Optimization of Parameterized Transport Models in COTSIM

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Introduction

COTSIM© (Control-Oriented Transport SIMulator) is a nonlinear, one-dimensional (1D) transport code based on MATLAB© and SIMULINK©, which makes it control-design friendly. It has a modular configuration, so the user can modify the complexity of the physics models in a functional manner depending on his/her particular needs. This also enables a trade-off between speed (when simpler models are used) and accuracy (when more complex models are utilized). Therefore, COTSIM© can execute off-line fast simulations, which makes it suitable for effective iterative control design. This includes the capabilities of testing control algorithms in closed-loop simulations and carrying out scenario planning by model-based optimization. Moreover, COTSIM© is capable of providing real-time and faster-than-real-time predictions, which makes it suitable for real-time control applications such as feedback control, state estimation, state forecasting, and real-time optimization.

In this work, COTSIM© is wrapped by an external optimizer in order to tailor parameterized transport models to device-specific experimental scenarios. As shown in Fig. 1, the optimizer adjusts the family of transport parameters α in order to minimize a cost function J subject to constraints. This cost function J is defined as a measure of the mismatch between the experimental plasma state x^{exp} and the COTSIM-predicted state $\bar{x}$ based on the associated experimental input u^{exp}. The optimization problem is solved by sequential quadratic programming (SQP), which is predicated on determining a local minimizer of the original nonlinear program by iteratively solving a sequence of approximated quadratic programs. The approach presented in this work is general and, in principle, can be employed to tune any current, heat, and particle-transport models. It is illustrated by using DIII-D experimental data to optimize transport models for the electron thermal-diffusivity, χ_e, such as the Chang-Hinton model [1, 2], the Bohm/gyro-Bohm model [3], and the Coppi-Tang [4, 5] model.

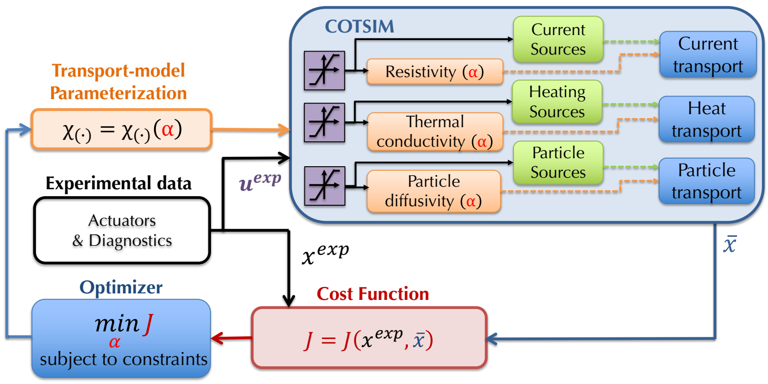

Figure 1: *Diagram of the optimization scheme for transport-model tuning using COTSIM©.*

Analytical Transport-Models: Neoclassical, Bohm/gyro-Bohm, and Coppi-Tang Models

In general, the model for χ_e implemented within COTSIM© is given by $\chi_e = \chi_e^{neo} + \chi_e^{ano}$, where χ_e^{neo} is the neoclassical (NC) contribution (neo), and χ_e^{ano} is the anomalous contribution (ano).

For NC ion transport, a model similar to that proposed in [2] is employed, $\chi_i^{neo} = \chi_i^{neo,banana} + \chi_i^{neo,PF}$. The banana-regime contribution, $\chi_i^{neo,banana}$, is given by

$$\chi_i^{neo,banana} = \sqrt{\varepsilon}\frac{\rho_\theta^2}{\tau_i}\frac{k_2^\star(\varepsilon)}{1+a_2\sqrt{\nu_i^*}+b_2\nu_i^*}. \quad (1)$$

