Data Assimilation for Particle and Heat Transport of LHD Plasma

Y. Morishita[1], S. Murakami[1], M. Yokoyama[2,4], G. Ueno[3,4,5]

[1] *Department of Nuclear Engineering, Kyoto University, Kyoto, Japan*

[2] *National Institute for Fusion Science, Toki, Japan*

[3] *The Institute of Statistical Mathematics, Tachikawa, Japan*

[4] *The Graduate University for Advanced Studies, SOKENDAI, Toki and Tachikawa, Japan*

[5] *The Joint Support Center for Data Science Research, Tachikawa, Japan*

Abstract

Data assimilation techniques are applied to the integrated transport simulation in Large Helical Device (LHD). We employ the ensemble Kalman filter (EnKF) and the ensemble Kalman smoother (EnKS) as data assimilation methods. The time series data of measured temperature and density profiles are assimilated into the particle and heat transport simulation by TASK3D. We apply ASTI to the time series data sets of the 12 NBI heated plasmas in LHD. The obtained profiles and temporal variations of temperature and density by the EnKF and EnKS agree well with measured ones due to the optimization of the employed model parameters. These results indicate the effectiveness and validity of ASTI for accurate prediction and analysis of the behavior of fusion plasmas.

1. INTRODUCTION

Integrated simulations for fusion plasmas have various uncertainties in each of the employed simulation models, specifically turbulent transport models. Because of these model uncertainties, the simulation results are not totally confident to predict and analyze the fusion plasma behavior. To solve this problem and develop a numerical system that can predict and control the behavior of fusion plasmas with high accuracy, data assimilation techniques are introduced to the integrated transport simulation. We are developing a data assimilation system, ASTI [1,2], based on the integrated transport simulation code, TASK3D [3]. ASTI can also estimate the variables that cannot be observed and the model parameters that can explain observation time series data.

Data assimilation is one of the statistical estimation methods. It optimizes the state vector to enhance the prediction capability and reproducibility of the simulation model based on the observation data obtained sequentially. In ASTI, as data assimilation methods, we employ the ensemble Kalman filter (EnKF) [4] for the prediction and the ensemble Kalman smoother (EnKS) [4] for the estimation of the model parameters. In this study, we apply ASTI to the particle and heat transport simulation of LHD plasmas for the data sets (temperature and density profiles) of 12 shots and investigate the prediction and estimation performance.

2. DATA ASSIMILATION SYSTEM

ASTI is a data assimilation system for integrated transport simulation of toroidal fusion plasmas based on the EnKF and the EnKS. The EnKF procedure is a loop of prediction and filtering. In the prediction step, the predicted distribution of state vector $\mathbf{x}_t$ with the observation data, $\mathbf{y}_{1:s} \equiv \{\mathbf{y}_1, \cdots, \mathbf{y}_s\}$, is approximated by an ensemble consisting of $\mathbf{x}^{(n)}_{t|t-1} = f_t(\mathbf{x}^{(n)}_{t-1|t-1}, \mathbf{v}^{(n)}_t)$ for the index of the ensemble members, $n = 1,\cdots, N$. In this study, TASK3D is employed as the temporal evolution model f_t. In the filtering step, each ensemble member of the predicted distribution is optimized based on Bayes' theorem using the observation data at time t (assimilated into the simulation) [4]. The optimization by EnKF is based on only past observation data (prior to the time of filtering). To estimate the state vector reasonably, not only spatially but also temporally, we employ the EnKS, which corrects the filtered estimates by the EnKF using future data (posterior to the time of filtering) [2,4].

In TASK3D, the particle and heat transport equations are solved in the radial direction for each electron and ion species [3]. For the turbulent transport models, we employ the constant model for particle diffusion, the gyro-Bohm model for electron thermal diffusion, and the gyro-Bohm+grad$T$ model for ion thermal diffusion, based on the previous study for NBI plasmas in LHD [3,5].

In this study, we consider the uncertainties of plasma density, temperature, transport coefficients, NBI heating model, and particle source model. The state variables are listed in Table 1. The standard deviations of initial ensemble and system noise are set to be proportional to the ensemble mean, and their rates are listed in Table 1. The standard deviation of observation noise is assumed to be proportional to the difference between the prediction and the observation data, and estimated before every filtering step. We employ the rate of 0.8 [1].

3. ASSIMILATION RESULTS

We apply ASTI (EnKF and EnKS) to 12 time series data sets of NBI heated light hydrogen plasma in LHD and investigate the capability of the prediction and the estimation. In this

Table 1. State variables and their rates of the standard deviation of initial distribution and system noise.

	State variable (target of optimization)	Initial	System noise
n	Density	10%	10%
T_e	Electron temperature	10%	10%
T_i	Ion temperature	10%	10%
d	Particle turbulent diffusivity	20%	20%
v	Convection velocity	1.0 [m/s]	1.0 [m/s]
c_e	Electron thermal turbulent diffusivity	20%	20%
c_i	Ion thermal turbulent diffusivity	20%	20%
ξ_{180keV}	Heat deposition (180 keV)	10%	5%
ξ_{40keV}	Heat deposition (40 keV)	10%	5%
ξ_c	Critical velocity	10%	5%
ξ_{sd}	Slow down time	10%	5%
n_n	Neutral density at plasma edge	10%	10%
T_n	Neutral temperature at plasma edge	10%	10%

