{E,plasma}\right)_{Lawson} = 1.5 \times 10^{20}\ \mathrm{s/\ m^3} = 5000\ n\tau_E(\mathrm{D-T\ plasma}), \tag{7}$$

which implies that it would be extremely difficult to reach the conditions satisfying the Lawson criterion in a laboratory D-T fusion reactor such as a tokamak.

Indirect experimental evidences: Direct experimental data are hardly available on the confinement time of ion (deuteron or tritium ion) kinetic energy in high-temperature plasmas. This is because most high-temperature experiments employed external heating such as neutral beam (NB) heating, electron cyclotron resonance heating (ECRH), etc. As a matter of fact, most reported measurements of confinement times were actually “sustaining time” because of use of external heating. However, there were indirect evidences of very short confinement times (order of 0.1 second or less). Although the measured very short confinement times may not be the actual confinement times of ion kinetic energy, it is worthwhile reviewing them. In particular, we review the experimental data from some of the most advanced thermonuclear fusion experiments, namely TFTR and Wendelstein 7-X.

In a D-T fusion experiment at TFTR, neutral beam heating and fusion powers were measured [1]. The fusion power dropped off on the order of 0.05 s once the neutral beam power was turned off at 2.85 s. The plasma density was $n = 0.5 \times 10^{20}\ \mathrm{m}^{-3}$, and the ion temperature was 3 to 4 keV. Based on this interpretation of the data, it is estimated that $n\tau_{E,plasma} = 0.05\ \mathrm{s} \times 0.5 \times 10^{20}\ \mathrm{m}^{-3} = 2.5 \times 10^{18}\ \mathrm{s/\ m^3}$.

In a recent experiment at Wendelstein 7-X, ECRH heating power, line plasma density, diamagnetic energy, and plasma current were measured [4]. As the ECRH power was turned off at 6 s, all of the measured plasma quantities dropped off on the order of 0.3 s or less. The plasma density was $n = 1 \times 10^{19}\mathrm{m}^{-3}$, and the ion temperature was ~ 5 keV at the center. Based on this interpretation of the data, it is estimated that $n\tau_{E,plasma} = 0.3\ \mathrm{s} \times 1 \times 10^{19}\ \mathrm{m}^{-3} = 3 \times 10^{18}\ \mathrm{s/\ m^3}$.

Proposed experimental program: Investigating ion beam in very dense laser-induced plasmas from solids is proposed as a promising method for further studying and potentially finding ways to greatly improve the confinement time of ion kinetic energy.

5. Conclusion

A theoretical model was presented for ion kinetic energy confinement time in a material where fusion reactions occur. An estimate was obtained for the confinement time of ion kinetic energy in a D-T plasma - and found to be orders of magnitude lower than required in the Lawson criterion. The estimate was compared qualitatively with indirect data from experiments at TFTR and Wendelstein 7-X. It is imperative to further study and greatly improve the confinement time of ion kinetic energy in fusion energy research.

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