where $a_2 = 1.03$ and $b_2 = 0.31$ are model parameters, $\varepsilon \triangleq a/R_0$ is the inverse aspect ratio (a and R_0 are the minor and major radiuses, respectively), $\rho_\theta \triangleq \frac{m_i v_{th}}{q_e B_\theta}$ is the poloidal gyroradius (where $v_{th} \triangleq \sqrt{\frac{2T_i}{m_i}}$ is the ion thermal velocity, T_i and m_i are the ion temperature and mass, respectively, B_θ is the poloidal magnetic field, and q_e is the electron charge), and $\tau_i = \frac{3}{4}\frac{\sqrt{m_i T_i^3}}{\sqrt{\pi} n_i Z^4 \log\Lambda}$ is the ion-ion average collision time [1]. In the expression for τ_i, n_i is the ion density, and $\log\Lambda = \log\frac{\lambda_D}{b_{\pi/2}}$ is the Coulomb logarithm, where λ_D is the Debye length and $b_{\pi/2}$ is the 90-degree impact parameter. In addition, the function $k_2^\star$ in (1) is given by $k_2^\star = \left(0.66+1.88\sqrt{\varepsilon}-1.54\varepsilon\right)\langle B_0^2/B^2\rangle$, where $\langle B_0^2/B^2\rangle = \left(1+\frac{3}{2}\varepsilon^2\right)$ (assuming circular flux surfaces and small Shafranov shift $R_0' << \varepsilon$ [2]). Also, the normalized ion-ion collision frequency is given by $\nu_i^* = \frac{1}{\tau_i}\frac{\sqrt{2}aB_T}{v_{th}\varepsilon^{3/2}B_\theta}$, where B_T is the toroidal magnetic field [1]. The Pfirsch-Schluter (PF) contribution, $\chi_i^{neo,PF}$, is given by

$$\chi_i^{neo,PF} = \varepsilon^2\frac{\rho_\theta^2}{\tau_i}\frac{\frac{c_2}{b_2}\nu_i^*}{1+c_2\nu_i^*\varepsilon^{3/2}}F^{PF}, \quad (2)$$

where $c_2 = 0.36$, $F^{PF} = \frac{1}{2\sqrt{\varepsilon}}\left(\langle B_0^2/B^2\rangle - \langle B^2/B_0^2\rangle^{-1}\right)$ is the PF factor, and $\langle B^2/B_0^2\rangle^{-1} \approx \sqrt{1-\varepsilon^2}$. In this work, the approximation $\chi_e^{neo} \approx C^{neo}\chi_i^{neo}$ is used, where C^{neo} is a tunable constant (it must be kept in mind that in most cases, and in particular in the tuning example shown in this paper, $\chi_e^{neo} << \chi_e^{ano}$).

The mixed Bohm/gyro-Bohm model [3] is given by

$$\chi_e^{GB} = \alpha_0\frac{T_e}{B_T}q^{\alpha_1}\left(a\frac{\nabla p_e}{p_e}\right)^{\alpha_2} + \alpha_3\frac{T_e}{B_T}\rho^*\left(a\frac{\nabla T_e}{T_e}\right)^{\alpha_4}, \quad (3)$$

where α_i ($i = 0,\ldots,4$) are tunable constants, q is the safety factor, T_e and p_e are the electron temperature and pressure, respectively, and $\rho^* \triangleq \frac{\rho_L}{a} = \frac{m_e v_{th}}{a q_e B_T}$ is the normalized gyroradius.

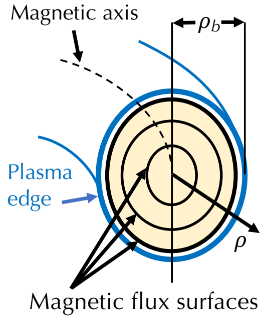

Figure 2: *1D magnetic-flux surface configuration in tokamaks.*

The Coppi-Tang model [4, 5] is given by

$$\chi_e^{CT} = \left[a_{121}\left(\frac{P_{tot}}{n_e^0}\right)^{0.6}(R_0B_Tq_{95})^{-0.8}a^{-0.2} + a_{122}\frac{a}{n_e^0}(R_0B_T)^{0.3}Z_{eff}^{0.2}\left(1+\frac{1}{4}\alpha_n\right)R_0^{-2.2}q_{95}^{-1.6}\right]F, \quad (4)$$

where a_{121}, a_{122} and α_n are tunable parameters, P_{tot} is the total heating power, n_e^0 is the central electron density, q_{95} is the value of q at 95% flux surface, Z_{eff} is the effective charge of the plasma ions, and $F \triangleq 8\pi^2\frac{P}{P_{tot}}\frac{n_e^0}{n_e}\frac{2R_0(\pi\rho_b^2B_T)^2}{\frac{\partial V}{\partial\hat{\rho}}|\nabla\Phi|^2}\hat{\rho}\, e^{\frac{2}{3}\alpha_q\hat{\rho}^2}$ is a geometric factor, where P is the power injected within the magnetic-flux surface whose mean-effective minor radius is $\hat{\rho} \triangleq \rho/\rho_b$ (see Fig. 2), ρ_b is the value of ρ at the last closed magnetic-flux surface, n_e is the electron density, V is the plasma volume, Φ is the toroidal flux, and α_q is a tunable constant.