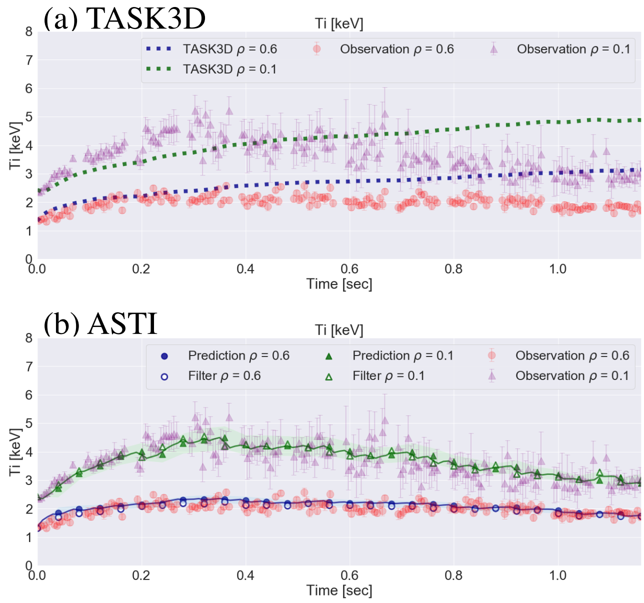

Figure 1. Prediction results of the time evolution of T_i by TASK3D (a) and by ASTI (b) for shot number: 119802.

study, all assimilations are performed with 500 ensemble members for the assimilation cycle τ_{DA}=40 ms.

Figure 1(b) shows the prediction and the filtered estimates of ion temperature by ASTI (EnKF) at the normalized minor radius ρ=0.1 and 0.6 for a time series data. For comparison, the simulation results by TASK3D (without data assimilation) are shown in Fig. 1(a). It can be seen that ASTI's predictions agree well with observation data, while TASK3D's predictions have large errors. Figure 2 shows the comparisons between the observed value and the prediction by TASK3D and by ASTI. The points in Fig. 2 are the values at ρ=0.1 and 0.6 for all 262 timings in the 12 shots. Table 2 lists the root mean square error (RMSE) and root mean square percentage error (RMSPE) between the observed value and the prediction. It is confirmed that the prediction errors are significantly reduced by using data assimilation. The RMSPEs of ion temperature in the prediction by ASTI are larger than those of electron temperature and density. This is because the temporal variation and observation error of ion temperature are larger than those of electron temperature and density in the 12 data sets.

The smoothed estimates of model parameters have been obtained by the EnKS, which can reproduce the observation time series data. Table 3 lists the RMSE and RMSPE between the smoothed estimates and the simulation results using the smoothed estimates of model

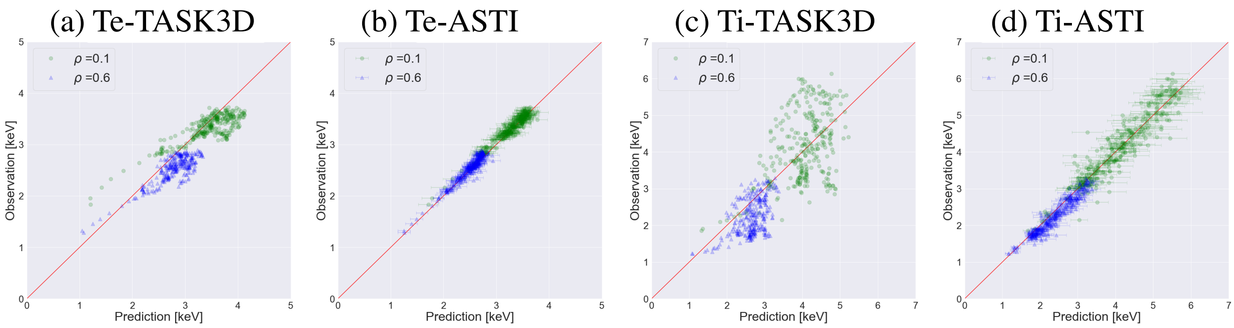

Figure 2. Comparisons between the observed values and the predicted values by TASK3D: (a) and (c), and by ASTI : (b) and (d), at ρ=0.1 and 0.6 for all the 12 shots.

Table 2. RMSE and RMSPE between the observed values and the predicted values at ρ=0.1.

	RMSE		RMSPE	
	TASK3D	ASTI	TASK3D	ASTI
T_e [keV]	0.310	0.0869	9.99%	2.86%
T_i [keV]	0.860	0.310	20.85%	7.97%
n [10^{18}m^{-3}]	3.22	0.534	26.49%	4.56%

Table 3. RMSE and RMSPE between the smoothed estimates and the TASK3D simulation using the smoothed estimates of the model parameters at ρ=0.1.

	RMSE	RMSPE
T_e [keV]	0.123	3.17%
T_i [keV]	0.323	5.29%
n [10^{18}m^{-3}]	1.09	11.32%

parameters. It can be confirmed that the simulation can reproduce the experimental time series data with low error, and these results indicate the validity of the estimation by ASTI (EnKS).

4. SUMMARY

We have developed the data assimilation system, ASTI, for the integrated transport simulation of fusion plasmas. We have applied ASTI to the particle and heat transport simulation for 12 experimental time series data sets of NBI heated plasmas in LHD. The predicted radial profiles by ASTI (EnKF) have agreed well with the observed profiles for all the data sets, and it has been confirmed that the TASK3D simulation using the smoothed estimates of the model parameters reproduces the experimental time seres data with high accuracy. These results indicate that the effectiveness and validity of ASTI (data assimilation approach) for accurate prediction and analysis of particle and heat transport in fusion plasmas.

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