In this work, χ_e^{ano} is taken either as $\chi_e^{ano} = \chi_e^{GB}$ or as $\chi_e^{ano} = \chi_e^{CT}$, depending on the simulation case (see the Section entitled "Tuning Results for a DIII-D, High-q_{min} Discharge").

Transport-parameter Tuning by Means of Nonlinear Optimization

The vector of tunable parameters in (1)-(4) is given by $\alpha \triangleq [C^{neo}, \alpha_1, ..., \alpha_5, a_{121}, a_{122}, \alpha_n, \alpha_q]$, whereas $T_e^{exp} \in \mathbb{R}^N$ contains experimental values for T_e at N spatial locations during a particular shot, i.e. T_e depends on time. The tunable parameters in α are determined by solving the following nonlinear-optimization problem,

$$\min_{\alpha} J = \int_{t_0}^{t_f} \left[(\bar{T}_e(\alpha, u^{exp}) - T_e^{exp})^T \, Q \, (\bar{T}_e(\alpha, u^{exp}) - T_e^{exp}) \right] dt, \tag{5}$$

$$\text{subject to } C^{neo} > 0, \alpha_n > 0, \quad \alpha_1, ..., \alpha_5, a_{121}, a_{122} \geq 0, \quad \alpha_q \geq 2.5, \tag{6}$$

where t_0 and t_f are the initial and final simulation times, respectively, $\bar{T}_e \in \mathbb{R}^N$ is the electron temperature during a given shot as calculated by COTSIM©, the bounds in (6) arise from the definitions of the tunable parameters [1]-[5], and $Q \in \mathbb{R}^{N \times N}$ is a design matrix that determines how the different spatial locations are weighed within the optimization problem. Therefore, the objective of this nonlinear optimization process is to find α such that COTSIM© yields an evolution for $\bar{T}_e$ that is as close to T_e^{exp} as possible. It can be noted that the experimental input u^{exp} must also be provided to COTSIM© for the calculation of $\bar{T}_e(\alpha, u^{exp})$.

Tuning Results for a DIII-D, High-q_{min} Discharge

Tuning of the analytical transport-models (1)-(4) in COTSIM© has been carried out using the optimization scheme in (5)-(6) for a DIII-D Advanced Tokamak (AT) scenario. The experimental inputs u^{exp} from the high-q_{min} shot 147634 are employed. In order to use transport models that are as physically relevant as possible, $C^{neo} = 1$, $\alpha_1 = 2$, $\alpha_2 = 1$, and $\alpha_n = 0.5$ are fixed and, therefore, not included in the optimization process. Two cases are presented using the same NC model (1)-(2), but different anomalous-transport models: (i) with the Bohm/gyro-Bohm model given by (3), $\chi_e^{ano} = \chi_e^{GB}$, and (ii) with the Coppi-Tang model given by (4), $\chi_e^{ano} = \chi_e^{CT}$.

The optimization is solved in a matter of a few minutes for each case, and yields $\alpha_0 = 4.63 \times 10^{-4}$, $\alpha_i \approx 0$ ($i \geq 3$, i.e. pure Bohm-like transport), $a_{121} \approx 1$, $a_{122} \approx 0.4$, and $\alpha_q \approx 2.5$, showing good agreement with [1]-[5]. The values of $\bar{T}_e$ are compared with T_e^{exp} in Fig. 3 (Bohm/gyro-Bohm) and Fig. 4 (Coppi-Tang) at t = 0.9, 2.0, 3.7, and 5.3 s, together with χ_e, χ_e^{neo} and χ_e^{ano} (shown only for $\hat{\rho} \in [0, 0.85]$, i.e. from the plasma center to the the top of the pedestal at $\hat{\rho} \approx 0.85$). Although a perfect match is not achieved, good qualitative agreement between $\bar{T}_e$ and T_e^{exp} is obtained in both cases. Also, $\chi_e^{neo} << \chi_e^{ano}$, correlating well with usual experimental observations (see e.g. [3]).

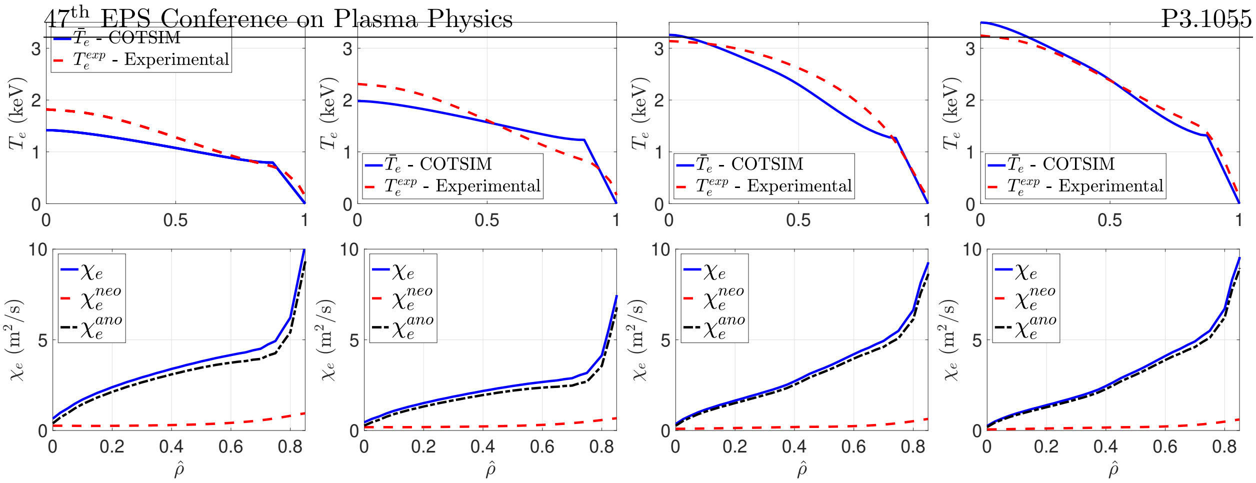

Figure 3: *Profiles for $\bar{T}_e$ and $\chi_e = \chi_e^{neo} + \chi_e^{GB}$ compared with T_e^{exp} (from left to right, $t = 0.9, 2.0, 3.7, 5.3$ s).*

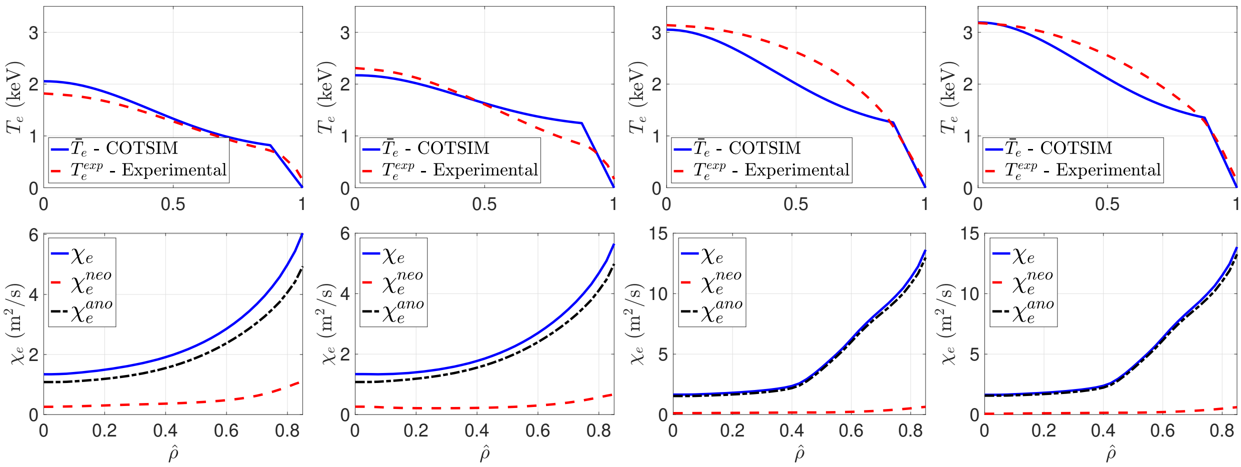

Figure 4: *Profiles for $\bar{T}_e$ and $\chi_e = \chi_e^{neo} + \chi_e^{CT}$ compared with T_e^{exp} (from left to right, $t = 0.9, 2.0, 3.7, 5.3$ s).*

Conclusion and Future Work

By using a nonlinear optimization approach, fast tuning of analytical transport models for control design has been demonstrated within the nonlinear 1D code COTSIM© and illustrated for a DIII-D scenario by using two χ_e models that are substantially different in their physics. This optimization-based tuning method can be a powerful tool for control modeling and scenario planning. Future work may include using other analytical models (for both the confinement and the pedestal regions), machines, and plasma scenarios, and simultaneously optimizing both transport and pedestal models.

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